STABILITY THRESHOLDS FOR TRANSVERSE DIPOLE MODES WITH NONLINEAR SPACE CHARGE, CHROMATICITY AND OCTUPOLES

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Abstract

Transverse stability due to combinations of chromaticity effect, nonlinear space charge and octupoles of different polarities plays an important role in the determination of the impedance budget for the FAIR synchrotrons. Different analytic approaches have been suggested, for which no direct comparison has been made so far. In order to clarify this issue we perform numerical investigations employing the particle tracking code PATRIC and compare results of simulation scans with predictions of a dispersion relation.

INTRODUCTION

Space charge effects play an important role in many existing and future high intensity ring accelerators, especially at injection energies or close to transition. The role of space charge is particularly important for the envisaged operation with high quality and high intensity beam in the FAIR synchrotrons [1]. Landau damping due to the finite momentum spread and chromaticity belongs to the damping mechanisms of transverse dipole modes, alternatively octupoles can be installed. For accurate predictions of stability thresholds the effect of the space charge nonlinearity—i.e., the amplitude-dependent incoherent tune shift due to non-homogenous transverse beam profile—can become important. One of the first attempts to describe the influence of nonlinear space charge was made in Ref. [2] with a dispersion relation based on a heuristic derivation. It was concluded that nonlinear space charge alone does not provide damping, but combined with an external nonlinearity (e.g., an octupole) it results in a significantly larger stability area than that due to the octupole alone. It was also shown that for one of the octupole polarities the stability enhancement is stronger than for the other polarity. This model was later applied to the LHC at injection in [3] and has been extended in [4], where tune shifts due to momentum spread and chromaticity have been included. The transverse stability for the combination of nonlinear space charge with octupoles has been also studied using a more elaborated analytic model in [5], where modifications of the stability boundary due to nonlinear space charge were confirmed.

However, there has been still some uncertainty about the role of nonlinear space charge. Doubts have been raised whether the nonlinearity in the space charge force should have an effect, or linear betatron oscillations can be assumed for a stability analysis. The predictions of the dispersion relations have not been directly compared with experiments, computer simulations, or with each other.

To resolve uncertainties we perform particle-in-cell simulations with the code `PATRIC` [6]. Combinations of nonlinear space charge with chromaticity and with an octupole are considered. In parallel, we solve the dispersion relation from Ref. [4] in an approximate manner and use it as a guide to understand the physical effects and to choose the most interesting cases for direct comparisons. In particular, we study the case of strong space charge and weak octupoles, as it resembles the situation in the FAIR synchrotrons.

PARTICLE-IN-CELL SIMULATIONS

The 3D particle-in-cell tracking code `PATRIC` [6] is used for simulation scans with self-consistent 2D space charge, coupling to transverse impedances, chromaticity and nonlinear lattice effects. For self-consistent space charge, two solvers of the Poisson equation are implemented, for rectangular and elliptic boundary conditions with arbitrary side ratio. The transverse impedance implementation allows to model arbitrary impedance spectra. Details of the implementation can be found in Ref. [6]. For coasting beams, the horizontal impedance kick per turn can be formulated as

$$\Delta x'(t) = \frac{Nq^2}{p_0}\left[-\,\overline{x}(t)\,\mathcal{I}m(Z^{\perp}) + \frac{\mathrm{d}\overline{x}}{\mathrm{d}t}(t)\frac{\mathcal{R}e(Z^{\perp})}{\mathcal{R}e(\Omega)}\right], \quad (1)$$

where $\overline{x}$ is the beam mass center in the horizontal plane, $Z^{\perp}(\Omega)$ is the transverse impedance which causes the coherent tune shift $\Delta Q_{\rm coh}$, Ω is the coherent frequency which we observe. Note that under the impedance $Z^{\perp}$ we understand only the interaction with the beam surrounding, which does not include e.g. the direct space charge force.

The aim of our simulation studies is to obtain the stability boundary in the complex $Z^{\perp}$ impedance plan. Hence, series of simulations have been performed varying both $\mathcal{R}e(Z^{\perp})$ and $\mathcal{I}m(Z^{\perp})$. As a result, a direct comparison of PIC simulations with a dispersion relation can be conducted. For the shift of the coherent frequency the effect of the image charges from the conducting wall must be taken into account.

The simulation model was chosen in a way to resemble the beam physics model underlying the dispersion relation: the beams were matched in rms size, a constant focusing model was used for the lattice. As the initial transverse distribution we use a waterbag distribution. For the longitudinal momentum, a Gaussian distribution is assumed. Coasting beam parameters similar to the ones foreseen in the SIS 18 have been assumed, with the factor 5 larger intensity to make growth times smaller for reasonable computing times. The other parameters (momentum spreads etc.) were then scaled to achieve the same normalized impedances $V+iU \propto Z^{\perp}$. External nonlinearity was modelled with the cubic component of the octupole magnetic field $B_x = -K_3 \frac{B\rho}{6} y^3$, etc. This cubic component of the magnetic field causes an incoherent tune shift, which grows in the absolute value with amplitude $\Delta Q_{\rm oct}(a) \propto -K_3 a^2$. Since space charge reduces the tune shift, there is a certain octupole polarity (in this case a negative K_3), which enhances the effect of nonlinear space-charge and is referred here as the advantageous polarity. The opposite case we denote as the disadvantageous polarity.

