

3D PIC METHOD DEVELOPMENT FOR SIMULATION OF BEAM-BEAM EFFECTS IN SUPERCOLLIDERS

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Abstract

A new Beam-Beam simulation code based on a 3D PIC method has been developed. Taking into account to the full extent the three-dimensional nature of interaction can be useful for studies of a pinch effect at large crossing angles in ILC and Crab Waist properties in SuperB Factory. The numerical examples of the electron and positron bunches movement and collision simulation are presented.

INTRODUCTION

Today the most elaborated method of numerical simulation of beam-beam effects for relativistic charged particles is based on solution of Poisson equation with boundary conditions [1]. Both colliding bunches are divided into macroparticle slices, and then the two-dimensional field of transverse forces is calculated for each slice. This quasi-three-dimensional approach is used for studying the problem of "strong-strong" interaction [2].

However, reduction to the 2D problem, where the longitudinal motion is simulated by "rearrangement" of slices, can not completely cover the longitudinal effects, which are of particular importance for colliding beams with superhigh densities such as in ILC and SuperB Projects [3, 4]. For example, at the disruption parameter [5] $D > 5$ a single pass collision makes possible strong compression (pinching) and even disruption of the bunch. In this case it is of interest to study a role of longitudinal electric fields induced due to bunch compression at very short lengths $\sim D^{-1}L_b$ (the beam length in a liner supercollider may be $L_b \sim 100$ microns). 3D PIC (Particle-In-Cell)-code can include in principle all classical electromagnetic effects (for example, a near-zone radiation) in beam-beam collisions in contrast to the slice-based models. This will enable to account accurately spreading of the particle energy distribution during a single pass of counter bunches through each other. Energy spreading may be harmful in the viewpoint of physical experiment performance (SuperB) [6]. 3D PIC-code will let simulate most closely a pinch effect at large crossing angles in ILC as well as Crab Waist [7] properties. With advances of the code and with the advent of its parallel supercomputer version it will be possible to apply it for beam-beam simulation at large number of particle turns in a cyclic collider.

Main difficulty in 3D PIC charged particle beam simulation arises from a large Lorentz-factor $\gamma \gg 1$. This circumstance makes the 2D approximation justified for the

most part, but while it makes 3D PIC code realization very hard.

In the paper the algorithm developed to solve the Beam-Beam problem by the 3D method of macro-particles are described. The numerical simulation examples with a model beam movement as well as with a strong single pass interaction of bunches are presented.

FORMULATION OF THE PROBLEM

In parallelepiped region $[0, L_x] \times [0, L_y] \times [0, L_z]$ the relativistic charged particle beams move along the z -axis in the self-consistent electromagnetic fields. Each of the beams is characterized by a shape, a particle density distribution, linear sizes as well as a position inside the region. To describe such a motion one can use the Vlasov's kinetic equation for the distribution function f of particles (positrons e^+ and electrons e^-) and the Maxwell's equation system in Gauss units. Let denote L , a characteristic size of the region; c , the light speed; $T = L/c$, a characteristic time; $p_0 = m_e c$; $F_0 = \frac{m_e c^2}{L}$, $E_0 = \frac{m_e c^2}{eL}$, $H_0 = \frac{m_e c^2}{eL}$, $\rho_0 = \frac{E_0}{4\pi L}$, $j_0 = c\rho_0$. Then we can write necessary equations in dimensionless form:

$$\frac{\partial f_{e^{+,-}}}{\partial t} + \bar{p}_{e^{+,-}} \frac{\partial f_{e^{+,-}}}{\partial \bar{r}} + \bar{F}_{e^{+,-}} \frac{\partial f_{e^{+,-}}}{\partial \bar{p}} = 0, \quad (1)$$

