QUADRUPLE-BEND ACHROMATIC LOW EMITTANCE LATTICE STUDIES[1]

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Abstract

A quadruple-bend-achromatic (QBA) cell, defined as a super cell made of two double-bend (DB) cells with different outer and inner dipole bend angles, is found to provide a factor of two in lowering the beam emittance of electron synchrotron light sources. The ratio of bending angles of the inner dipoles to that of the outer dipoles is numerically found to be about 1.5 to 1.6 for an optimal low beam emittance in the isomagnetic condition. The QBA lattice provides an advantage over the double-bend achromat or the double-bend non-achromat in performance by providing some zero dispersion straight-sections and a small natural beam emittance. A lattice with 12 QBA cells with a preliminary dynamic aperture study serves as an example.

INTRODUCTION

In the last ten years, many low emittance electron storage rings were constructed to provide high brilliance x-ray photon sources[2]. Double-bend (DB) cells were often used to accommodate many straight sections for insertion devices (IDs), such as the wigglers and undulators. Two operation modes for the DB lattice are either the achromatic condition to achieve the minimum emittance double-bend achromat (MEDBA) or the nonachromatic condition aiming for the theoretical minimum emittance (TME). Theoretically, the natural emittance attained in the nonachromatic mode is three times smaller than that attained in the DBA mode.

The horizontal (natural) emittance of an electron beam in an electron storage ring is determined by the equilibrium between quantum fluctuations and radiation damping. The minimum emittances for the DBA and the TME lattices are

$$\varepsilon_{\mathrm{MEDBA}} = \frac{C_q \gamma^2}{4\sqrt{15} J_x} \theta^3 \qquad (1)$$

$$\varepsilon_{\mathrm{TME}} = \frac{1}{3} \varepsilon_{\mathrm{MEDBA}} = \frac{C_q \gamma^2}{12\sqrt{15} J_x} \theta^3 \qquad (2)$$

where $C_q = 55\hbar/(32\sqrt{3}mc) = 3.83\times10^{-13} m$, $J_x \sim 1$ is the horizontal damping-partition number, and θ is the bending angle of dipoles in one bending section. Relaxing the achromatic condition, the emittance can generally be reduced by a factor of 3.

The bottom plot of Fig. 1 in reference 1 shows the emittance of existing storage rings scaled to 3 GeV beam energy versus the number of pairs of dipole magnets. The top plot of Fig. 1 in reference 1 shows the ratio of the attained emittance to its corresponding theoretical minimum for these storage rings. The diamond symbols in Fig. 1 in reference 1 are associated with lattices at the achromatic condition, while the square symbols are results of nonachromatic lattices. We note that the emittances attained in the lattice design are a factor of 2–4 larger than their theoretical minima of Eq. (2), as shown in the top plot of Fig. 1 in reference 1. A simple rule to attain small emittance is to increase the number of cells, so that the bending angle θ is reduced, but the number of cells and the circumference of the accelerator will increase. The question is how to attain a minimum emittance with a fixed circumference and fixed number of cells in order to accommodate an ever-increasing number of IDs.

This article studies the option of the quadruple-bend achromat (QBA) lattice, which is defined as a supercell composed of two double-bend cells with unequal bending strengths for the outer and inner dipoles. We will compare the emittances attained in the QBA design and the DB(A) design. There were two published QBA lattice designs[2,3]. This work differs from the previous studies in (1)retaining the dispersive straight sections so that the optical matching is easier and (2) providing an optimal length ratio between the inner and outer dipoles. There are also multibend achromat lattices proposed to attain low emittances, as shown in Fig. 1 (labeled as *n*BA). These lattices lack the available straight sections for the ever-increasing need for many insertion devices. Our concept can also be considered as a possible upgrade for some existing facilities that presently use the DB(A) structure. This upgrade is most beneficial for low to medium energy storage rings that can benefit emittance damping from high field wigglers and wavelength shifters.

