COMPACT LONG WAVELENGTH FREE-ELECTRON LASERS

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Abstract

The idea of using the Smith-Purcell effect to build a compact (table-top) long wavelength (0.1 -1 mm) free-electron laser is quite old. However, it is only recently that a complete theory for the operation of such devices has been proposed. The current state of the theoretical and experimental efforts to understand these devices will be summarized.

INTRODUCTION

Compact narrow-band far-infrared, or terahertz (THz), sources have potential applications in a large number of fields including biology, chemistry, and materials science[1, 2]. The current THz sources in existence either produce very short-pulsed broadband radiation, or require very large facilities. The exception to these are CO_2 pumped FIR lasers and backward-wave oscillators (BWOs). FIR lasers only have discreet lines, making them impractical for spectroscopy, and BWOs do not reach short enough wavelengths. The theory of operation of a free-electron laser (FEL) based on the Smith-Purcell effect has progressed significantly in recent years. This paper reviews the theoretical and experimental results to date.

OVERVIEW

When an electron passes over a periodic conducting surface, two types of radiation are emitted. The first is spontaneous Smith-Purcell (SP) radiation [3] whose wavelength λ is determined by the relation

$$\lambda = \frac{L}{|n|}\left(\frac{1}{\beta} - \cos\theta\right) \tag{1}$$

in which L is the grating period, n is the grating order, negative integers only, βc is the electron velocity, and θ is the angle from the direction of electron beam travel to the angle of emission (see figure 1). The second kind of radiation is a bound or evanescent wave. The wavelength of this radiation is always longer than wavelengths allowed in the SP band, and is determined by matching the phase velocity of the wave to the electron beam velocity [4, 5].

When the electron beam is bunched, the spontaneous radiation becomes coherently enhanced for wavelengths longer than the bunch length[6, 7]. This makes the radiation a useful diagnostic for the electron beam[8]. When the bunches are periodic, the coherent SP emission becomes superradiant and the spectrum is characterized by narrow, intense lines at harmonics of the bunching frequency[9]. SP radiation at wavelengths other than the harmonics is

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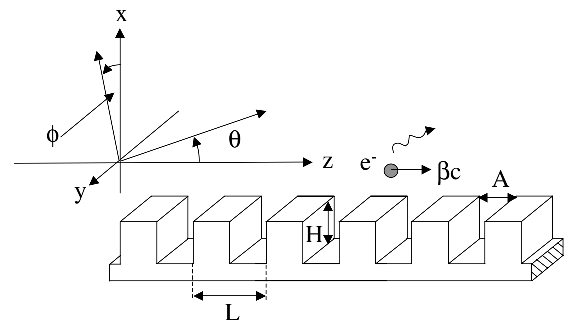

Figure 1: The lamellar grating geometry used in the two and three dimensional theories shown with an electron (e^-) and emitted photon (squiggly line).

suppressed. These predictions are borne out by numerical simulations [6, 10] and by experiments with prebunched beams from rf linacs[11].

It is not necessary for the electron beam to be prebunched to observe superradiant SP emission. When the electron-beam current exceeds a threshold value, called the start current, the electrons interact nonlinearly with the evanescent wave and are bunched at its frequency.

The direction of energy flow of the evanescent wave is determined by the group velocity, $v_g = \beta_g c = \frac{\partial \omega}{\partial k}$ which is found from the dispersion relation (shown in figure 2). For positive v_g the energy flows in the same direction as

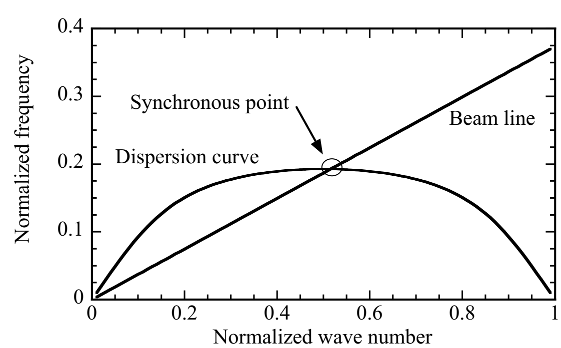

Figure 2: Dispersion relation for the grating used in the Dartmouth College experiments, grating parameters listed in Table 1. The intersection of the dispersion curve and beam line determines the operating frequency and wavenumber at which the phase velocity of the evanescent wave matches the velocity of the electron beam. The slope of the dispersion curve at the intersection determines the group velocity of the evanescent wave, $v_g = \frac{\partial \omega}{\partial k}$.