$$\text{rot} \bar{E} = -\frac{\partial \bar{H}}{\partial t}, \quad (2)$$

$$\text{rot} \bar{H} = \bar{j} + \frac{\partial \bar{E}}{\partial t},$$

$$\text{div} \bar{E} = n_{e^+} e^+ + n_{e^-} e^-,$$

$$\text{div} \bar{H} = 0,$$

where: $\bar{F}_{e^{+,-}} = \bar{E} + [\bar{v}_{e^{+,-}}, \bar{H}]$ is a Lorentz force; $\bar{\gamma}_{e^{+,-}} = 1/\sqrt{1-v_{e^{+,-}}^2}$ is a relativistic factor; $\bar{p}_{e^{+,-}} = \bar{\gamma}_{e^{+,-}} \bar{v}_{e^{+,-}}$ is a particle momentum. Particle and current densities in the equations above are determined through the volume integrals and the distribution functions:

$$n_{e^+} = \int_{V_0} f_{e^+} d\bar{p}$$

$$n_{e^-} = \int_{V_0} f_{e^-} d\bar{p}$$

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$$\bar{j} = \int_{V_0} (f_{e^+} \bar{v}_{e^+} e^+ + f_{e^-} \bar{v}_{e^-} e^-) d\bar{p}.$$

Traditional way of numerical solution relates to using a well-milled computational grid in longitudinal direction and increasing the computational region in transverse direction. But an increase of grid is proportional to γ and is unacceptable even for the modern computers regarding memory volumes and operating speed. One can reduce the work content thanks to bringing a computational region boundary sufficiently near the beam. In this case, a transverse size of region must be cut down in γ and more times with the aim to not attain a wave zone. At that the difficulties arise in determination of self-consistent initial and boundary conditions and, in fact, - in numerical experiment conduction. We consider the ways to get over these difficulties in [8]. Initial and boundary conditions are found with the help of the fundamental solution for the potential and the electric field of a moving freely single charge with subsequent summation of the contributions from all charges. The boundary position corresponds to a near wave zone.

ALGORITHM DESCRIPTION

To solve the systems (1) and (2) we use the Particle-in-Cell (PIC) method which is an universal method for collisionless plasma simulations including a relativistic one [9, 10]. The leap-frog scheme [11] is applied providing the second order of approximation in space and time. After initialization a following sequence of operations is performed at every step of the simulation. The electromagnetic fields are re-counted through a half-step forward. If a particle crosses a mesh boundary its path is divided by two ones and the formulas are applied for each of these paths. Such a method allows to satisfy automatically the differential equation of continuity and thus to fulfil accurately the differential Gauss law. This substantially decreases the approximation errors and makes the algorithm more stable. Then, using Maxwell's equations, the electric and magnetic fields are re-counted through one step and one half-step more forward respectively. The cycle is repeated till destination of a required point of time.

SIMULATION EXPERIMENT RESULTS

The algorithm described has been tested in some characteristic examples of a relativistic beam motion simulation. In the first example the bunch consists of the monoenergetic electrons moving strictly along the z-axis at $\gamma = 10^4$. The particle density is distributed according to the Gaussian law with the root-mean-square beam sizes $\sigma_x = \sigma_y = \sigma_z$. The spatial distribution in a plane $x - z$ of the electromagnetic energy density carried by the electron bunch is plotted in Fig.1 at two points of time: $t = 0$ (Fig.1a) and after 320 time increments (Fig.1b) that corresponds to a shift compared with the bunch length. It is shown from the figures that the structure and amplitudes of the electromagnetic impulse are well conserved during

the motion in accordance with the task features in spite of an exceptionally small mesh number ($32 \times 32 \times 32$). This example is calculated with the help of middling-power (~ 1000 MHz) PC in 15 minutes.

