NONLINEAR DYNAMICS OF ELECTROMAGNETIC PULSES IN COLD RELATIVISTIC PLASMAS*

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Abstract

In the present analysis we study the self consistent propagation of nonlinear electromagnetic pulses in a one dimensional relativistic electron-ion plasma, from the perspective of nonlinear dynamics. We show how a series of Hamiltonian bifurcations give rise to the electric fields which are of relevance in the subject of particle acceleration. Nonlinear coupling of plasma waves and electromagnetic pulses triggers strong chaotic dynamics which may detrap the plasma wave from the electromagnetic pulse, leading to wave breaking. Connections with results of earlier analysis are discussed.

INTRODUCTION

Recent SLAC plasma wakefield accelerator experiments [1], operated in a metre-scale plasma, have demonstrated the ultra-high gradients provided by this technology. To maintain these gradients over longer and longer plasmas, it is important to have a deeper understanding on the processes of wakefield destruction (wave breaking).

Using the same model that Kozlov et al. [2] investigated numerically the propagation of coupled electromagnetic and electrostatic modes in cold relativistic electron-ion plasmas and Mofiz & de Angelis [3] applied analytical approximations, we shall construct a canonical representation to examine some key points like the way small amplitude localized solutions are destroyed and when isolated pulses are actually free of smaller amplitude trails (this is related with the existence of wakefields following the leading wave front which is of relevance for particle acceleration). This will be done in association with techniques of nonlinear dynamics [4], since we intend to establish connection between the pulses of radiation and fixed points of the corresponding nonlinear dynamical system (Lichtenberg and Lieberman 1992).

THE MODEL

We follow previous works and model our system as consisting of two cold relativistic fluids: one electronic, the other ionic. Electromagnetic radiation propagates along the z axis of our coordinate system and we represent the relevant fields in dimensionless forms [5]. In addition, we suppose stationary modulations of a circularly polarized carrier wave for the vector potential in the form $\mathbf{A}(z,t) = \psi(\tilde{\xi})[\hat{\mathbf{x}}\sin(kz-\omega t) + \hat{\mathbf{y}}\cos(kz-\omega t)]$ with $\tilde{\xi} \equiv z - Vt$, whereupon introducing the expression for the vector potential into the governing Maxwell's equation one readily obtains $V = c^2 k/\omega$. Manipulation of the governing equations finally takes us to the point where two coupled equations must be integrated [2, 3]:

$$\psi'' = -\frac{1}{\eta}\,\psi + \frac{V_0}{p}\,\psi\left[\frac{1}{r_e(\phi,\psi)} + \frac{\mu}{r_i(\phi,\psi)}\right], \qquad (1)$$

$$\phi'' = \frac{V_0}{p}\left[\frac{(1+\phi)}{r_e(\phi,\psi)} - \frac{(1-\mu\phi)}{r_i(\phi,\psi)}\right], \qquad (2)$$

where the primes denote derivatives with respect to $\xi \equiv (\omega_e/c)\,\tilde{\xi}$, $r_e(\phi,\psi) \equiv \sqrt{(1+\phi)^2 - p(1+\psi^2)}$, $r_i(\phi,\psi) \equiv \sqrt{(1-\mu\phi)^2 - p(1+\mu^2\psi^2)}$, $\eta \equiv \omega_e^2/\omega^2$, $\mu \equiv m_e/m_i$, $V_0 \equiv V/c$, and $p \equiv 1 - V_0^2$, with $\omega_e^2 \equiv 4\pi n_e e^2/m_e$ as the plasma frequency, and $n_e = n_i$ as the equilibrium densities. We further rescale $\omega/ck \to \omega$ and $\omega_e/ck \to \omega_e$ in V_0, η and p, which helps to simplify the coming investigation: η preserves its form, $V_0 \to 1/\omega$, and $p \to 1 - 1/\omega^2$. A noticeable feature of the system (1) - (2) is that it can be written as a Hamiltonian system of a quasi-particle with two-degrees-of-freedom. Introducing the momenta $P_\psi \equiv \psi'$ and $P_\phi \equiv -\phi'/p$, the equations for ψ and ϕ takes the form