THE EFFECT OF DIPOLE LENGTH RATIO

Although the theoretical necessary matching condition is L_2/L_1=1.44(based on the small angle approximation[4]), the actual optimal matching condition may vary. The left plot in Fig. 2 shows the emittance versus L_2/L_1 for lattices with 12 QBA cells and L1+L2=2.0 m (straight sections: 9.6 and 6.0 m_), 16 QBA cells with L1+L2=4.8 m (straight sections: 10.0 and 5.77 m), 20 QBA cells with L1+L2=7.9 m (straight sections: 10.4 and 6.0 m), and 24 QBA cells with L1+L2 =4.8 m (straight sections: 7.8 and 5.6 m). The circumference of these accelerators are 518.4 m for the TPS, 780.3 m for for the NSLS-II, and 1104 m for the APS. The right plot of Fig.2 shows the emittance

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normalized by $\gamma^2\theta^3$ to 3 GeV and 16 QBA cells. A small spread in the emittance normalized by this scaling law indicates the effect of dipole length. It is harder to reach a small emittance with short, high field dipoles. We also note that there is a broad minimum emittance at L2 /L1~1.5–1.6. For a properly matched lattice, the *H*-function at the dispersive straight section, H_{DID}, increases with the ratio L2 /L1 parameter.

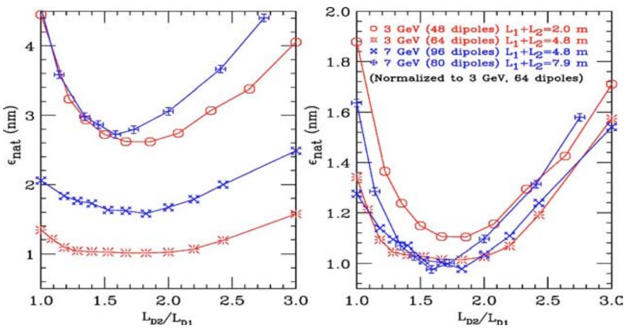

Figure 1: (Color online_) Left: The emittance of QBA lattices vs L_2/L_1 while keeping L_1+L_2=const. For the 3 GeV storage rings, we choose lattices with 12 and 16 super QBA cells. For the 7 GeV ring, we choose lattices with 20 and 24 super QBA cells for a possible retrofitting of the APS. Right: Emittances normalized to 3 GeV and 16 QBA cells. Note that it is harder to reach a small emittance for lattices with short high-field dipoles.

EMITTANCE OF THE DB(A) LATTICES

The horizontal natural electron beam emittance of a storage ring arises from the balance of quantum fluctuations and radiation damping. The resulting emittance of electron beams is[5]

$$\varepsilon_x = C_q\gamma^2 \frac{I_5}{I_2 - I_4} \qquad (3)$$

where γ is the relativistic energy factor. The radiation integrals are I_2= $\oint 1/\rho^2 ds$, I_4= $\oint D_x/\rho(1/\rho^2+2K)ds$ and I_5= $\oint H/|\rho|^3\, ds$, where *s* is the length coordinate along the accelerator, ρ is the bending radius of dipoles, D_x is the dispersion function, *K* is the focusing function, and the *H*-function is

$$H = \frac{1}{\beta_x}\{D_x^2 + (\alpha_x D_x + \beta_x D_x')^2\} \qquad (4)$$

Here, D'_x is the derivative of the dispersion function with respect to *s*, β_x is the betatron amplitude function, and α_x= $-1/2(d\beta_x/ds)$. The minimization of beam emittance requires minimizing the average *H*-function through the dipoles. Using a small angle approximation, we find $\langle H\rangle_{TME}=1/4H_{TME}$ for the TME lattice, where $H_{TME}=\rho\theta^3/(3\sqrt{15})$ is the *H*-function outside dipoles. For the MEDBA, we find $\langle H\rangle_{MEDBA}$= $3/4H_{TME}$. In this case, the *H*-function in the dispersion matching section is $3H_{TME}$. Since the *H*-function is invariant outside dipoles, the most basic three-bend achromat (TBA) design can never achieve its intended minimum emittance because the *H*-functions from both TME and MEDBA cannot be matched in the dispersion matching section[4]. In order to achieve dispersion matching, the outer dipole must be made shorter or have a weaker magnetic field.

Because the DB(A) lattice is easily tunable and yields a large number of straight sections, many storage rings use the DB(A) design. In the DB(A) design, there are achromatic and non achromatic modes of operation. Theoretically, the non achromatic mode can reach an emittance three times smaller than that at the achromatic condition. Thus, most synchrotron light sources operate in the non achromatic mode in order to reach a smaller emittance.