The incoherent tune shifts are different for each individual particle. Hence, there are characteristic tune spreads which we denote as $\delta Q_{\rm sc}$ for the space-charge tune spread, $\delta Q_{\rm oct}$ for that induced by the octupole nonlinearity and δQ_ξ for the tune spread due to chromaticity. To indicate the strength of the octupole nonlinearity in comparison with nonlinear space charge we use the parameter $\chi_{\rm oct} = \delta Q_{\rm oct}/\delta Q_{\rm sc}$, and for the chromaticity effect we use $\chi_\xi = \delta Q_\xi / \delta Q_{\rm sc}$.

DISPERSION RELATION

By following closely the approach given in Ref. [4] the dispersion relation for the coherent mode frequency Ω in the horizontal plane is constructed,

$$\int \frac{\Delta Q_{\rm coh} - \Delta Q_{\rm inc}}{\Omega/\omega_0 - (Q_{\rm ex} + \Delta Q_{\rm inc})} \left(-\frac{a^2}{2} \frac{{\rm d}\psi_a}{{\rm d}a} \right) \times \times b\, \psi_b(b)\, \psi_p(p)\, {\rm d}a\, {\rm d}b\, {\rm d}p = 1 \ , \qquad (2)$$

where ψ_a, ψ_b and ψ_p are the corresponding distribution functions normalized as $\int a \psi_a {\rm d}a = 1$, $\int b\, \psi_b {\rm d}b = 1$ and $\int \psi_p {\rm d}p = 1$; ω_0 is the revolution frequency. $Q_{\rm ex}$ includes the bare tune plus tune shifts due to external nonlinearities and chromaticity effects,

$$Q_{\rm ex}(a,b,p) = Q_0 + \Delta Q_{\rm oct}(a,b) + \Delta Q_\xi(p) \ . \qquad (3)$$

For the case with nonlinear space charge only, the dispersion relation Eq. (2) predicts no damping, even if the coherent tune overlaps the incoherent tune spectrum.

The dispersion relation Eq. (2) is solved numerically by direct three-dimensional integration. The amplitude-dependent incoherent tune due to space charge is estimated numerically for all individual amplitude combinations by averaging over particle trajectories, which is an approximation. The results are validated for distributions with analytic solutions. Arbitrary distributions ψ_a and ψ_b can be taken, but in this work we only consider the waterbag distribution. The change in the density distribution due to space charge is neglected. Including the external nonlinearity for $Q_{\rm ex}$ described above, the frequency shift of the classical anharmonic oscillator can be taken into account analytically, $\Delta Q_{\rm oct}(a) = -(\beta c)^2/(16 \omega_{\beta 0} \omega_0)\, K_3 a^2$.

Examples for the solutions of the dispersion relation Eq. (2) are presented in Fig. 1. These stability diagrams show the contour level for $\mathcal{I}m(\Omega) = 0$ in the normalized impedance plane. The stable areas are the regions enclosed by the curves and by the $U-$axis. In Figure 1 (top) the stability boundary for an octupole alone (dashed line) is presented. It is compared with the stability boundary for the combination (solid line) of the same octupole with nonlinear space charge for the waterbag distribution. An octupole of the advantageous polarity is considered, the relative strength of the nonlinearities is characterized by the parameter $\chi_{\rm oct} \approx 0.12$. Figure 1 (bottom) compares stability diagrams for the chromaticity effect only (dashed line) and for the combination (solid line) with nonlinear space charge for the waterbag distribution. The incoherent tune spreads correspond here to $\chi_\xi \approx 0.2$.

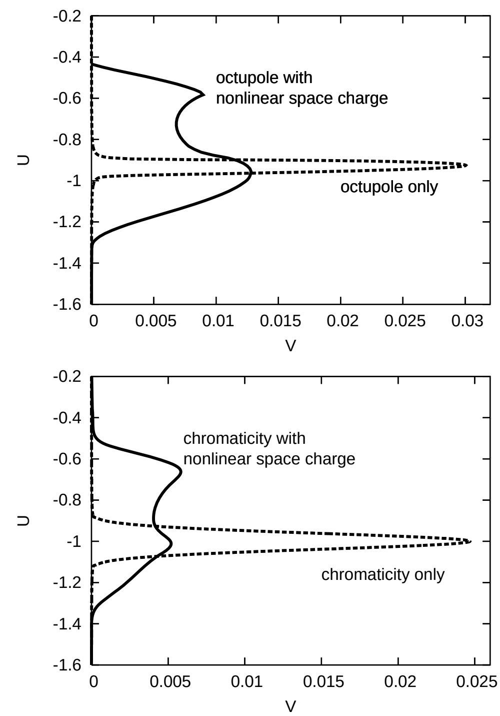

Figure 1: Stability diagrams obtained with the dispersion relation Eq. (2). Top: for an octupole only and for a combination of this octupole with nonlinear space charge (waterbag distribution); bottom: for the chromaticity effect only and for the combination with nonlinear space charge.

