NUMERICAL ALGORITHMS FOR MODELING ELECTRON COOLING IN THE PRESENCE OF EXTERNAL FIELDS*

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Abstract

The design of the high-energy cooler for the Relativistic Heavy Ion Collider (RHIC) recently adopted a nonmagnetized approach. To prevent recombination between the fully stripped gold ions and co-propagating electrons, a helical undulator magnet has been proposed. In addition, to counteract space-charge defocusing, weak solenoids are proposed every 10m. To understand the effect of these magnets on the cooling rate, numerical models of cooling in the presence of external fields are needed. We present an approach from first principles using the VORPAL parallel simulation code. We solve the n-body problem by exact calculation of pair-wise collisions. Simulations of the proposed RHIC cooler are discussed, including fringe field and finite interaction time effects.

ELECTRON COOLING IN RHIC-II

The primary component of the RHIC-II upgrade is an electron cooling section which serves to increase the luminosity of the ion beam. In this section of the ring, an electron beam co-propagates with the ions, and the resulting binary collisions between ions and electrons cool the ion beam.

RHIC operates using a variety of ions, the heaviest of which is a fully stripped gold ion with 100 GeV/nucleon. In the cooling section, 54 MeV electrons co-propagate with the gold ions. Although the motion is highly relativistic in the lab frame, in a frame of reference translating uniformly with the ions and electrons their velocity is non-relativistic. Working in this "beam frame" the particle dynamics are described by classical Coulomb collisions.

The density of the ion beam is low enough that ion-ion interactions are not significant in the cooling section. The interaction time is also short enough that Debeye shielding is not significant.

In the absence of external fields, the cooling rate is easily estimated by a classical Coulomb collision model. However in the RHIC-II beam line there may be a number of magnets: first a 10 Gauss helical undulator whose function is to suppress recombination of the electrons and ions. Second, weak solenoids may be used to provide some focusing. These solenoids are not strong enough to provide so-called "magnetized cooling", as discussed in [1]. Third,

all of these magnets may have misalignment errors, resulting in fields slightly different from the design. The purpose of this paper is to study the impact of the weak solenoids on the cooling rate.

To model Coulomb collisions under arbitrary fields, we use the general purpose 3D code VORPAL [2], using a special binary collision algorithm which models collisions between electron-ion pairs exactly (with no external field). The external magnetic field is Lorentz transformed to the beam frame, where it becomes a time-dependent electric and magnetic field. The particle motion is resolved using operator splitting, with a binary collision particle push alternating with the standard Boris push[3]. The details of this algorithm are given in [4, 5].

THEORETICAL ESTIMATES OF COOLING RATES

Cooling theory in the absence of fields is well established [6]. In this section we consider the problem in the beam frame co-propagating with the particles, where all velocities are non-relativistic. By convention the beam is moving in the $\hat{z}$ direction, with $\hat{x}$ and $\hat{y}$ the transverse directions.

By integrating a single collision in time over a straight line trajectory, we can estimate the net force on an ion. We then integrate between the minimum impact parameter ρ_{min} and maximum impact parameter ρ_{max} to obtain the mean friction force on an ion [7]

$$\mathbf{F} = -\frac{4\pi n_e Z^2 e^4}{(4\pi\epsilon_0)^2 m_e} \int \ln\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{\mathbf{v}}{|\mathbf{v}|^3} f(\mathbf{v}_e) d^3 v_e. \quad (1)$$

Here n_e is the electron density, $\mathbf{v}_i$ is the velocity of the ion, $\mathbf{v}_e$ is the velocity of an electron, $\mathbf{v} = \mathbf{v}_i - \mathbf{v}_e$, and $f(\mathbf{v}_e)$ is the 3D velocity distribution of the electrons. The velocity distribution is assumed to be Gaussian, with different RMS values in each dimension. Equation 1 is easily evaluated using numerical integration, as in the code BETACOOL [8].

The parameter ρ_{min} is the impact parameter for a $90°$ collision,

$$\rho_{min} = \frac{Ze^2}{4\pi\epsilon_0 m_e |\mathbf{v}|^2}. \quad (2)$$

Often the maximum impact parameter ρ_{max} is taken as the Debeye length, however the interaction time over the 80m cooling section is much less than a plasma period, so ρ_{max} is controlled by the interaction time, $\rho_{max} = 0.5|\mathbf{v}|\tau$.