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instability, or as an amplifier, in the manner of a traveling-wave tube (TWT). To achieve oscillation, some sort of external feedback must be provided, although if the gain is high enough, sufficient feedback can be provided by parasitic reflections from the ends of the grating. For negative v_g, the energy flows opposite to the electron beam and the device operates on an absolute instability, or as an oscillator, similar to a BWO[4, 5, 12]. Above the start current, no external feedback is required. In addition to energy transferred to the evanescent wave, the electrons emit SP radiation at the wavelengths given by (1). Because the electrons are bunched periodically at the frequency of the evanescent wave, the SP emission is superradiant at the harmonics of the bunching frequency [9]. When the bunching is strong, SP emission is suppressed at other frequencies. In this configuration, the device is called a Smith-Purcell free-electron laser (SP-FEL).

It should be pointed out that so-called SP-FELs come in two configurations, sometimes called the Fabry-Perot and evanescent-wave versions of a ledatron [13]. In the Fabry-Perot configuration, also called an orotron, a mirror is placed above the grating to reflect the SP radiation back to the grating [14]. This provides feedback and permits the orotron to oscillate at the wavelength of the SP radiation given by (1) with $\cos\theta = 0$. The wavelength must, of course, also be an eigenfrequency of the resonator. Despite their low gain, orotrons have proved useful for spectroscopy in the millimeter-wave region [15]. In the other configuration, which is the subject of the present report, there is no mirror. The electrons are bunched by the evanescent wave that is excited by the electron beam and travels along the grating.

TWO DIMENSIONAL THEORY

The model used in our two dimensional calculations is shown in figure 1. We start with a grating having period L, groove depth H and groove width A. We start by expanding the fields above the grating into Floquet modes, and finding the fields within the grating grooves. In the region above the grating, the electron beam is considered to be a plasma dielectric moving only in the $\hat{z}$ direction which extends from the top of the grating teeth upwards. Matching the fields at the top of the grating teeth and taking the no-beam limit leads to the empty grating dispersion relation. The dispersion relation for the grating used in experiments at Dartmouth College [17, 18, 19] is shown in figure 2. Parameters for this grating are given in Table 1. The intersection of the dispersion relation and the beam line in the dispersion plane determine the operating frequency and wave number of the evanescent wave. This is called the synchronous point. Treating the electron beam as a perturbation of empty grating modes, we expand the dispersion relation about the synchronous point to first order, and find the frequency and wavenumber shifts due to the presence of the beam. Additionally, because the grating metal is not a perfect conductor, it will have some small resistive loss.

Table 1: Grating and operating parameters used in calculations. These are the parameters used in the Dartmouth experiments published in [17]

Grating period	173 μm
Groove width	62 μm
Groove depth	100 μm
Beam energy	35 keV
Beam thickness	25 μm
Current density	1.6 MA/m^2

These losses are also included as a frequency shift in the dispersion relation. The resulting dispersion relation is

$$(\delta\omega - \beta c\delta k)^2 \left[\delta\omega - \beta_g c\delta k + \frac{\omega_0}{2Q_c}(1+i)\right] = \frac{\omega_p^2 S}{\gamma^3 R_\omega} \quad (2)$$

in which $\delta\omega$ is the complex frequency shift, δk is the complex wavenumber shift, ω_0 is the operating frequency, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, ω_p is the plasma frequency, Q_c is the quality factor of the grating and S and R_ω are functions of the grating parameters [16].

The dispersion relation admits three roots. These are generally referred to as the structure wave, and the fast and slow space-charge waves. In the simple case where the waves are excited at a real frequency we take $\delta\omega = 0$. In the absence of losses, the gain, or the negative imaginary component of the slow space charge wave, is given by

$$\mu_\infty = Im\,(\delta k) = \frac{\sqrt{3}}{2}\left|\frac{\omega_p^2 S}{\gamma^3 R_\omega \beta^2 \beta_g c^3}\right|^3 \quad (3)$$

