TRANSVERSE SELF-CONSISTENT MODELING OF A 3D BUNCH IN SIS100 WITH MICROMAP *

C. Benedetti†, S. Rambaldi, G. Turchetti, Dip. Fisica & INFN, Via Irnerio 46, 40126, Bologna, Italy
G. Franchetti, I. Hofmann, GSI, Planckstr.1, 64291, Darmstadt, Germany

Abstract

We present the upgrade of the MICROMAP beam dynamics simulation library to include a $2\ \frac{1}{2}$ D space charge modeling of a 3D bunch using local slices in z. We discuss the parallelization technique, the performances, several tests and comparison with existing well-established analytical/numerical results in order to validate the code. An application to the SIS100 synchrotron of the FAIR project at GSI is outlined.

INTRODUCTION

The increasing interest in high-intensity beams for future accelerators requires to improve our understanding of the behaviour of a high intensity beam in a real accelerator where both lattice nonlinearities and space charge (SC) effects need to be taken into account in order to provide an accurate description of the system[1]. In particular beam loss prediction and control is a key issue for the new generation of high-intensity heavy ion rings, such as the SIS100 synchrotron of the FAIR project. Even if the basic mechanisms of beam loss related to the interplay between SC, external nonlinearities and the synchrotron motion can be studied by using simplified particle-core models[2], a complete (numerical) fully self consistent approach is required. This study can be undertaken by using 3D PIC codes. However, even with present computer capabilities, the use of a 3D Poisson solver[1] to compute the electrostatic force acting on a particle inside a long and thin bunch in a synchrotron is extremely demanding in term of memory and CPU time and, as far as transverse dynamics is concerned, its usage is even unnecessary. If we consider a bunch whose transversal dimension is much smaller compared to its length, then the longitudinal component of the electrostatic field can be neglected compared to the transverse one[3]. To compute the latter we assume that locally the charge distribution can be approximated by a coasting beam. We define few $(20\div30)$ longitudinal slices along the beam, then we collect the charge density on each slice and we solve the Poisson equation in two dimensions. The transverse electric field acting on a particle is finally obtained by interpolating the value of the field from two adjacent slices. This approach is commonly referred as the $2\frac{1}{2}$D approximation.

In this paper we present the new features of the MICROMAP library ($2\frac{1}{2}$D SC modules, parallelization). After discussing the performances and the accuracy of the code we consider a preliminary application to the SIS100.

UPGRADE OF MICROMAP

The standard MICROMAP library[4] is a *suite* of FORTRAN routines organized into two levels. The first one (single particle mode) performs the tracking of a particle in an arbitrary lattice. Linear transport is carried out by symplectic matrix composition. Nonlinear elements, described in the thin lens approximation, are modeled by symplectic maps. The second level (multiparticle 2D mode) is nested on the first one and performs the tracking of a multiparticle beam including the SC forces in the 2D approximation. The solver for space charge calculations is a spectral Poisson Solver with Dirichelet boundary conditions ($\phi = 0$) on a square box or on an arbitrary closed domain describing the beam pipe. Static tests on the accuracy of the 2D solver are discussed in[5].

New space charge modules

A new modality devoted to the SC calculations for a bunched beam in the $2\frac{1}{2}$D approximation has been added. One of the key issue in this approximation is the location of the longitudinal slices and, consequently, the positioning of the (transverse) 2D computational grids along the beam. In this respect, we have chosen to place most of the computational grids close to the bunch center, "tuning" their position according to the longitudinal charge distribution, in order to give a good description of the SC forces in the region where they are more intense. This solution is functional in the case of an AG lattice where we need to resolve the variation of the beam envelope (and consequently the one of the SC forces) through the lattice. This is the default option. The user can easily define and use its own positioning routine. Concerning the charge deposition scheme and the interpolation of the electric field from the grid nodes to particle position, new algorithms based on quadratic shape functions have been added besides the standard linear interpolation (linear shape function). The smoothness properties of the quadratic shape function, involving three grid points in the interpolation procedure, helps the reduction of the noise in the electrostatic field evaluation (see the section on the single particle tune tests for a comparison).

Together with the self consistent SC module also a frozen and a semi-frozen SC module for the $2\frac{1}{2}$D calculations

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† benedetti@bo.infn.it

[1] All the SC calculations are performed in the bunch reference frame where the field generated by the bunch can be considered purely electrostatic.

have been introduced. In the former the semiaxes of the charge distribution are determined directly from the local values of the beta functions and from the emittances. The electric field is computed analytically (KV, WB) or numerically (for Gaussian or another arbitrary distribution) by using Gaussian integration ensuring an adequate accuracy with few integration points. The latter is an extension of the first one where the semiaxes and, if necessary, the tilt angle of the beam section in each slice are determined from the second order moments of the actual particle distribution.