SIMULATIONS USING VORPAL

In VORPAL only a small section of the beam is modeled, a cube $L = 0.8$mm on a side. This domain size was selected as a trade-off between having a sufficient number of both small and large impact collisions.

In this domain we place a small number of gold ion test charges, which are well separated and do not interact.

Table 1: Parameters used in RHIC-II simulations [9]. Simulations do not always include the solenoid or undulator.

Parameter	Frame	Value
Electron Density, n_e	Beam	$9.50 \times 10^{13} \, e^-/m^3$
RMS e^- x, y-velocity	Beam	$2.8 \times 10^5 \, m/s$
RMS e^- z-velocity	Beam	$9.0 \times 10^4 \, m/s$
Interaction time	Beam	2.47 nanoseconds
Relativistic $\beta = v/c$	Lab	0.99957
Relativistic γ	Lab	108
Cooling length	Lab	80 m
Undulator wavelength	Lab	8 cm
Undulator strength	Lab	10 Gauss
Solenoid wavelength	Lab	10 m
Solenoid length	Lab	10 cm
Solenoid strength	Lab	220 Gauss

Table 1 gives the parameters for the runs. In the absence of external fields, the cooling rates predicted by VORPAL are 10-20% lower than those predicted by equation (1). There are several reasons why this may be so:

- The theoretical calculations assume all collisions are complete (they occur over all time), while in the RHIC-II cooler (and in the VORPAL simulations) collisions are incomplete.

- The theoretical calculations assume all collisions are symmetric in time, while in the RHIC-II cooler (and in the VORPAL simulations) this is not true.

- The VORPAL simulations may be undersampling the collisions with very small impact parameter. Note that in the VORPAL simulations there is no lower limit to the impact parameter which can occur, but collisions with impact parameter less than ρ_{min} are very rare.

The solenoid field has a wavelength of 10m, to first approximation the field is zero from 0–9.8m, then has magnitude $B_0 \hat{z}$ for 10cm, and then $-B_0 \hat{z}$ for 10cm. A magnetic field in the longitudinal ($\hat{z}$) direction is the same in both the lab and beam frames. Therefore, the simplest model is to apply this (discontinuous) field in VORPAL. Simulations indicate that to within numerical error, the friction with the solenoids present is the same as that for zero fields (Figures 1 and 2).

The friction values shown in Figures 1 and 2 were obtained by simulating eight test particles, and averaging. The error bars shows are ± 1 RMS. It takes about 10 hours to calculate each point in the figures using a parallel simulation with 32 processors.

A final set of simulations compares RHIC-II with the undulator to the undulator and solenoids. Simulations where only the undulator is present show that the effect of the undulator is to reduce the cooling by a constant factor f. This can be understood as an increase of the minimum impact parameter from that given by equation (2) to r_0, the oscillation amplitude of an electron in the undulator.

$$r_0 = \frac{eB_0\lambda^2}{4\pi^2 m_e \gamma \beta c}. \tag{3}$$

The factor f is the ratio of the friction from (1) where the minimum impact parameter is replaced by r_0, which gives a factor of

$$f = \frac{\ln(\rho_{max}/\rho_{min})}{\ln(\rho_{max}/r_0)}. \tag{4}$$

For the parameters in the simulation, we have $\rho_{max} = 3.7 \times 10^{-4}$m, $\rho_{min} = 2.2 \times 10^{-7}$m, and $r_0 = 8.8 \times 10^{-7}$m which gives a friction reduction factor of $f = 1.23$. This captures the mean reduction seen in the VORPAL simulations of Figure 1 and Figure 2, up to numerical error.

CONCLUSIONS

VORPAL simulations indicate that the friction when the undulator is present is reduced by the factor f in equation (4) compared to the friction with no external field. Further VORPAL simulations show that the presence of short (10cm), weak (220 Gauss) solenoids will not significantly impact the cooling rate, with or without the undulator. Additional runs are currently being completed to check that the result does not change with a more realistic solenoid fringe field model.

