FINITE ELEMENT ANALYSIS OF METALLIC THIN WINDOW: AN ITERATIVE PROCESS

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Abstract

Thin windows are devices required by some particle accelerator physics experiments. These windows must be thin and light enough so they have a minimum effect on the beam. However, due to the boundary and loading conditions a window might observe; nonlinear structural behavior can occur from a number of different causes, such as geometric and material nonlinearities. If a structure experiences large plastic deformation, its changing geometric relationship can cause the structure to respond in a nonlinear manner. Material nonlinearities occur when the material's stress-strain relation depends on the load history as in plasticity models. The method of analysis for this study entails an FEA analysis, in which the stress and displacement are solved for a metallic membrane; these results are then compared to results obtained from an iterative process in relating the stress and strain with respect to the deformed geometry of the membrane. In addition, experimental tests will be carried out to determine the membrane displacement from a prescribed load. The study is conducted on 1145-O series Al.

Introduction

A stress analysis was carried out on the thin window by means of FEA and analytical modeling to determine if the window will sufficiently handle the differential pressure load caused by a vacuum boundary in the beam line (vacuum on one side of the window and atmospheric pressure on the other). Analysis confirmed the window will undergo plastic deformation due to the pressure differential across the window which will transition out of the elastic regime and into the plastic. Figure 1 shown below, illustrates a uniformly loaded circular membrane with clamped edges.

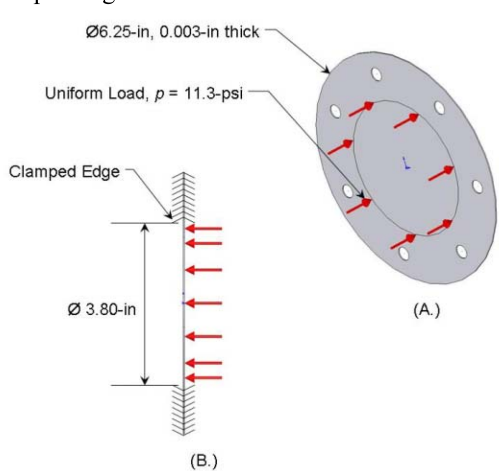

Figure 1: Aluminum 1145-O 0.003-in thick window.

FEA Analysis of Thin Window

An FEA model of the aluminum window was created using parabolic triangular shell elements. A nonlinear study was performed to take into account nonlinear material and geometric nonlinearities. The model is axisymmetric; therefore analysis is done for only a quarter of the model.

The window is fabricated out of aluminum 1145-O* and the geometric dimensions are the following: 3.25-in radius and .003-in thick. The circular plate is clamped at the edge and is subjected to a uniformly distributed load of 11.3-psi. The results of the finite element analysis are shown in the subsequent figures consisting of the von Mises Stress and Deflection plots for the thin window.

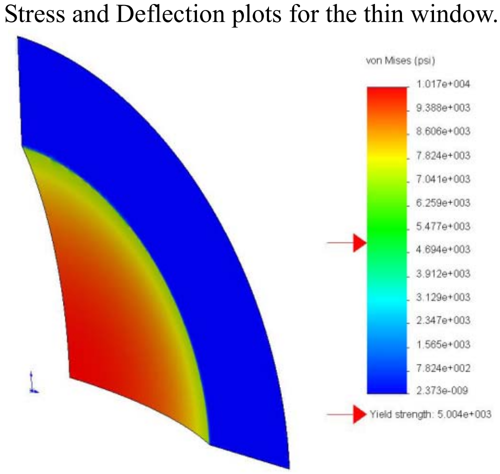

Figure 2: Nonlinear Nodal Membrane Stress Plot.