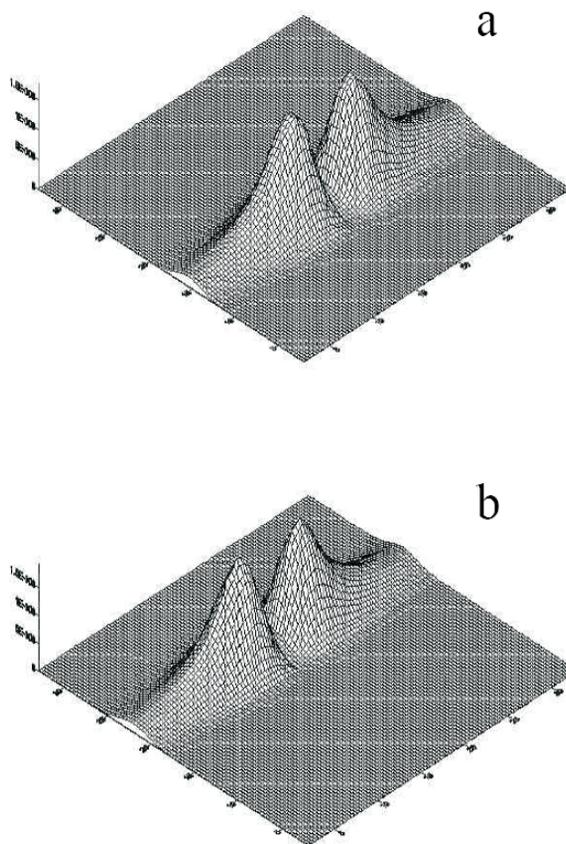


Figure 1: Electromagnetic energy density distribution of the spherical ultra-relativistic bunch ($\gamma = 10^4$) at two instants of time.

Another example relates to the simulation of collision of the electron and positron bunches focusing in the center of interaction region (in I.P.). The mesh number is $60 \times 60 \times 120$, the computational region sizes in arbitrary units are $L_x = L_y = 10^{-2}$, $L_z = 1$. The Gaussian density distribution for a undisturbed beam in the crossover plane is described by the parameters $\sigma_x^* = \sqrt{\beta_x^* \varepsilon_x}$, $\sigma_y^* = \sqrt{\beta_y^* \varepsilon_y}$ where ε_x and ε_y are the radial and vertical beam emittances, respectively, β_x^* , β_y^* are the corresponding beta-function values. The distribution function with transverse momentums x' and y' in I.P. is also Gaussian with the parameters $\sigma_{x'}^* = \sqrt{\varepsilon_x / \beta_x^*}$ and $\sigma_{y'}^* = \sqrt{\varepsilon_y / \beta_y^*}$. In the simulation the following basic parameters were used: $\gamma = 6.85 \cdot 10^3$; the bunch population $N_- = 2.63 \cdot 10^{11}$ (the electrons), $N_+ = 1.31 \cdot 10^{11}$ (the positrons); $\varepsilon_x = \varepsilon_y = 5 \cdot 10^{-9}$ cm; $\beta_x^* = \beta_y^* = 0.1$ cm; $\sigma_x^* = \sigma_y^* = 224$ nm; $\sigma_z = 0.1$ cm. Such a combination of the parameters corresponds to a supercritical regime of the beam-beam interaction since the disruption parameter takes a very large value:

$D_{x,y} = 2r_e N \sigma_z / [\gamma \sigma_{x,y} (\sigma_x + \sigma_y)] \approx 2.2 \cdot 10^3$ ($N = N_+$ or $N = N_-$). In Fig.2a the calculated profiles of colliding beams (the red is electrons, the blue is positrons) are presented at the instant $t = 0.5$ when the optical focusing has just become apparent, but a collision has not begun yet. Strong focusing effect of the counter beam field is illustrated in Fig.2b. The transverse sizes of both bunches become shrunk several times at a range of their mutual penetration as compared with the case of only optical focusing ($t = 2$). In succeeding instants of time, a structure of the bunches in a given example is completely destroyed in accordance with the theoretical prediction based on a large value of the D parameter.