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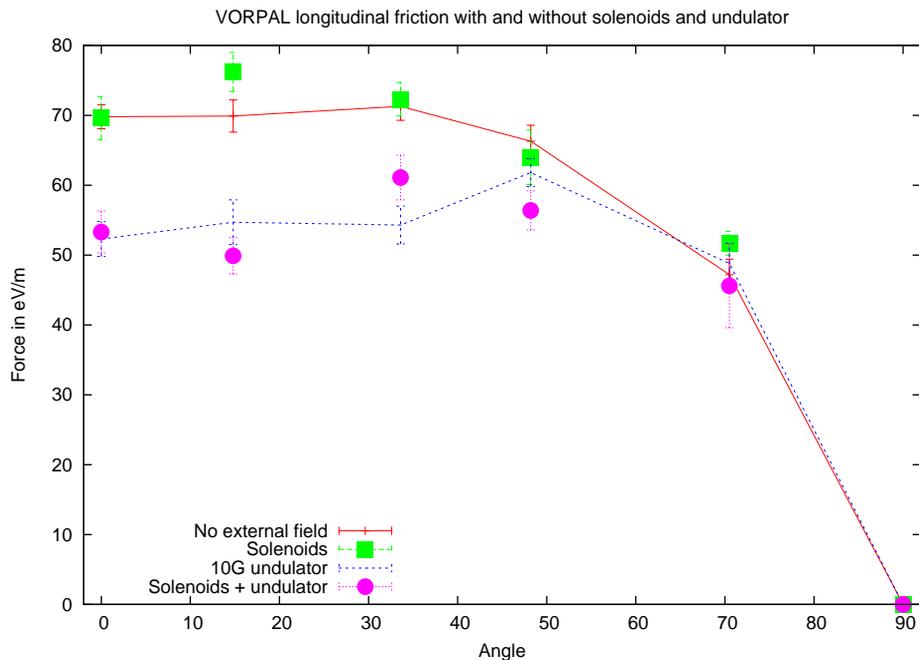[9] RHIC-II Feasibility Study, http://www.bnl.gov/cad/ecooling/

Figure 1: Longitudinal friction on an ion calculated in a VORPAL simulation. An ion with an angle of $0°$ is moving longitudinally, that with an angle of $90°$ is moving transversely. The ion speed is always 3.0×10^5 m/s. The two curves show the results for no field and undulator, while the other points add the solenoids.
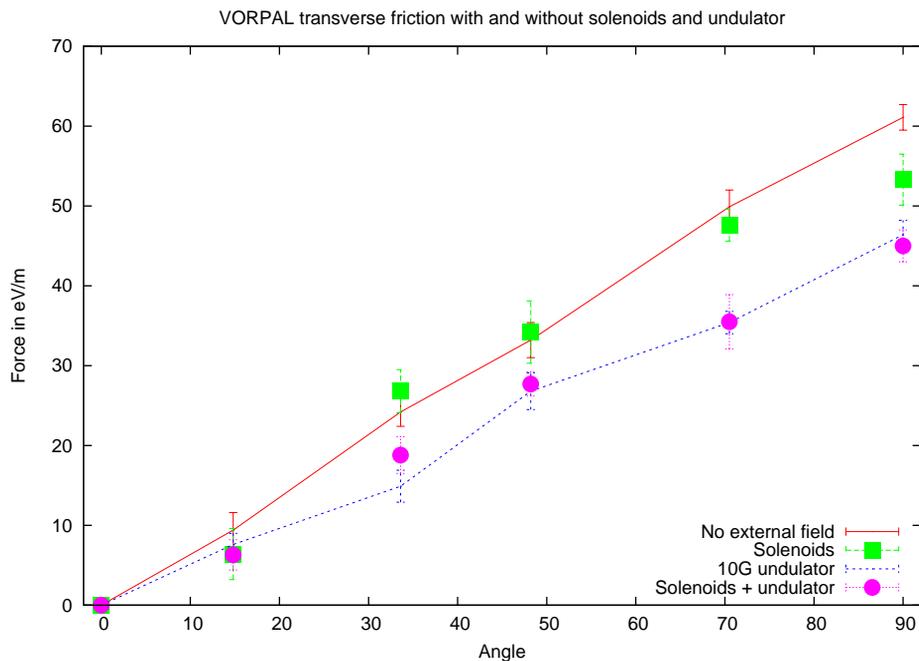

Figure 2: Transverse friction on an ion calculated in a VORPAL simulation. The two curves show the results for no field and undulator, while the other points add the solenoids.