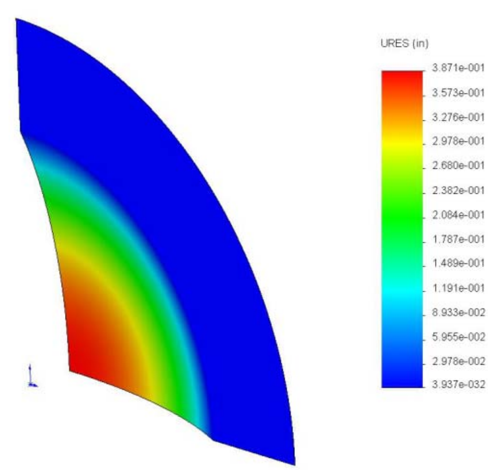

Figure 3: Nonlinear Displacement Plot.

* The author was not able to obtain a true stress-strain curve for 1145-O aluminum alloy; 1100-O aluminum was used instead.

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Figure 2 is the resultant von Mises stress plot for the thin window. The peak stress is 10,170-psi, which occurs in the crown of the window. The stress exceeds the yield strength of the material, and is reasonably near the ultimate strength of both 1100-O and 1145-O. The ultimate strength for 1100-O and 1145-O aluminum is 13.0-ksi and 10.9-ksi, respectively. Maximum deflection occurs in the window with a value of 0.3871-in, as shown in Figure 3.

The distinguishing limits which separate thick plate, thin plate, and membrane theory according to text by Ventsel and Krauthammer are stated as the following: Plates can be classified into three groups according to the ratio a/h, where ***a*** is the typical length dimension of a plate in a plane and ***h*** is the thickness of the plate. The groups are the following [1]:

- Thick plates having ratios $8 \le a/h \le 10$.
- Thin plates with ratios of $10 \le a/h \le 80$.
- Membranes have ratios of $80 \le a/h$.

In this application one would treat this as a membrane with a ratio of a/h = 1083. Also, known are the characteristics of membranes: First, membranes do not have any flexural rigidity, and hence cannot resist any bending loads. Second, membranes can only sustain lateral loads by axial tensile loads acting in the plate middle surface.

Figure 4 illustrates the nonlinear response of stress versus time at three specified nodes whose locations are shown in Figure 5.

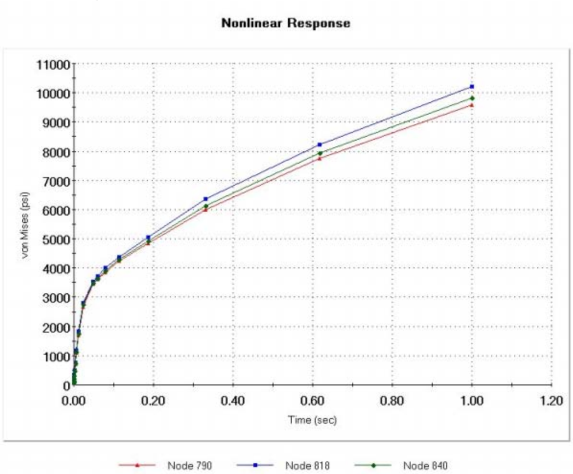

Figure 4: Time History Graph for Nonlinear Response; von Mises vs. Time.

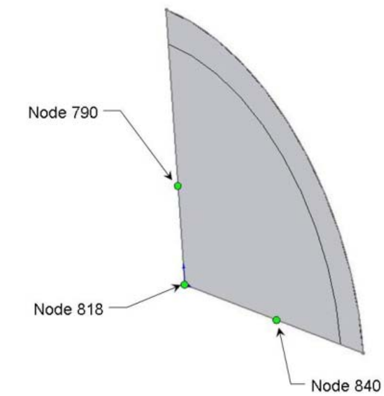

Figure 5: Location of specified nodes for Nonlinear Response.

Analytical Analysis

One could also calculate the stress by doing an iterative process in relating the stress and strain with respect to the deformed geometry of the membrane [2]. Figure 6, shown below illustrates the plastic deformation due to the uniform pressure load.