However, a more important quantity for the SP-FEL is the start current. For an electron-beam current above the start current, the evanescent wave grows enough to cause bunching of the electron beam. To find the start current we must first find the growth rate of the three waves. We assume both $\delta\omega$ and δk are complex, and numerically solve the dispersion relation in conjunction with three boundary conditions for the three waves. The first and second boundary conditions require that all density and velocity modulations in the electron beam vanish at the upstream end of the grating. Finally, we require that the amplitudes of the three waves sum to zero at the downstream end of the grating. This is equivalent to stating that there is neither an incident wave traveling upstream on the grating, nor reflections from the end of the grating. To include reflections, we instead require that the wave amplitudes sum to a non-zero value determined by upstream and downstream reflection coefficients, which must be determined from simulations. The system of equations is recast in terms of dimensionless parameters, δ_i. After finding numerical values for these parameters, δ_0 is solved for the growth rate. Including losses and reflections, the growth rate is

$$Im\left(\delta\omega\right) = \frac{2}{\sqrt{3}}\frac{\beta\beta_g c\mu_\infty}{\beta_g - \beta}\left[Im\left(\delta_0\right) - \frac{\sqrt{3}\nu_\infty}{2\mu_\infty}\right] \quad (4)$$

where ν_∞ is the empty-grating loss. Our predicted growth rate compares favorably with the growth rate found in 2D particle-in-cell (PIC) simulations [16]. A comparison of these results is shown in figure 3.

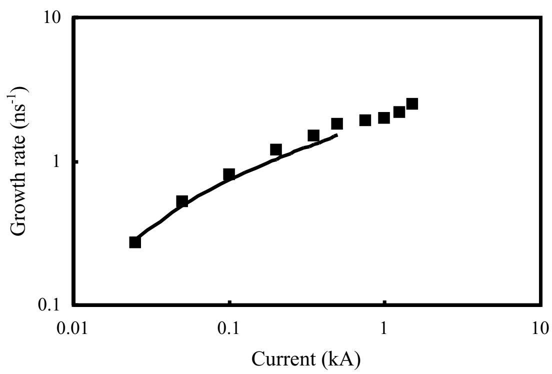

Figure 3: The predicted growth rate (solid line) agrees well with the growth rate observed in PIC simulations.

We calculate the start current as a function of electron accelerating voltage for the grating and operating conditions listed in Table 1. We find that in this regime neither losses nor reflections have a profound impact. Losses increase the start current by a small percentage, and reflections cause oscillations in start current. These effects can be seen in figure 4. It is expected that for operation at shorter wavelengths, losses will raise the start current of these devices substantially.

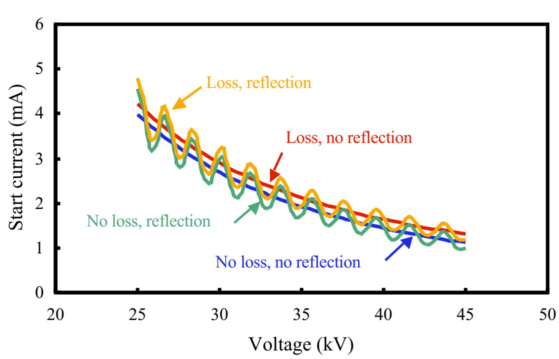

Figure 4: Start current with and without losses and reflections as a function of voltage for the parameters shown in Table 1. In this regime the losses and reflections do not have a strong impact on device operation, however, at shorter wavelengths it is expected that losses will raise the start current substantially.

THREE DIMENSIONAL THEORY

The three-dimensional model is much the same as the 2-D theory, except we restrict the electron beam to a width W. The approach is similar to that used for the 3-D Cerenkov FEL[20]. We expect from diffraction arguments that the mode width will be large compared with the evanescent scale height but small compared with the gain length. The modes are expected to peak at the center of the grating and electron beam, $y = 0$, and decay to zero as $y \rightarrow \pm\infty$.

In this treatment we consider only transverse-magnetic (TM) modes of the grating for two reasons. First, to lowest order in the electron-beam perturbation, the mode structure should be nearly the same as that of the empty grating. For the infinite-width grating considered here, the surface currents required to support the longitudinal magnetic fields of a transverse-electric (TE) mode cannot exist. Therefore the lowest-order mode structure will be approximately that of a TM mode. Secondly, the electron beam is introduced as a linear dielectric. To first order in the fields, the electron beam resonantly exchanges energy with only the TM mode's longitudinal electric field component. Energy exchange with TE modes is of higher order in the fields and can be ignored.

