

# STUDY OF THE BEAM DYNAMICS IN A LINAC WITH THE CODE RETAR\*

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## Abstract

The three-dimensional fully relativistic and self-consistent code RETAR has been developed to model the dynamics of high-brightness electron beams and, in particular, to assess the importance of the retarded radiative part of the emitted electromagnetic fields in all conditions where the electrons experience strong accelerations. In this analysis we evaluate the radiative energy losses in the electron emission process from the photocathode of an injector, during the successive acceleration of the electron beam in the RF cavity and the focalization due to the magnetic field of the solenoid. The analysis is specifically carried out with parameters of importance in the framework of the SPARC and PLASMONX projects.

## INTRODUCTION

RETAR is a point to point 3D parallel tracking code for the beam dynamic of charged particles. The code is fully relativistic and calculates the self-fields directly from convenient integral forms that can be obtained from the usual retarded expressions [1].

In this paper we will analyze the radiation emitted by a high charge, high brilliance bunch of electrons moving in assigned electric and magnetic fields and under the effect of the self consistent field. We performed the analysis of the radiation emitted by the electron beam from the extraction at the photocathode to the emittance minimum, located at about 1.5 meters down the beam line. A solenoid field is located after the RF gun to perform the emittance compensation scheme [2, 3] and the overall configuration of external fields is the same improved in the SPARC project [4]. We focus our study on the amount and temporal profile of the radiation emitted in the acceleration and focusing process in order to assess its possible exploiting for a non destructive diagnostic of the beam [5].

In what follows, unless otherwise specified, we refer to a simulation where a 1 nC flat top bunch with a radius of 1 mm and without transverse inhomogeneities, is first accelerated in a 1.6 S-band cavity with a field gradient of 120 Mv/m, then is focused by a solenoidal magnetic field with a peak value of 0.27 T. The beam waist is located at about 1.3 m from the cathode and the transverse emittance shows the double minima feature, with a value between 2 and 4 mm mrad.

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## ACCELERATION AND RADIATION PRODUCTION

In a RF gun, the emitted radiation is mainly due to the acceleration process. The extraction of charge from the photocathode, though implying a sudden change of electrons state, is mainly dominated by space charge effects [6]. Furthermore, the creation of a free electron from a bound state inside matter, is a full quantum process, outside the capabilities of RETAR.

The consistent self fields, both electric and magnetic, calculated by RETAR (Lienard-Wickert retarded fields), are obtained by summing two terms. For example the electric field can be written as [1]:

$$\mathbf{E}_R(\mathbf{x}, t) = \int d\mathbf{x}' \left\{ \frac{f(\mathbf{n}, \boldsymbol{\beta}, \dot{\boldsymbol{\beta}})}{|\mathbf{x} - \mathbf{x}'|} + \frac{g(\mathbf{n}, \boldsymbol{\beta})}{\gamma^2 |\mathbf{x} - \mathbf{x}'|^2} \right\} \rho_{\text{ret}} \quad (1)$$

$$f(\mathbf{n}, \boldsymbol{\beta}, \dot{\boldsymbol{\beta}}) = \frac{\mathbf{n} \times ((\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{c(1 - \boldsymbol{\beta} \cdot \mathbf{n})^2}$$

$$g(\mathbf{n}, \boldsymbol{\beta}) = \frac{\mathbf{n} \cdot \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \mathbf{n})^2}$$

where

$$\mathbf{n} = \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|} \quad \gamma = (1 - \boldsymbol{\beta}^2)^{-\frac{1}{2}}$$

$$\boldsymbol{\beta} = \frac{\mathbf{v}}{c} \quad \dot{\boldsymbol{\beta}} = \frac{d\boldsymbol{\beta}}{dt}$$

The first term depends on the acceleration and is a relativistic bremsstrahlung term, while the second, depending only on velocity, is a static (coulombian like) term.

An evaluation of the radiative losses can be given by computing the flux of the Poynting vector through a surface (for instance a cylinder S) surrounding the electron beam. The power irradiated is in fact

$$P = \int_S (\mathbf{E} \times \mathbf{B}) \cdot \mathbf{n} da$$

and the total energy irradiated is given by

$$W = \int dt \int_S (\mathbf{E} \times \mathbf{B}) \cdot \mathbf{n} da.$$

When the bunch is extracted, the radiation is mainly produced by the high longitudinal acceleration. Indeed, being the RF gun a 1.6 cavity, the greatest amount of radiation

will be emitted before the first iris, where  $\langle \gamma \rangle$  grows from 1 to  $\approx 5.5$ , since afterward the bunch is already relativistic and does not experience any significant velocity increase. Anyway, some measurable contributions could be produced by transverse accelerations when the bunch goes through the solenoidal fields or close to the beam waist.