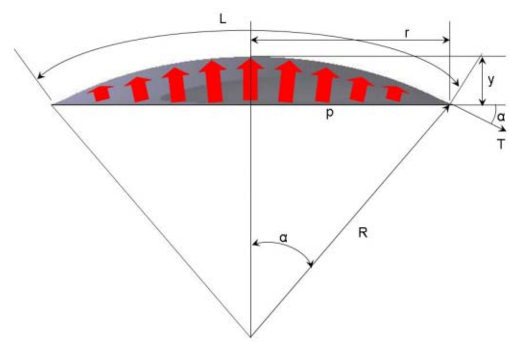

Figure 6: Aluminum window deformed diagram.

By doing a force balance in the vertical direction one would obtain the following [3]:

$$\sum F_y = 0; \quad 2 \cdot T \cdot \pi \cdot r \cdot \sin\alpha = \pi \cdot r^2 \cdot p \qquad (1)$$

- where T = σ·t
  - Force per unit length

The strain for a material is given by

$$\varepsilon = \frac{L - L_0}{L_0} \rightarrow L = L_0 \cdot (1 + \varepsilon) \qquad (2)$$

The arc length of the displaced window is

$$L = 2 \cdot R \cdot \alpha \qquad (3)$$

- where $\alpha = \sin^{-1}\left(\frac{r}{R}\right)$

The stress can be calculated by rewriting Eq. (1) in the form of Eq. (4)

$$\sigma = \frac{p \cdot r}{2 \cdot t \cdot \sin\alpha} \tag{4}$$

By choosing an initial condition for the strain and knowing that L_0 = 2r, Eq. (2) can be solved for L. Substitute the value found in Eq. (2) into Eq. (3) and solve for R. Once the value for R has been determined the angle can now be resolved. With Eq. (2) and (3) solved, the stress can be calculated by Eq. (4). Next, compare the initial strain to the true stain by using a full elastic-plastic stress-strain curve to relate the stress that was calculated in Eq. (4) to the corresponding true strain on the curve. If the stress is too low for the stress-strain relation, additional iterations will take place until the results converge.

The results between the FEA and analytical analysis correlate very well and show a percent difference of 9.14%. FEA produced a stress value of 10.17-ksi, while the analytical method yielded a stress value of 9.24-ksi with a strain initial condition value of 3.0% elongation. Results will converge as additional iterations take place.

Experimental Testing

Experimental testing was done on the 1145-0 foil to quantitatively measure the deflection results from the FEA analysis with actual experimental testing. Six windows were tested at room temperature (273K) with a nominal thickness of 0.003-in.

Figure 7: Deflection Experimental Setup.

Figure 7 depicts the experimental setup for measuring the deflection in the aluminum window. The load is created by a vacuum pump which creates a hermetic barrier between environment and the evacuated chamber. The vacuum pump has an inlet pressure of 10^{-3}-torr ($2e^{-5}$-psi), causing an estimated Δp of 11.3-psi.

Deflection measurements were taken once plastic deformation set in. As it is listed in Table 1, the numerical and experimental results correlate extremely well and show a percent difference of 3.3%.

Table 1: Deflection - FEA vs. Experimental Testing. Mean values are shown. The standard deviation in Mean Deflection = 0.011.

	Min. Window thickness (in.)	FEA Results	Experimental Results
		Deflection (in.)	Mean Deflection (in.)
Window	0.003	0.387	0.400

Summary

Both analytical and numerical results agree reasonably well in determining the membrane stress for circular thin window. From the analysis it is shown that the aluminum window will exceed the materials yield strength and enter the plastic regime. However, aluminum with the –O (annealed) temper has a large tensile ductility (lowest strength, highest ductility temper), so strains on the order of 25% are possible before failure at the ultimate load [2]. It is shown that with the current load conditions, the window observes strains on the order of 3.4%. The experimental results compares favorably with the numerical modeling using the finite element analysis in determining the deflection of the window.

Acknowledgments

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References

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