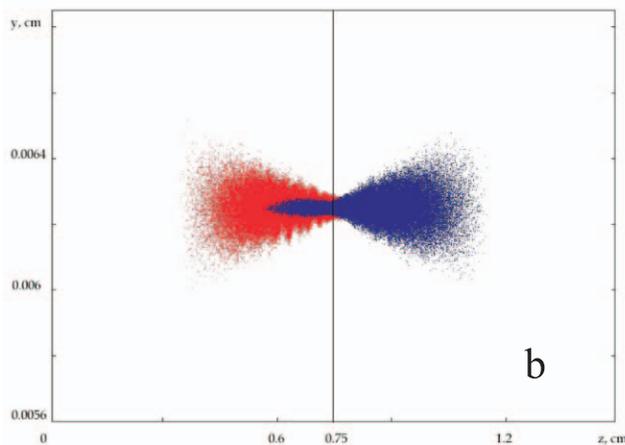
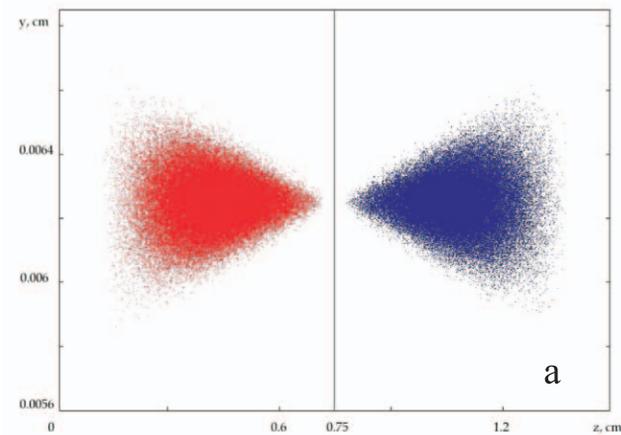


Figure 2: Results of the beam-beam collision simulation. The transverse scale in these figures are 1000 times greater than longitudinal one.

The calculation accuracy in our simulations is examined by tracing the energy value. In Fig.3 the upper curve presents the relative changes of $\sim 10^{-9}$ in the total electromagnetic field

$$W_F = \int \frac{E^2 + H^2}{8\pi} dx dy dz.$$

with a time. The lower curve shows the same in magnitude

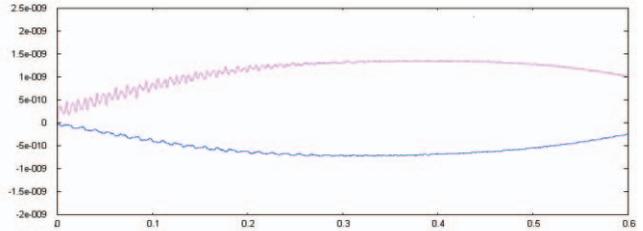


Figure 3: Test of the energy conservation.

but of opposite sign changes in the total kinetic energy of the particles

$$W_K = E - E_0 = m_e \sum_i^N (\sqrt{1 + |p_i|^2} - 1).$$

CONCLUSION

New fully three-dimensional algorithm for beam-beam simulation with rather high gamma-factors ($\sim 10^3 \div 10^4$ and higher) has been developed. Simulation results have verified appropriateness of the chosen model and algorithm efficiency. We attend to apply this computational method for study of beam-beam effects in the supercolliders projects.

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REFERENCES

- [1] D. Schulte/ Tesla Reports, Hamburg, 1997.
- [2] K. Ohmi //Phys. Rev. E, 62, 2000.
- [3] K. Yokoya /Report at ICFA Seminar, Daegu, Korea, 2005.
- [4] P. Raimondi /Report at 2nd Workshop on Super B Factory, LNF-INFN, Frascati, 2006.
- [5] P. Chen and K. Yokoya // Physical Review D, 38, 3 (1988), 987-1000.
- [6] M. Giorgi/Report at Super B-Factory Meeting at LNF (Frascati)11-12 November 2005.
- [7] P. Raimondi, D. Shatilov, M. Zobov / LNF-07/003(IR), 2007
- [8] M.A. Boronina, V.A. Vshivkov, E.B. Levichev, S.A. Nikitin, V.N. Snytnikov, submitted to Journal of Computational Technologies (Novosibirsk) in 2007.
- [9] R. Hockney and J. Eastwood, Computer simulation using particles, McGraw-Hill Int. Book Company, 1980.
- [10] Yu.Grigoryev and V. Vshivkov, The numerical methods "Particle-in Cell", Novosibirsk, Nauka, 2000.
- [11] A.B. Langdon and B.F. Lasinski// Meth. Comput. Phys., v.16, -1976.