$$\psi' = \partial H/\partial P_\psi,\; P_\psi' = -\partial H/\partial\psi, \qquad (3)$$

$$\phi' = \partial H/\partial P_\phi,\; P_\phi' = -\partial H/\partial\phi, \qquad (4)$$

where the Hamiltonian H reads

$$H = \frac{P_\psi^2}{2} - p\,\frac{P_\phi^2}{2} + \frac{1}{2\eta}\,\psi^2 + \frac{V_0}{p^2}\left[r_e(\phi,\psi) + \frac{1}{\mu}r_i(\phi,\psi)\right]. \qquad (5)$$

As we are interested in the propagation of pulses vanishing for $|\xi| \to \infty$, conditions $P_\psi = P_\phi = \phi = \psi = 0$ must pertain to the relevant dynamics, from which one concludes that $E = (V_0/p)^2\,(1 + 1/\mu)$. Considering wave breaking and instability criteria [5], the entire dynamics must evolve within the physical region

$$\sqrt{p(1+\psi^2)} - 1 < \phi < \frac{1}{\mu}[1 - \sqrt{p(1+\mu^2\psi^2)}] \qquad (6)$$

where we will define the limits as ϕ_{min} and ϕ_{max} respectively. Evaluating the linear frequencies of laser and wakefield small fluctuations $\psi'' = \Omega_\psi^2\psi,\ \phi'' = -\Omega_\phi^2\phi$, we have

$$\Omega_\psi^2 \equiv -1/\eta + 1/p\,(1+\mu), \quad \Omega_\phi^2 \equiv (1+\mu)/V_0^2. \qquad (7)$$

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The potential ϕ oscillates with a real frequency Ω_ϕ and, for the vector potential, $\Omega_\psi^2 > 0$ (necessary condition for the presence of instability to reach high-intensity fields from noise level radiation) and, consequently from relation (7),

$$1 < \omega^2 \leq 1 + \omega_e^2\,(1 + \mu), \quad (8)$$

The threshold $\Omega_\psi^2 = 0$ can be rewritten in the form $\omega = \omega_* \equiv \sqrt{1 + \omega_e^2(1 + \mu)}$, where ω_* is the linear dispersion relation for electromagnetic waves. If one sits very close to the threshold, amplitude modulations of ψ are tremendously slow, while the oscillatory frequency of ϕ remains relatively high. This disparity provides the conditions for a slow adiabatic dynamics where, given a slowly varying ψ, ϕ always accommodates itself close to the minimum of

$$U(\phi, \psi) \equiv -V_0/p^2 \left[r_e(\phi, \psi) + \mu^{-1} r_i(\phi, \psi)\right]. \quad (9)$$

When $\psi = 0$, U has a minimum at $\phi = 0$ which is thus a stable point in the adiabatic regime. As one moves away from the threshold, faster modulations and higher amplitudes may be expected to introduce considerable amounts of nonintegrable behavior and chaos into the system. There will be cyclic orbits while ϕ is such that the corresponding potential is not above the level $U(\phi_{min})$. At Fig. 1 the potential $\Delta U \equiv U(\phi, 0) - U(0, 0)$ is represented for $V_0 = 0.99$ and $\mu = 0.0005$, parameters characterizing high-velocity pulses with $U(\phi_{max}) \gg U(\phi_{min})$. Orbits of region I, $\phi_{min} < \phi < \tilde{\phi}$, will oscillate back and forth, but orbits in region II eventually reach ϕ_{min} where $r_e \rightarrow 0$. Since it can be shown that the electronic density depends on r_e in the form $n_e \sim r_e^{-1}$ [2, 3], break down of the theory indicates wave breaking on electrons.

Also shown in the figure is the wave breaking energy $\Delta U(\phi_{min}) \equiv E_{wbr}$ separating regions I and II

$$E_{wbr} = \frac{V_o^2}{p^2}\left[1 + \frac{1}{\mu} - \frac{1}{\mu V_o}\sqrt{(1 - \mu\phi_{min})^2 - p}\right] \quad (10)$$

The same figure suggests how nonintegrability affects localization of our solutions: as one moves away from adiabaticity and into chaotic regimes, trajectories initially trapped by U may be expected to chaotically diffuse towards upper levels of this effective potential, escaping from the trapping region, approaching E_{wbr} and eventually hitting the boundary at ϕ_{min} or, in general, attaining $r_e = 0$ for $\psi \neq 0$. If this is so, we have an explanation on how small amplitude solitons are destroyed, one of the issues of interest in the subject [9].