When the storage ring operates at the nonachromatic mode, the dispersion function is not zero in the ID straight sections, and thus the beam momentum spread can contribute to the horizontal beam width. We define the “one dimensional (1D) effective emittance” in the ID section as

$$\varepsilon_{x,1D} = \varepsilon_x + H_{1D}\sigma_\delta^2, \qquad (5)$$

where H_{ID} is the *H*-function at the ID locations, $\sigma_\delta^2 = C_q\gamma^2/(J_\varepsilon\rho)$ is the square of the rms momentum spread, and $J_E=2+I_4/I_2\sim2$ is the longitudinal damping-partition number. More commonly, in the light source community, the effective emittance is defined as

$$\varepsilon_{x,eff} = \sqrt{\varepsilon_x\varepsilon_{x,1D}}, \qquad (6)$$

Interestingly, the 1D effective emittances of the TME and MEDBA lattices are equal.

Since it is difficult to design a lattice reaching either the MEDBA or the TME condition, we ask a simple question: are we better off with the achromatic or the nonachromatic mode of operation? Let us assume we can design a lattice in the achromatic mode with $\varepsilon_{x,a}= f_a\varepsilon_{MEDBA}$ and another nonachromatic mode with $\varepsilon_{x,na}= f_{na}\varepsilon_{TME}$, where both f_a and f_{na} are typically about 2–4. The effective emittances of these two lattices become

$$\frac{\varepsilon_{x,1D,na}}{\varepsilon_{x,a}} \approx \frac{f_{n,a}}{3f_a}, \quad \frac{\varepsilon_{x,eff,na}}{\varepsilon_{x,a}} \approx \frac{\sqrt{f_{n,a}(f_{na}+2)}}{3f_a}, \qquad (7)$$

If both fa and fna are 2, the 1D and effective emittance of the nonachromatic lattice are about 0.66 and 0.47 of the achromatic one, respectively, i.e., the nonachromatic lattice can provide higher beam brilliance. However, the emittance reduction factor is not 1/3.

The effects of IDs on beam emittance of nonachromatic lattice and achromatic lattice and effects of IDs on momentum spread are discussed in reference 1. Illustrative example of effective emittance in different modes of operation is shown in figure2.

A QBA LATTICE EXAMPLE

To provide a concrete example, we study a QBA lattice with a circumference of 486 m and 12 QBA cells for the TPS design. We choose $L2/L1$=1.5 and $L1+L2$=2.5 m, i.e., $\theta1$ =6°, $\theta2$=9°, and B_0=1.048 T. The length of $L1+L2$ is chosen to lower the dipole radiation power loss, optimize emittance reduction from damping wigglers and wavelength shifters, and allow for low field, low power undulators in the dispersive straight sections. To maximize the number of insertion devices, we have achieved 6x10.91 and 18x5.31 m straight sections. The ratio of straight sections to the total circumference is 33%. The lattice design is thus very compact. There are ten families of quadrupoles with reflection symmetry in a superperiod. The betatron tunes are n*x* =26.28 and n*y*=12.25, and the natural chromaticities are −64 and −30, respectively. Figure 3 shows the optical functions. The emittance of this lattice is 3.0 nm rad

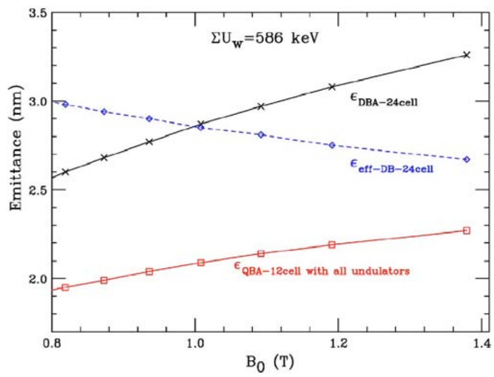

Figure 2: (Color online) The effective emittance, defined in Eq. (6) including the effect of all IDs with fh=0.62 with a natural emittance at 1.7 nm, plotted as a function of the main dipole field for various lattice types for the Taiwan Photon Source. The natural emittance of the DBA lattice without insertion devices is 5.2 nm. The effective emittance of the QBA lattice is calculated, assuming that high field IDs are installed in achromatic straight sections and medium field IDs are installed in the dispersive straight sections. The natural emittance of the QBA lattice is 3.0 nm.