In the limit where the electron beam is infinitely wide, the 2-D dispersion relation is recovered. However, when the beam is narrow compared with the optical mode we find that the gain is no longer cubic in beam current, but instead has a five-halves dependence. The dispersion relation for this case is

$$\left(\delta\omega - \beta c\delta k\right)^2\left[\frac{D_\omega}{D_y}\left(\delta\omega - \beta_g c\delta k\right)\right]^{\frac{1}{2}} \quad (5)$$

$$= \frac{\beta^3 c^2 W}{ALD_y}\frac{\omega_p^2}{\gamma^2\omega_0^2}\tan\left(\frac{\omega_0}{c}H\right)\left[1 - \cos\left(k_0 A\right)\right]$$

where k_0 is the operating wavenumber and D_ω and D_y are the derivatives of the empty grating dispersion relation with respect to ω and k_y^2, the $\hat{y}$ wavenumber squared, respectively.

When the device operates as an amplifier, with β_g positive and $\delta\omega = 0$, the dispersion relation becomes,

$$\delta k^{\frac{5}{2}} = \Gamma e^{\frac{i\pi}{2}} \quad (6)$$

where

$$\Gamma = \frac{W}{AL}\frac{\omega_p^2}{\gamma^2\omega_0^2}\frac{\beta\tan\left(\frac{\omega}{c}H\right)}{\sqrt{c\beta_g\left|D_\omega D_y\right|}}\left[1 - \cos\left(k_0 A\right)\right]. \quad (7)$$

There are five roots to this dispersion relation, however, only three of them satisfy the boundary conditions requiring that the fields decay to zero as y goes to $\pm\infty$. Again we find three roots corresponding to a growing wave, or slow space charge wave, a decaying wave, or a fast space charge wave, and a third wave which neither grows nor decays. The gain for the slow space charge wave is

$$\mu_{3-D} = -Im\left(\delta k\right) = \Gamma^{\frac{2}{5}}\sin\left(\frac{\pi}{5}\right) \quad (8)$$

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This value for gain is approximately a factor of 7 smaller than the 2-D gain as shown in figure 5. Basic diffraction

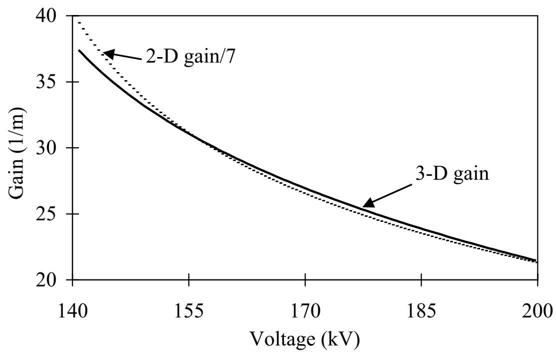

Figure 5: 3-D field gain coefficient for the amplifier mode is a factor of seven smaller than 2-D gain coefficient.

arguments support this result. For the two-dimensional case $gain \propto n_e^{1/3} \propto I^{1/3}$. In the three-dimensional result the average electron density over the area of the mode is approximately $n_e \approx \frac{I}{\Delta x \Delta y \beta c}$. The mode width $\Delta y \propto 1/\sqrt{gain}$, so $n_e \propto I\sqrt{gain}$. Combining this with the first, we find $gain \propto I^{2/5}$. This can be understood to be a manifestation of gain guiding in the SP-FEL.

COMPARISON WITH EXPERIMENT

Although BWOs are well known both theoretically and experimentally, only the group at Dartmouth College has operated a SP-FEL in the evanescent-wave configuration and reported observations of superradiant SP emission[17, 18, 19]. The experimental results they report are not in agreement with the theoretical predictions. In the experiments, superradiant SP radiation was observed on the first three Smith-Purcell orders. Theory predicts that the first order should be suppressed when the bunching is strong because it is not a harmonic of the bunching frequency. However, the Dartmouth experiments never reached the regime of strong bunching (the emission never achieved saturation), and the simple theory may not apply. Two further difficulties must also be addressed. First, despite the fact that strong emission at the frequency of the evanescent wave from the ends of the grating is predicted by theory and observed in numerical simulations (and the output of BWOs appears at precisely this frequency), radiation at wavelengths longer than first order-SP emission was never observed. Second, theoretical predictions of the start current are higher than the superradiant threshold observed in the Dartmouth experiments[12, 21]. Although the discrepancy is not large for the two-dimensional theory, diffraction effects in the three-dimensional theory worsen the comparison.

CONCLUSIONS

We summarize theoretical work describing the operation of an SP-FEL. The two-dimensional theory compares very well with PIC simulations. The inclusion of losses and reflections does not change the results substantially for the operating parameters considered. Diffraction effects included in the three-dimensional theory change the dependence of gain on current from one-third to five-halves. Experimental results to date are not in agreement with predictions.

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