NONLINEAR DYNAMICS

We introduce our Hamiltonian phase space in the form of a Ponicaré surface of section mapping where the pair of variables (ϕ, P_ϕ) is recorded each time the plane $\psi = 0$ is punctured with $P_\psi < 0$. The Newton-Raphson method was used to locate periodic orbits and evaluate the corresponding stability index α which satisfies $|\alpha| < (>)1$ for stable

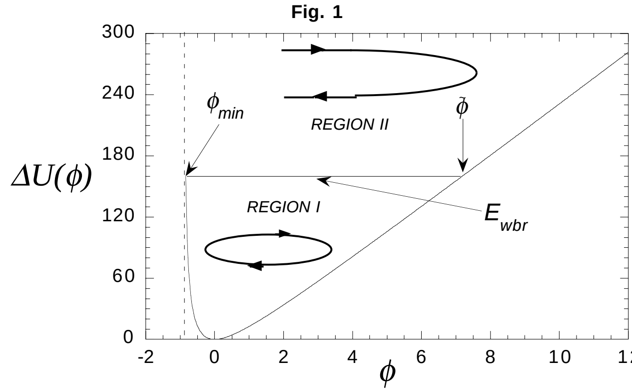

Figure 1: Oscillating (I) and wave breaking (II) regions for the electric potential at $\psi = 0$.

(unstable) trajectories [7]. To have a convenient setting of parameters for fast electron acceleration by wakefields, we shall keep V_o close to the unit, and thus ω slightly larger than one (representing wave modes propagating nearly at the speed of light). After V_o is established, the electron plasma frequency is calculated as $\omega_e^2 = \eta\omega^2$, η satisfying condition (8) again.

In all cases analyzed here we take $\mu = 0.0005$ as in [2] and $V_o = 0.99$ to represent the high speed conditions of wakefield schemes. Since *isolated* pulses cannot be seen in *periodic* plots we alter slightly the energy E to $E = V_0/p^2(1 + 1/\mu)\,(1 + \epsilon)$, $\epsilon \ll 1$ so the vanishing tail $P_\psi = P_\phi = \psi = \phi = 0$ is avoided. With this we convert isolated pulses into trains of *quasi*-isolated pulses. The instability threshold for the vector potential is obtained in the form $\eta_* = p/(1 + \mu) = 0.0198$ so $\omega_e \ll \omega$ as it must be in the underdense plasmas. To investigate the adiabatic regime of the relevant nonlinear dynamics we examine phase portraits for η slightly larger than η_*. In panel (a) of Fig. 2 we set $\eta = 1.00001\,\eta_*$. With such a relatively small departure from marginal stability, modulations are slow with $|\Omega_\phi| \gg |\Omega_\psi|$, adiabatic approximations are thus fully operative and what we see in phase space is just a set of concentric KAM surfaces rendering the system nearly integrable. The central fixed point corresponds to an isolated periodic orbit since it represent a phase locked solution that return periodically to $\psi = 0$, $\phi \rightarrow 0$, and the surrounding curves depict regimes of quasiperiodic, non-vanishing fluctuations of ϕ. Resonant islands are already present but still do not affect the central region of the phase plot where the solitary solution resides. When η grows the behavior of the central fixed point can be observed in terms of its stability index: initially it oscillates within the stable range marking the existence of a central elliptic point near the origin; then, when it reaches $\alpha = +1$, no central orbit is found. This indicates a tangent bifurcation with a neighbouring orbit which terminates the existence of the central point [8]. Immediately after tangency, the phase plot at $\psi = 0$ is still constricted to small values of ϕ as seen in Fig. 2(b) where