The IDs in the dispersive straight sections should be emittance neutral; i.e., they must have low field or low total power. Figure 2 shows an effective emittance for the

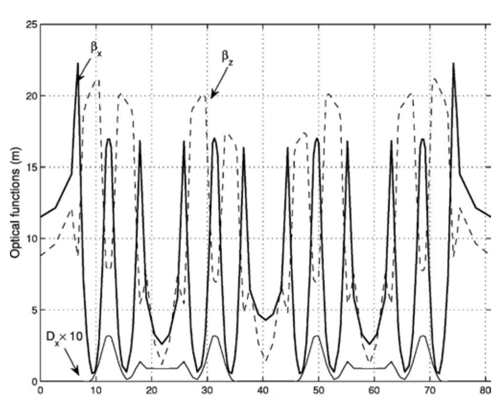

Figure 3: Optical function of a single QBA cell. The resulting emittance is about 3.0 nm, or about a factor of 3 larger than the minimum theoretical emittance.

QBA lattice. Note that if all or 60% of the IDs are installed in the achromatic straight sections of a QBA lattice, the effective emittance is smaller than that of the DB or the DBA lattices. Although the QBA emittance (3.0 nm) is larger than the emittance of the nonachromatic DB lattice (1.7 nm), the effective emittance of the QBA lattice in the achromatic straight section is smaller after including the insertion devices. This effect is shown as red lines in Fig. 2. Control of the dynamic aperture is a difficult problem in all low emittance lattices. We chose to use eight-families of sextupoles to correct the chromaticity and to improve the dynamic aperture. The lattice contains two-families of chromatic sextupoles located in the dispersion-function matching section and six families of harmonic (geometric) sextupoles elsewhere. Two families of harmonic sextupoles are located in zero-dispersion regions. These families can be used to correct geometric aberration without affecting the chromaticities. At zero chromaticity in both planes, the dynamic aperture is shown in Fig. 4 for on-momentum and ±3% off momentum particles.

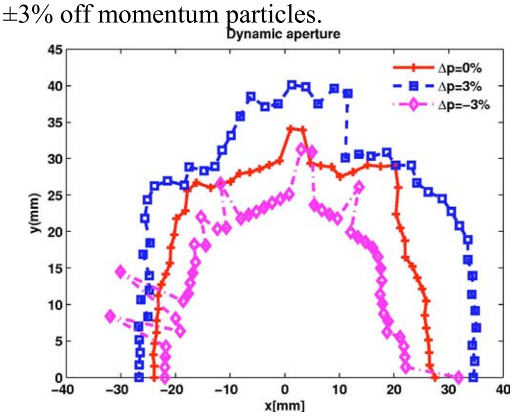

Figure 4: (Color online) Dynamic aperture in units of millimeters based on tracking calculations with no random errors. The rms beam sizes at this location are σ_x~0.19 mm and σ_z~0.016 mm.

DISCUSSION

The low emittance QBA-lattice concept has been studied for a few low emittance storage rings and compared with DBA and nonachromatic DB lattices. We find that the emittance of the QBA lattice is smaller than that of the corresponding DBA lattice by nearly a factor of 2, and yet the QBA lattice retains the same flexibility as that of the DB(A) lattice. The effective emittance of the QBA lattice is also smaller than that of the nonachromatic DB lattice, while keeping some zero-dispersion sections for high field damping wigglers.

REFERENCE

[1] M.H. Wang, S. Y. Lee, et. el., Review Scientific Instruments 78, 055109(2007).
[2] K. Tsumaki, R. Nagaoka, K. Yoshida, H. Tanaka, and M. Hara, Proceedings of PAC, 1989, p. 1358.
[3] D. Einfeld and M. Plesko, Nucl. Instrum. Methods Phys. Res. A **335**, 402 (1993).
[4] S.Y. Lee,Phys. Rev. E 54, 1940 (1996)
[5] M. Sand, Report No. SLAC-PUB-121, 1979 (unpublished)

1-4244-0917-9/07/$25.00