## SIMULATION RESULTS

In this section we show some numerical results supporting the use of retarded fields as a means of non destructive beam diagnostics.

### Emitted Power

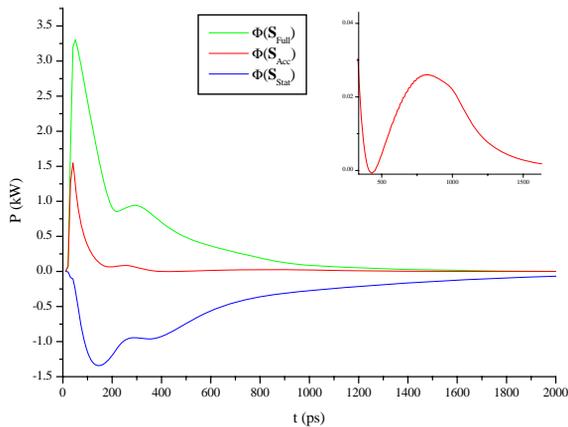


Figure 1: The emitted power (flux of  $\mathbf{S}$ ) collected on a cylinder around the bunch as a function of time, with separated contribution from total fields, acceleration fields and static fields. In the inset a third maximum shown by  $\Phi(\mathbf{S}_{Acc})$  due to transverse accelerations in the solenoid.

The power on a cylinder surrounding the beam and with a radius  $r = 7$  mm as function of time is represented by the green line in Fig 1. There are also shown the emitted powers if either the coulombian (red line) or the acceleration (blue line) parts of the full fields are turned off. While the former situation represents a rough approximation of the results expected using a collecting cylinder with larger radius, the latter is in no way physical, since the bunch did accelerate. In a drift region, without the 'memory' of past accelerations, its value is zero.

The power is initially zero until a time  $t \approx r/c$ , due to the delay in arriving on the surface of the cylinder, is elapsed, followed then by a sequence of maxima and minima. From the comparison between the shape in time of the power emitted, the shape of the external electric and magnetic fields as seen by the electrons of the bunch and the bunch transverse dimension, we can say that the first peak of radiation is due to the first maximum of the accelerating electric field reshaped by the retard effects, while the second peak corresponds to the second peak of acceleration, superimposed with the entrance into the magnetic field and

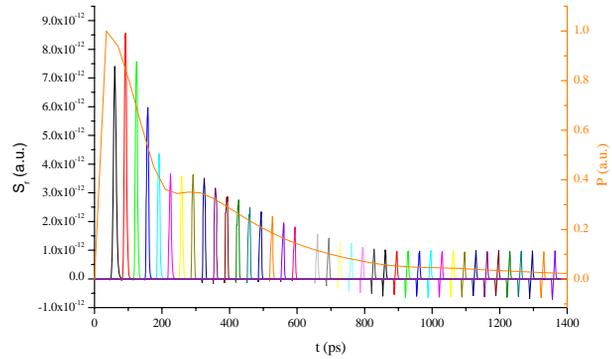


Figure 2: The value of  $|\mathbf{S}|$  measured in many points (see text) and the derived value of  $S_r$  rescaled to 1.

a focalizing effect due to the radio frequency. Afterward there is a less accentuated peak of radiation, only visible when the static (coulombian) component is turned off due to the small radius of the cylinder, corresponding to the inversion of radial velocity at the maximum value of bunch envelope. Note that, since the acceleration is radial in this situation, the radiation is emitted with a small angle in the positive  $z$  direction, so its detection is due to the closeness between the bunch radius and the position of the collecting surface (see Figure 3).

From this kind of measurement we can extract two important information [6]: the total emitted energy is proportional to the square of the bunch charge, while the time evolution of the emitted power can be related to the kinematics of the beam by the Liénard result, that generalizes the Larmor formula for point-like charge far from the source:

$$P = \frac{2}{3} \frac{q^2}{c} \gamma^6 \left[ (\dot{\beta})^2 - (\boldsymbol{\beta} \times \dot{\boldsymbol{\beta}})^2 \right],$$

where the term  $\boldsymbol{\beta} \times \dot{\boldsymbol{\beta}}$  is usually negligible whenever the acceleration is mainly longitudinal. Since an experimental