COMPARISON RESULTS

For the case with nonlinear space charge only, i.e. without external nonlinearities, extensive simulations scan with the code `PATRIC` for different $\mathcal{R}e(Z^\perp)$ and $\mathcal{I}m(Z^\perp)$ did not indicate stability for finite real impedances, which supports the prediction of the dispersion relation Eq. (2).

In order to compare the predictions of the dispersion relation Eq. (2) with PIC simulations for the interplay of an octupole with nonlinear space charge, we consider the same conditions as those assumed to obtain results in Fig. 1. Results of the comparison for the case with nonlinear space charge are presented in Fig. 2, top. The curve is the stability boundary from Fig. 1, the simulations scans are shown with squares (unstable) and crosses (stable), where each of these symbols is the outcome of a different simulation run. The octupole has the advantageous polarity here. `PATRIC` simulations confirm the enlargement of the stability along $\mathcal{I}m(Z^{\perp})$ (regarding the case with an octupole only) and the extent of the stability area in $\mathcal{R}e(Z^{\perp})$, as it is predicted by the dispersion relation Eq. (2).

For the case of an octupole with the disadvantageous polarity the dispersion relation Eq. (2) yields the stability boundary, which is presented in Fig. 2, bottom. Here, an octupole with the same modulus strength $|K_3|$, but with the opposite polarity with respect to Fig. 2 (top) is assumed. A comparison between the plots in Fig. 2 shows that the usage of an octupole with the disadvantageous polarity substantially reduces the stability area along the $\mathcal{R}e(Z^{\perp})$ axis.

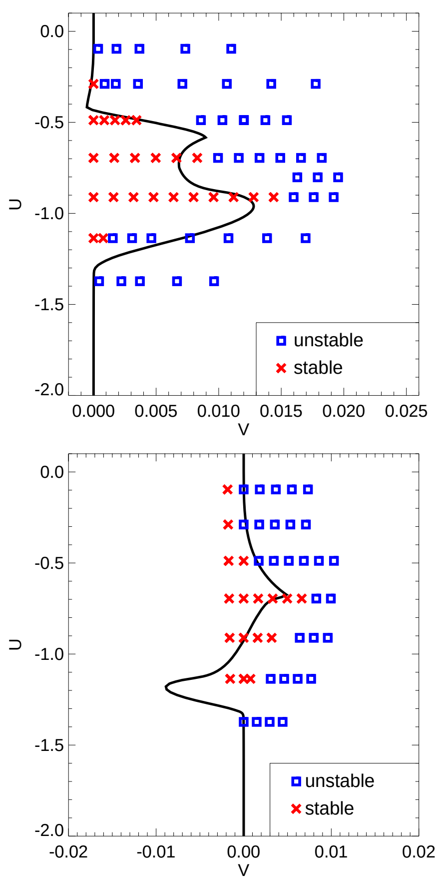

Figure 2: Combination of nonlinear space-charge and an octupole with the advantageous polarity (top), disadvantageous polarity (bottom). The symbols are the results of simulations, the lines are stability boundaries from the dispersion relation.

As it was already mentioned above (see Fig. 1, bottom), the dispersion relation Eq. (2) suggests that inclusion of nonlinearity due to space charge can significantly modify the stability due to chromaticity and momentum spread. Results of our simulation scans with the `PATRIC` code for this case are presented in Fig. 3. There is a good agreement in the extent of the stability area in $\mathcal{R}e(Z^{\perp})$, but for the U-axis our `PATRIC` simulations reveal only approximately half of the stability width as given by the dispersion relation.

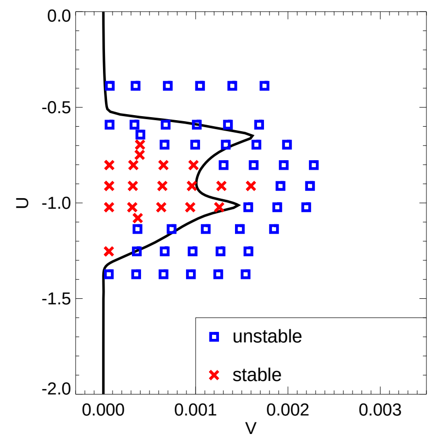

Figure 3: Combination of chromatic effects and nonlinear space charge.

CONCLUSIONS

The interplay of an octupole with nonlinear space charge can strongly modify the stability properties, especially for a weak octupole (which induces smaller tune spreads than that due to nonlinear space charge). The width of the stability area along the imaginary impedance axis is determined by the tune spread caused by nonlinear space charge, while the extent of the stability area in $\mathcal{R}e(Z^{\perp})$ depends on the octupole strength. For weak space charge the stability area is determined by the octupole. Our `PATRIC` simulation scans confirm these conclusions. Also, simulations confirm the prediction of the dispersion relation that using an octupole with the disadvantageous polarity substantially reduces the stability area by decreasing the stability thresholds in $\mathcal{R}e(Z^{\perp})$. Landau damping due to chromaticity, which often plays a central role in the transverse stability analysis, can also be strongly modified by the inclusion of nonlinear space charge.

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