NONLINEAR δf PARTICLE SIMULATIONS OF ENERGY-ANISOTROPY INSTABILITIES IN HIGH-INTENSITY BUNCHED BEAMS*

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Abstract

The self-consistent Vlasov-Maxwell equations and a generalized δf particle simulation algorithm are applied to high-intensity finite-length charge bunches. The nonlinear δf method exhibits minimal noise and accuracy problems in comparison with standard particle-in-cell simulations. For bunched beams with anisotropic energy, there exists no exact kinetic equilibrium because the particle dynamics do not conserve transverse energy and longitudinal energy separately. A reference state in approximate dynamic equilibrium has been constructed theoretically. The electrostatic Harris instability driven by strong energy anisotropy relative to the reference state have been simulated using the generalized δf algorithm for bunched beams. The observed growth rates are larger than those obtained for infinitely-long coasting beams. The growth rate decreases for increasing bunch length to a value similar to the case of a long coasting beam. For long bunches, the instability is axially localized symmetrically relative to the beam center, and the characteristic wavelength in the longitudinal direction is comparable to the transverse dimension of the beam.

INTRODUCTION AND THEORETICAL MODEL

Collective effects with strong coupling between the longitudinal and transverse dynamics are of fundamental importance for the applications of high-intensity bunched beams [1]. The self-consistent theoretical framework for studying collective effects is provided by the nonlinear Vlasov-Maxwell equations [2]. A corresponding numerical method, the δf particle simulation method, has been developed [3] to solve the nonlinear Vlasov-Maxwell equations with significantly reduced noise. This theoretical and numerical framework has been successfully applied to study stable beam propagation [4], electron-ion two-stream (electron cloud) instabilities [5–8], and collective instabilities driven by large energy anisotropy [9] for coasting beams. In this paper, we report recent progress in developing a generalized nonlinear δf simulation method to study collective effects for finite-length charge bunches with nonlinear space-charge fields in both the longitudinal and transverse directions. For high-intensity bunched beams, the equilibrium and collective excitation properties are qualitatively different from those for coasting beams. As a consequence of the coupling between the transverse and longitudinal dynamics, the transverse energy and longitudinal energy are not conserved separately, and there exists no exact kinetic equilibrium $(\partial/\partial t = 0)$ which has anisotropic energy in the transverse and longitudinal directions. On the other hand, for charged particle beams accelerated to high energy, energy anisotropy in the beam frame develops naturally as a result of phase space volume conservation. We have developed a reference state for beams with anisotropic energy, which is not an exact, but an approximate equilibrium solution of the Vlasov-Maxwell equations. The difference between the exact solution and the reference state is simulated by the generalized δf particle simulation algorithm described in this paper. Numerical simulations of the electrostatic Harris instability driven by large energy anisotropy are carried out. The effects of finite bunch length are investigated, and the results are compared with previous simulation results for infinitely-long coasting beams.

We consider a single-species, bunched beam confined in both the $r-$ and $z-$ directions by an external smooth-focusing force in the beam frame

$$\mathbf{F}_{foc} = -m\omega_\perp^2 \mathbf{x}_\perp - m\omega_z^2 z \mathbf{e}_z \,. \tag{1}$$

Here, $\omega_\perp$ and ω_z are the constant transverse and longitudinal applied focusing frequencies in the smooth-focusing approximation. In the beam frame, the dynamics of the bunched beam is described by the nonlinear Vlasov-Maxwell equations [2]

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left[m\left(\omega_\perp^2 \mathbf{x}_\perp + \omega_z^2 z \mathbf{e}_z\right) \right.\right. \tag{2}$$

$$\left.\left. + e(\nabla\phi - \frac{v_z}{c}\nabla_\perp A_z)\right]\right\}\cdot\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0,$$

$$\nabla^2\phi = -4\pi e\int d^3p f(\mathbf{x},\mathbf{p},t), \tag{3}$$

$$\nabla^2 A_z = -\frac{4\pi}{c} e\int d^3p v_z f(\mathbf{x},\mathbf{p},t), \tag{4}$$

where f is particle distribution function in phase space.