Parallelization

Meaningful self-consistent 3D PIC simulations require at least $N_{part} \sim 10^6$ numerical particles, in order to have low statistical errors[6], so the parallelization of the code is a must. The parallelization is performed by decomposing the bunched beam into longitudinal computational domains, each one containing $\sim N_{part}/n_{proc}$ particles, n_{proc} being the number of available processors (procs). The end points of each domain are obtained looking at the actual longitudinal cumulative distribution function for the particles. The domains are dynamically and automatically modified in order to keep a good load-balance between the procs. At every time step the following operations are performed in parallel: 1. charge deposition in all the slices pertaining to a given proc[2]; 2. computation of the transverse electric field on all the slices by solving the Poisson equation in 2D; 3. evaluation of the electric field acting on each particle by linearly interpolating the value of the field from two adjacent slices; 4. tracking of the particles through the external lattice. 5. particles exchange between procs because of the synchrotron motion. Since this migration is slow the number of particles to exchange at each time step is small compared to the total number of particles. The parallel framework in MICROMAP, managed with MPI[7], is almost transparent for the user, few routines control all parallel operations (longitudinal "chopping" of the beam, data collection from different procs, test particles management).

PERFORMANCES

In figure 1 we show the time needed (in microseconds) for one particle push[3] multiplied per n_{proc} as a function of n_{proc} for two different sizes of the grid of the 2D Poisson solver. The tests have been run on a Linux cluster with CPUs Intel Xeon (32 bits) 2.8 GHz. The speedup is close to the ideal one. The results (obtained for $N_{part} = 10^6$, $N_{slices} = 25$) are weakly dependent on N_{part} and N_{slices}.

[2]Particles belonging to the same longitudinal slice can be managed by different procs, a data reduction between neighbouring procs is required in order to get the correct value of the charge density on each slice.

[3]The particle push includes one space charge kick and one tracking step through a simple constant focusing lattice.

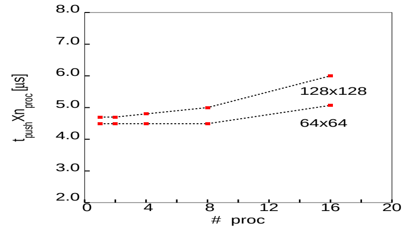

Figure 1: Time for one particle push multiplied per n_{proc} as a function of n_{proc} for two different sizes of the grid of the Poisson solver. See the text for details.

TESTS ON SINGLE PARTICLE TUNE

The adequate numerical description of phenomena such as resonant halo, particles trapping/detrapping requires a correct representation of the single particle dynamics which is characterized by the single particle tune (SPT). We have computed the tunes for a set of test particles in several controlled simulations, comparing the results with analytical results. In all the simulations we have taken a symmetric constant focusing channel ($Q_{x0} = Q_{y0} = 1.655$, $L_{ring} = 104$ m. The bunch (U^{73+}, $N_{bunch} = 10^9$ ions/bunch, $E_{inj} = 11.4$ MeV/u) has a longitudinal Gaussian profile ($\sigma_z = 10$ m), the transversal r.m.s. emittances are $\epsilon_x = \epsilon_y = 10$ mm mrad, several transverse distributions have been considered. In all the simulations we have $N_{part} = 10^6$, for the grid we take $(256 \times 256) \times 25$ and we use 37 SC kicks per betatron wavelength.

Static tests. We first consider a case without synchrotron motion in order to investigate solely the correctness of the interpolation procedure. In figure 2 we show the longitudinal (left) and the transverse (right) behaviour of SPT for Gaussian, WB and KV transverse distribution. Black curves are the theoretical predictions. The tunes are inferred by taking the FFT. The agreement with theoretical values is very good.

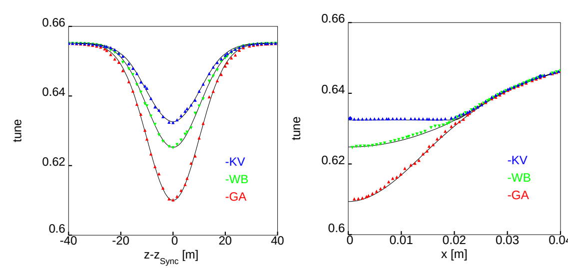

Figure 2: Longitudinal (left) and transversal (right) behaviour of the tune without synchrotron motion.

