ADAPTIVE IMPEDANCE ANALYSIS OF GROOVED SURFACE USING THE FINITE ELEMENT METHOD

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Abstract

Grooved surface is proposed to reduce the secondary emission yield in a dipole and wiggler magnet of International Linear Collider. An analysis of the impedance of the grooved surface based on adaptive finite element is presented in this paper. The performance of the adaptive algorithms, based on an element-element h-refinement technique, is assessed. The features of the refinement indictors, adaptation criteria and error estimation parameters are discussed.

INTRODUCTION

The strategy of Finite Element Method (FEM) is to divide the solution space into a large number of area or volume elements and derive the linear equations based on the physics problem.

Generally in finite element analysis or other mesh based on method, as the mesh is refined, the accuracy of the solution, as well as its cost, goes up. However, whenever refinement is located in areas where the solution has high error, the increase in accuracy is relatively high than the increase in cost. In adaptive mesh generation, error estimates are used to refine the mesh where the error is higher than an acceptable value and to make coarse mesh where the error is lower than an acceptable value. Adaptive meshing is one of the key research topics being investigated to produce more robust and user-friendly finite element analysis environments in many disciplines. The adaptive method is applied to the RF cavity or wave-guide in accelerators [1]. Here, the similar method is applied to the calculation of the impedance of triangular grooved chamber, which is used to suppress the secondary electron emission in a dipole magnet [2]. After giving a brief summary of the FEM (Finite Element Method), we derive a rigorous posteriori bound on the error estimation and adaptive refinement. Examples are also given of the use of adaptive refinement.

ADAPTIVE STRATEGY

The adaptive refinement procedure is based on the use of two key quantities, evaluated on the basis of a tentative solution: the refinement indictor and the convergence parameter. In addition, an estimate of the error of the solution is evaluated.

The usual continuity assumption used in the field based finite element formulations results in a continuous field from element to element, but a discontinuous field gradient. Therefore, the reasonable error norm of the field for each element can be defined as follows

$$\|e\|_e = \left[\int_\Omega \left(\frac{\partial \phi}{\partial x} - \frac{\partial \hat{\phi}}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} - \frac{\partial \hat{\phi}}{\partial y} \right)^2 d\Omega \right]^{1/2}, \quad (1)$$

where ϕ is the exact field, $\hat{\phi}$ is the finite element solution. The actual err norm is calculated from the smoothed values of the element nodal gradient by the recovery process instead of the exact field. In this smoothing process, it is assumed that the approximation quantities are interpolated by the same basis function ϕ and that they fit the original ones in a least square sense. This method is better than the averaging of the element nodal gradient which is used by ANAYSIS[3].

A more practical representation of the error norm in term of a percentage error is

$$\varepsilon_e = \frac{\|e\|_e}{\|q\|_e} \times 100\%, \quad (2)$$

where

$$\|q\|_e = \left[\int_\Omega \left(\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} \right)^2 d\Omega \right]^{1/2}. \quad (3)$$

The maximum permissible error for each element can be calculated from the average of $\|q\|_e^2$ over all the elements $\|q\|_{av}^2$, $\|\bar{e}\|_e^2 \le \bar{\varepsilon}\|q\|_{av}^2$, here, $\bar{\varepsilon}$ is the specified maximum value. The $\|e\|_e$ values can be used for adaptive mesh refinement. It has been shown by Babuka and Rheinboldt [4] that if $\|e\|_e$ is equal for all elements, then the model using the given number of elements is the most efficient one. This concept is also referred to as "error equilibration".

We define refinement indictor $\xi_e = \|e\|_e / \|\bar{e}\|_e$, if ξ_e>1, the size of element *e* must be reduced and the mesh will require refinement, otherwise, the size of element must be increased and the mesh will be coarsened. Thus the predicted size of the new element based on an element-element h-refinement technique can be calculated from the current element size as $\bar{h}_e = h_e / \xi_e^{1/P}$, where $\bar{h}_e$ is the predicted element size, h_e is the current element size and P is the order of the shape functions.

The estimate of the error of the solution can be evaluated as

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$$\varepsilon = \sqrt{\frac{\sum_e \|e\|_e^2}{\sum_e \|q\|_e^2}} \times 100\% . \quad (3)$$

The summation in above formula is carried on the all elements.