According the the δf method, the total distribution function is divided into two parts, $f = f_0 + \delta f$, where f_0 is a *known* reference distribution function, and the numerical simulation is carried out to determine the detailed nonlinear evolution of the perturbed distribution function δf. This is accomplished by advancing the weight function defined by $w \equiv \delta f/f$. The dynamical equation for w is given by

$$\frac{dw}{dt} = -(1-w)\frac{1}{f_0}\left[\left(\frac{df_0}{dt}\right)_\delta + \left(\frac{df_0}{dt}\right)_0\right], \tag{5}$$

* Research supported by the U.S. Department of Energy.

where

$$\left(\frac{df_0}{dt}\right)_\delta = -e(\nabla\delta\phi - \frac{v_z}{c}\nabla_\perp\delta A_z)\cdot\frac{\partial f_0}{\partial \mathbf{p}}, \quad (6)$$

$$\left(\frac{df_0}{dt}\right)_0 = \left\{\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left[m\left(\omega_\beta^2\mathbf{x}_\perp + \omega_z^2 z\mathbf{e}_z\right)\right.\right. \quad (7)$$

$$\left.\left. + e(\nabla\phi_0 - \frac{v_z}{c}\nabla_\perp A_{z0})\right]\right\}\cdot\frac{\partial}{\partial \mathbf{p}}f_0\,. \quad (8)$$

Here, $\delta\phi \equiv \phi - \phi_0$, and $\delta A_z \equiv A_z - A_{z0}$. For the perturbed fields, Maxwell's equations are given by

$$\nabla^2\delta\phi = -4\pi e\int d^3p\, wf(\mathbf{x},\mathbf{p},t)\,, \quad (9)$$

$$\nabla^2\delta A_z = -\frac{4\pi}{c}e\int d^3p\, v_z wf(\mathbf{x},\mathbf{p},t)\,, \quad (10)$$

where the reference potentials (ϕ_0, A_{z0}) satisfy

$$\nabla^2\phi_0 = -4\pi e\int d^3p f_0(\mathbf{x},\mathbf{p},t), \quad (11)$$

$$\nabla^2 A_{z0} = -\frac{4\pi}{c}e\int d^3p v_z f_0(\mathbf{x},\mathbf{p},t). \quad (12)$$

It is desirable to pick (ϕ_0, A_{z0}, f_0) as self-consistent solutions to the Vlasov-Maxwell equations (2)-(4), such that the $(df_0/dt)_0$ term in Eq. (5) vanishes. Even though it is a perturbative approach, the δf method is *fully nonlinear* and simulates completely the original nonlinear Vlasov-Maxwell equations. Compared with conventional particle-in-cell simulations, the noise level in δf simulations is significantly reduced. The δf method reduces the noise level of the simulations because the statistical noise for the total distribution function in the conventional particle-in-cell (PIC) method, is only associated with the perturbed distribution function in the δf method. For most applications, (ϕ_0, A_{z0}, f_0) are chosen to correspond to an equilibrium solution with $\partial/\partial t = 0$. For bunched beams, if the energy is isotropic in the beam frame, the reference state can be chosen to be an exact equilibrium solution. Detailed study of the collective excitations in bunched beams with isotropic energy can be found in Ref. [10]. However, for bunched beams with energy anisotropy, exact equilibrium solutions do not exist due to the lack of independent longitudinal and transverse invariants of the particle dynamics, and we can only choose a reference distribution f_0 that is close to a quasi-equilibrium state. Furthermore, for a single-species beam, we neglect A_z in the beam frame because $|A_z| \ll |\phi|$.