Dynamic tests. We consider now the synchrotron motion taking $T_{sync} = 2000$ turns. In this case a test particle initially far from the bunch center (*i.e.* far from the synchronous particle) will explore regions of the beam with different space charge. We have inferred the "instantaneous tune" of the test particle by taking the turn by turn phase ad-

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vance. In figure 3 (left) we show the typical behaviour of the SPT for one particle that at the beginning of the simulation is in $z_p - z_{sync} = -40$ m. After 1000 turns it reaches the head of the bunch. Because of the statistical fluctuations of the density, the tune is, in general, different from the theoretical value (black curve). The red and cyan plots refer to simulations performed respectively with linear shape and quadratic shape functions in the interpolation process. The reduction of the fluctuation in the second case is evident. We have also verified if the noise level in the SPT is consistent with the theoretical one (no systematic effects related to synchrotron motion, tracking or interpolation). In this respect we have taken a set of 100 test particles with the same amplitude for the synchrotron motion but different initial phases. We have then computed the average value and the r.m.s. spread of the tune as a function of the longitudinal position. Results are shown in 3 (right). The average value (red solid) is well interpolated by the theoretical curve (black solid), also the amplitude fluctuation (red dashed) is fully consistent with the expected one (black dashed). Similar tests have been done with different transverse distribution (KV, WB). In all cases the results were good.

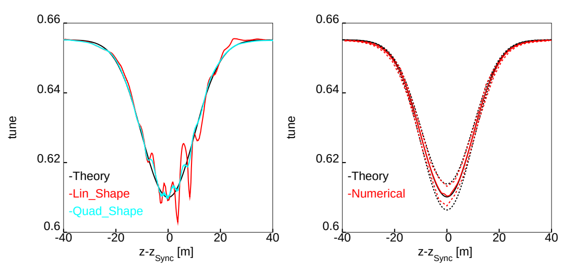

Figure 3: Left: SPT for one particle with linear (red) and quadratic (blue) shape functions. Right: average value (solid line) and r.m.s. spread (dashed line) of the SPT.

APPLICATION TO THE SIS100

We outline here a first application of MICROMAP ($2\frac{1}{2}$D) to a relevant case which is of concern about the problem of beam losses in SIS100. We consider a bunched beam with sharp edge (WB transverse profile) injected from the SIS18 into the bucket of the SIS100. We assume that the injection causes a moderate ($\sim 5\%$) transverse mismatch. As a preliminary fully self-consistent study we consider a simplified version of the SIS100 lattice where all nonlinearities are removed. In the working point we have chosen the bare tunes are $Q_{x0} = 18.83$, $Q_{y0} = 18.73$. The bunch (U^{28+}, $N_{bunch} = 7.5 \cdot 10^{10}$ ions/bunch, $E_{inj} = 200$ MeV/u) has a longitudinal Gaussian profile with $\sigma_z = 27$ m, the initial edge emittances are $\epsilon_x/\epsilon_y = 35/14$ mm mrad. The maximum tune shifts are $\Delta Q_x \simeq -0.12$, $\Delta Q_y \simeq -0.2$. We also consider the synchrotron motion taking $T_{sync} = 1000$ turns. In the simulation we set $N_{part} = 10^6$, $N_{slices} = 29$ and we use 399 SC kicks/turn. In figure 4 (left) we show the behaviour of the r.m.s. amplitudes of the beam. Because of the mixing effect of the synchrotron motion and, secondly, of the space charge, the mismatch oscillations are damped on a time scale of ~ 100 turns. In figure 4 (right) we plot the

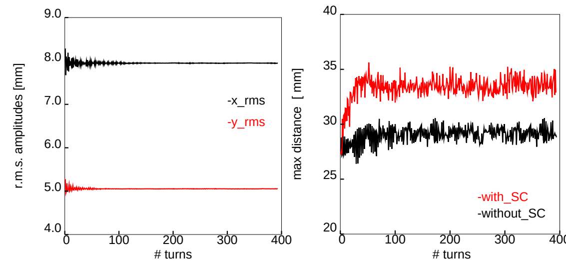

Figure 4: Left: behaviour of the r.m.s. amplitudes in the case study. Right: maximum distance along the x-axis reached by the particles with (red) and without (black) SC.

maximum distance (along the x-axis) reached by the particles in two simulations respectively with (red) and without (black) SC. The larger distance reached by the particles in the first case is due to the formation of a tail. The tail can be easily recognized looking at the plot of the single particle emittances (SPE), see figure 5 (left plot). Black points are the SPEs at the beginning of the simulation, the red ones are the emittances after 300 turns. In figure 5 (right plot) we show the percentage of the particles in the halo as a function of the time, the asymptotic value is $\sim 0.3\,\%$. We point out that once we take into account lattice nonlinearities all these particles are good candidates to be lost[8].

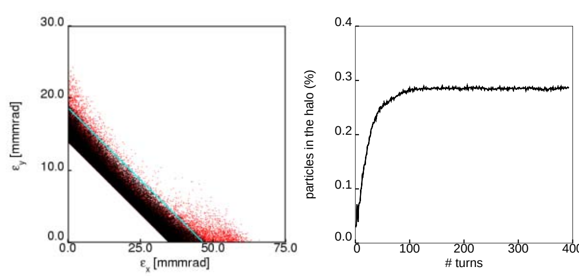

Figure 5: Left: SPEs at the beginning (black) and after 300 turns (red). Right: percentage of particles in the halo.

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