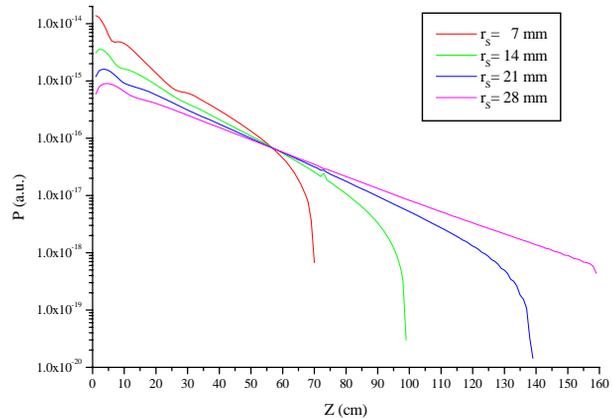


Figure 3: The power emitted collected on different radius cylinders.

measurement of the whole power emitted is not conceivable, a possible implementation consists in measuring the

flux of  $\mathbf{S}$  in many different fixed point-like places along the  $z$  axis; assuming cylindrical symmetry, derive the total flux by multiplying the values of  $\mathbf{S}$  times the considered area. In Figure 2 we show a numerical result illustrating this proposal: we calculated the radial component of  $\mathbf{S}$  in different points spaced by 10 mm along  $z$  and 7 mm off axis, and averaged the measurement in time, obtaining an emitted power time dependence fairly equivalent to the one shown in Figure 1, though with a lesser time resolution.

Following this proposal, we simulated the emitted power collected at increasing distances from the  $z$  axes. The results are shown in Figure 3. Increasing the distance at which the radiation is collected, implies the disappearance of the third maximum due to the inversion of radial velocity (see above) and a decreasing contribution from static, coulomb like, field components. As a consequence the tail, in drift region, assumes the characteristic exponential decay of a wake field [7].

### Electric Field Measurements

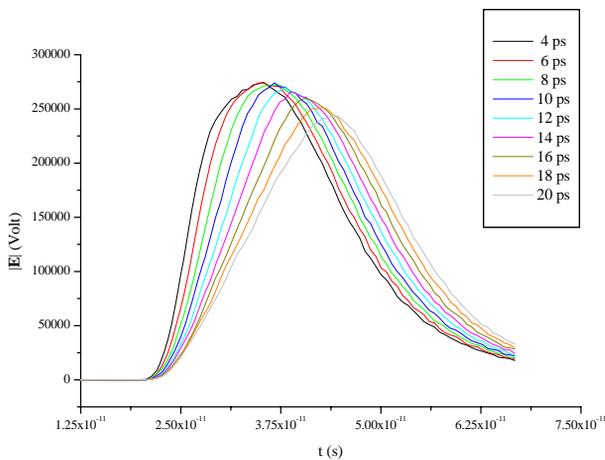


Figure 4: The radial component of the electric field  $E_r$ , calculated in a fixed points for different bunch lengths.

Another possible use of retarded fields measured in a point like spot, is the determination of the bunch length [5] and its longitudinal charge profile. In Figure 4 the time signal for the electric field, taken in a point placed at  $z = 10$  mm and  $r = 5$  mm, is plotted for different bunch lengths, going from 2 up to 20 ps. It is apparent that the signal increase is linear, while the peak values decrease changing the bunch length. The slope of the signal or, equivalently, its rising time, can then be used to determine the bunch length. In Figure 5 a calibration has been done, showing a fairly linear correlation between the signal rising time and the bunch length.

### CONCLUSION

In this paper we showed how it is possible to perform nondestructive beam diagnostics by performing measurements on the retarded fields. To this end we made use of

the code RETAR. It must be underlined that all the situations we analyzed were in a low energy context. That was due to the fact that computational time increase dramatically in high energy simulations. This does not prevent, in future, the study of situations in which the bunch energy is so high to require exclusively nondestructive diagnostics.

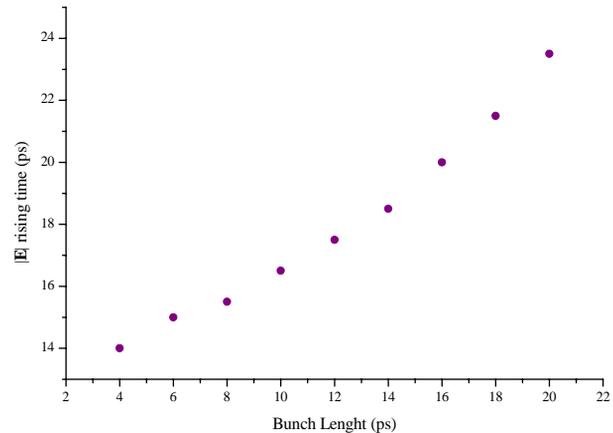


Figure 5: The signal rising time as a function of the bunch length. A fairly linear dependence is shown.

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