COLLECTIVE EXCITATIONS FOR BUNCHED BEAMS WITH ENERGY ANISOTROPY

Approximate kinetic equilibria with anisotropic energy can be constructed for long bunches, or other cases where the coupling induced by the nonlinear space-charge field is relatively weak. For these cases, the transverse energy $H_\perp$ and longitudinal energy H_z defined by [10]

$$H_\perp = \frac{p_\perp^2}{2m} + \frac{m}{2}\omega_\perp^2 r^2 + e\widetilde{\phi_0}(r,z), \quad (13)$$

$$H_z = \frac{p_z^2}{2m} + \frac{m}{2}\omega_z^2 z^2 + e\left<\phi_0\right>(z)\,, \quad (14)$$

are approximately conserved. Here, $\left<\phi_0\right>$, $\widetilde{\phi_0}$, and $\overline{\phi_0}$ are defined by

$$\left<\phi_0\right>(z) = \overline{\phi_0}(z) - \overline{\phi_0}(0)\,, \quad (15)$$

$$\widetilde{\phi_0}\,(r,z) = \phi_0(r,z) - \left<\phi_0\right>(z)\,, \quad (16)$$

$$\overline{\phi_0}(z) = \frac{\int_0^{r_w} r\phi_0(r,z)dr}{r_w^2/2}\,. \quad (17)$$

For present purposes, we choose the reference distribution function f_0 in the beam frame to be the anisotropic thermal equilibrium distribution

$$f_0 = \frac{\hat{n}}{(2\pi m T_\perp)\,(2\pi m T_z)^{1/2}}\exp\left(-\frac{H_\perp}{T_\perp} - \frac{H_z}{T_z}\right)\,. \quad (18)$$

Here, $T_\perp$ and T_z are the constant transverse and longitudinal temperatures, respectively. The reference density profile $n_0(r,z)$ and reference potential $\phi_0(r,z)$ are determined self-consistently from Eq. (11).

There are two terms that determine the dynamics of w in Eq. (5). The $(df_0/dt)_\delta$ term is explicitly related to the perturbed fields, and the second term $(df_0/dt)_0$ is related to the fact that the reference state f_0 is not an exact equilibrium solution of the Vlasov-Maxwell equations. To carry out the δf particle simulations, we need to calculate the $(df_0/dt)_0$ term first. Some straightforward algebra gives [10]

$$\frac{1}{f_0}\left(\frac{df_0}{dt}\right)_0 = -\frac{\dot{H}_\perp}{T_\perp} - \frac{\dot{H}_z}{T_z} = \dot{H}_z\left(\frac{1}{T_\perp} - \frac{1}{T_z}\right)\,, \quad (19)$$

$$\dot{H}_z = ev_z\frac{\partial\widetilde{\phi_0}\,(r,z)}{\partial z}\,, \quad (20)$$

where super-dot $\dot{()}$ denotes $(d/dt)_0$. For a well-chosen reference state (f_0,ϕ_0), the dynamics associated with $(df_0/dt)_0$ has a longer time-scale for variation than that of $(df_0/dt)_\delta$. Numerical study of the dynamics associated with $(df_0/dt)_0$ can be found in Ref. [10].

We present here initial simulation results for the electrostatic Harris instability driven by large temperature anisotropy in a finite-length charge bunch. The large temperature anisotropy characteristic of charged particle beams in particle accelerators has long been thought to be a possible free energy source to drive collective instabilities. Recently, a systematic study has been carried out for this instability in long coasting beams [9], showing that both sufficiently large temperature anisotropy (small $T_z/T_\perp$) and sufficiently large beam intensity (s_b) are required for instability. The essential physics of this instability is the coupling between the transverse and longitudinal particle

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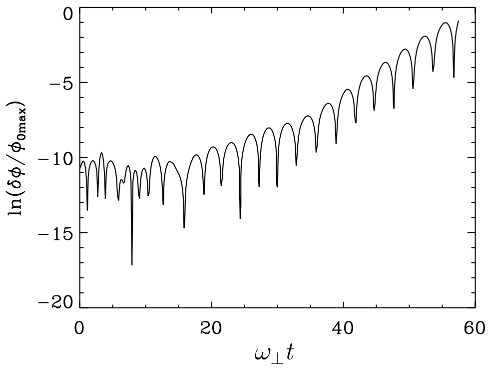

Figure 1: Time history of an unstable perturbation at one spatial location for a high-intensity anisotropic charge bunch with $s_b = 0.8$, $T_z/T_\perp = 1/36$, $z_b/r_b = 15$.