APPLICATIONS

Figure 1 shows the geometry of the triangular grooved surface. The period of the surface is *W*. A triangular surface with sharp tip is desirable to reduce the secondary emission [5]. During fabrication, the tip of the triangle is likely to be rounded. Therefore, the grooved surface with rounded tip is considered in this paper.

The energy loss induced by the electromagnetic field inside the wall in the small skin depth approximation is proportional to the square of the magnetic field on the metal surface. Therefore, the enhancement η of the resistive wall wake effect (both transverse and longitudinal) for the finned beam pipe, compared to a normal beam pipe, can be written as [5]

$$\eta = \frac{\int H^2 ds}{H_0^2 W} \quad (4)$$

where H is the magnetic field of the beam on the surface of the metal, H_0 the magnetic field in the case of a flat (non-grooved) surface, and integration follows the grooved surface over one period in a plane of constant z (beam direction). The magnetic field can be represented as $\mathbf{H} = \hat{z} \times \nabla\phi$, with $\hat{z}$ the unit vector in z and the magnetic potential ϕ satisfying the two-dimensional Laplace equation $\nabla^2\phi = 0$. Note that using the Laplace equation for the magnetic field is valid for frequencies ω such that $c/\omega >> W$; for example, for W~3mm this means $\omega \le 2\pi \cdot 10^{11}$ Hz.

In the calculation of the field, the initial mesh size and the desired error ε are used to control the whole calculation procedure. The program automatically finds the best mesh distribution with a minimum number of mesh nodes to get the desired accuracy. When the accuracy is reached, the calculation stops. Figure 2 shows the example of the adaptive mesh. There is a high mesh density near the rounded tips due to the larger error there. Figure 3 shows the magnetic field lines near the grooved surface. The field lines can't deeply penetrate inside the grooved surface. There is strong field near the tip as shown in Figure 4. The magnetic field is normalized by the field of the flat surface. The field near the tip is enhanced by a factor of 7, which is the main source of the impedance. The field doesn't change much with angle α as shown in Figure 4. Figure 5 shows the impedance enhancement factor for various sizes and shapes of the grooved surface. The impedance roughly linearly increases with the angle α and there is a smaller impedance for a larger scale of the surface. The peak impedance enhancement factor in the region we are interested in is 1.6.

Figure 6 compares the calculated impedance with uniform mesh and adaptive method. The uniform mesh method uses 300k mesh nodes and the calculated error ε is 10^{-3}; On the other hand, the adaptive method uses only 20k nodes and the error ε is 10^{-5}. The calculated impedance with adaptive method is larger because of its higher accuracy, which is one of the characters of the finite element method.

Figure 7 shows the sketch of a test chamber in dipole magnet. The width of the multipacting region where the grooved surface would be required is only 10 *mm* and the required grooved surface is only 15% of the total surface [2]. Therefore, the overall impedance enhancement due to the grooved surface is small.

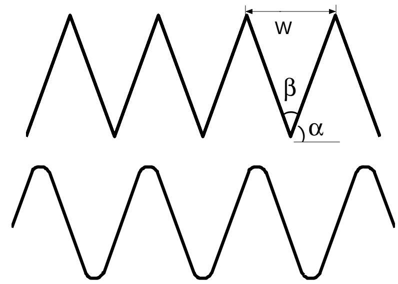

Figure 1: Triangular grooved surface: triangular surface with sharp tips (top) and triangular surface with rounded tips (bottom).

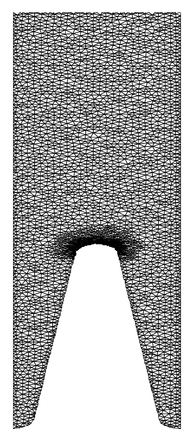

Figure 2: Adaptive mesh of the grooved surface.

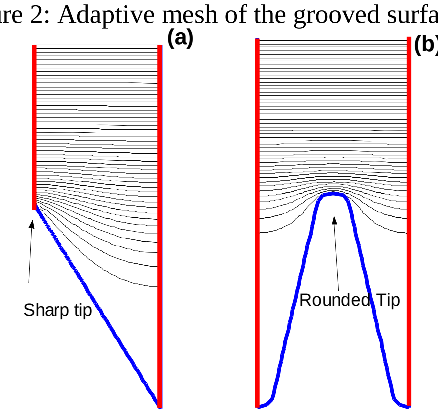

Figure 3: Magnetic field lines penetrating in a groove. A half period of a groove with sharp tip is shown in (a) and

a period of a groove with rounded tip is shown in (b). The blue line shows the metal surface. The red lines are the boundaries in the calculation.

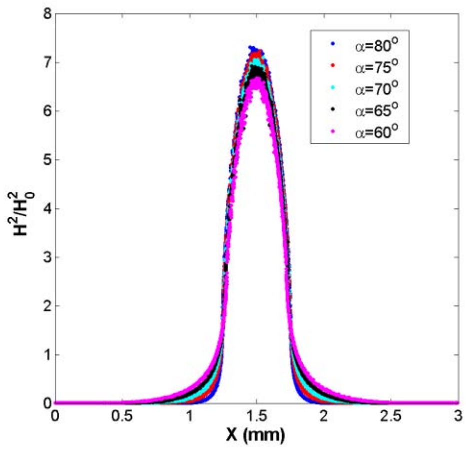

Figure 4: Magnetic fields on the grooved surface with rounded tip. A period of a groove is shown. The geometry of the surface is shown in Figure 2. The field is normalized by the field with flat surface. The peak field locates at the surface near the tip.

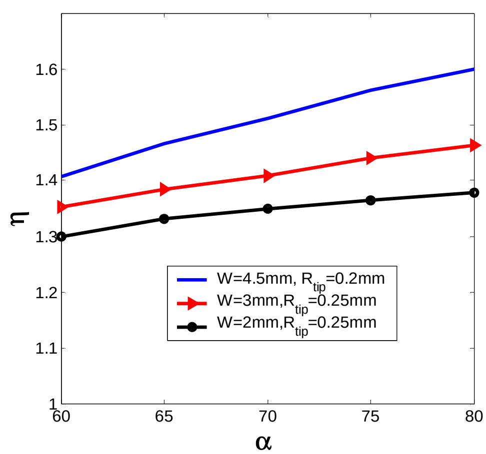

Figure 5: Impedance enhancement factor of the triangular grooved surface with round tips.

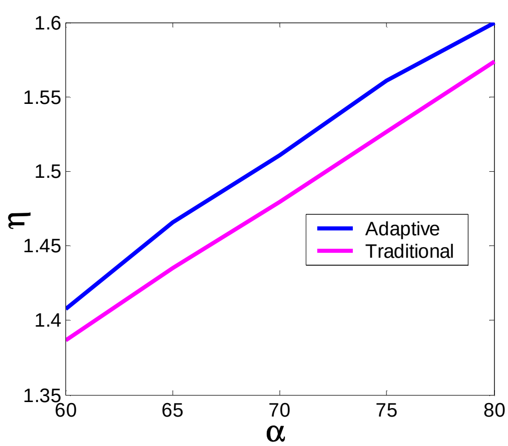

Figure 6: Comparison of the impedance calculated using different methods. The traditional method uses uniform mesh with about 300k mesh nodes; the adaptive method uses 20k mesh nodes.

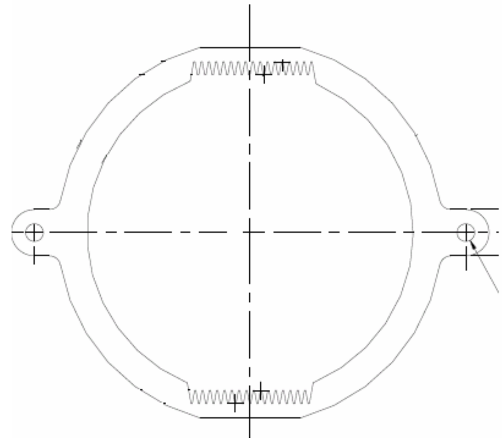

Figure 7: Sketch of the beam pipe with grooved surface in a dipole magnet.

SUMMARY

The adaptive refinement method is successfully applied to the electromagnetic field analysis. The refinement greatly helps the improvement of the accuracy and CPU time. The impedance enhancement due to the grooved surface is less than 10%.

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