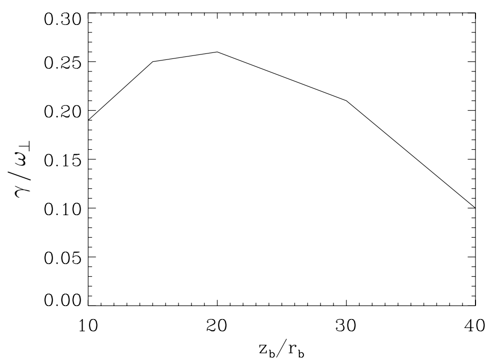

Figure 2: Growth rate γ as a function of bunch aspect ratio z_b/r_b for high-intensity anisotropic charge bunchs.

dynamics. For long coasting beams, the coupling is provided by the wave excitation generated by the instability. For bunched beams, the reference state for a finite-length charge bunch provides an extra channel for the coupling to take place. Indeed, we expect to see additional features of the instability due to the finite bunch length.

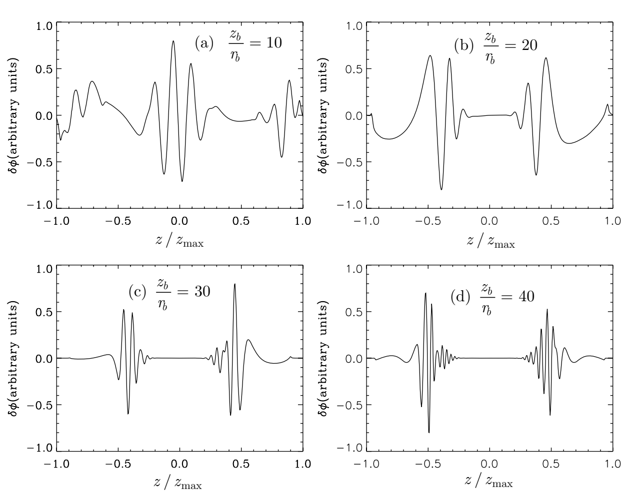

Figure 3: Unstable perturbed potentials $\delta\phi$ as functions of z for different bunch aspect ratios z_b/r_b.

Shown in Fig. 1 is the time history of an unstable, azimuthally-symmetric perturbation relative to the reference state (f_0, ϕ_0) at one spatial location for a high-intensity anisotropic charge bunch with $s_b \equiv 4\pi\hat{n}e^2/2m\omega_\perp^2 = 0.8$, $T_z/T_\perp = 1/36$, and $z_b/r_b = 15$, where z_b and r_b are the RMS half bunch length and radius of the bunch. The instability growth rate is measured to be $\mathrm{Im}\,\omega = \gamma = 0.25\omega_\perp$, and the real frequency is $\omega_r = \mathrm{Re}\,\omega \approx \omega_\perp$. Simulations were performed for different bunch lengths corresponding to $z_b/r_b = 10$, 15, 20, 30, and 40 to investigate the effects of finite bunch length on the instability.

The growth rate γ as a function of bunch aspect ratio is plotted in Fig. 2. The measured growth rates are somewhat larger than those in long coasting beams [9], which can be attributed to the stronger coupling between the longitudinal and transverse dynamics produced by the finite bunch length. The longitudinal structure of the instability shown in Fig. 3 demonstrates interesting variations as well [10]. For $z_b/r_b = 10$, the unstable structure maximizes at the beam center [Fig. 3(a)]. For larger bunch aspect-ratios, the unstable structure localizes symmetrically in the vicinity of $z/z_{\max} = \pm 0.5$ [Fig. 3(d)]. The localization is more prominent for larger bunch length. As $z_b/r_b \to \infty$, the unstable structure becomes highly localized such that the beam intensity is approximately uniform across the unstable structure in the longitudinal direction, and the coupling due to the non-uniformity of the equilibrium in the longitudinal direction is significantly reduced. This explains why the growth rate decreases for increasing bunch length. In addition to the growth rate, the characteristic wavenumber at maximum growth in Fig. 3(d) is $k_z r_b \sim 5.2$, which agrees well with the results obtained from the study for long coasting beams [9].

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