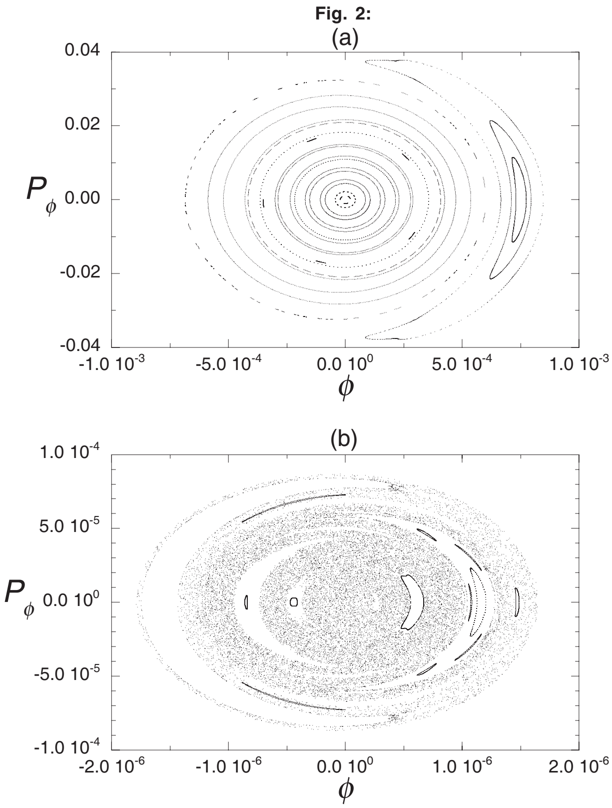

Figure 2: (a) Phase plot near the modulational instability threshold, with $\eta = 1.00001\eta_*$; (b) phase plot after the inverse tangency , with $\eta = 1.0001\eta_*$. $\epsilon = 10^{-11}$.

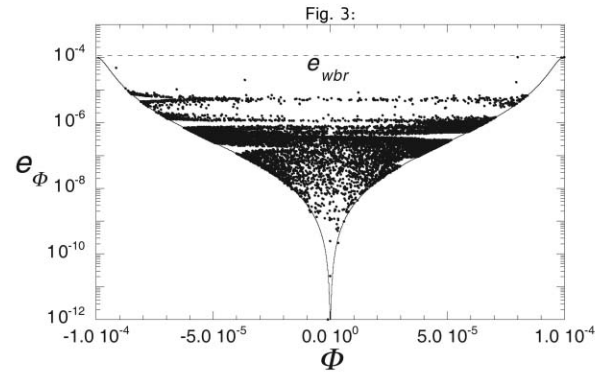

Figure 3: Dynamics as represented in the e_ϕ versus Φ space: $\eta = 1.00021\eta_*$, $e_{wbr} \equiv \chi_e E_{wbr}/(\chi_e + E_{wbr})$.

$\eta = 1.0001\eta_*$. To show that larger values of η cause diffusion towards upper levels of $U(\phi)$, we have investigated the behaviour of the energy $E_\phi \equiv pP_\phi^2/2 + \Delta U$ corresponding to the electrostatic field ϕ, working with the compact variables $e_\phi \equiv \chi_e\, E_\phi/(\chi_e + E_\phi)$ *and* $\Phi \equiv \chi_\phi\, \phi/(\chi_\phi + |\phi|)$, where $\chi_{e,\phi}$ represent the scale above which the corresponding variables are compactified (both setted to 0.0001). For $\eta = 1.00021\eta_*$ as in Fig. 3 , the central fixed point no longer exist. In addition to that, KAM surfaces no longer isolate the central region of the phase plot and diffusion is observed. The quasi-particle moves toward E_{wbr} and eventually arrives at this critical energy producing wave breaking on electrons. At this point the simulation stops with the electron density diverging to infinity. Diffusion is initially slow and becomes faster as energy increases. One sees voids in the diffusion plots which correspond to resonant islands in the phase space, so as diffusion proceeds the quasi-particle escalates along the contours of the resonances that become progressively larger as already mentioned - this is why the process is initially slow, becoming faster in the final stages. For larger values of η no resonance is present and the quasi-particle moves quickly toward E_{wbr}. In case of Fig. 3 one can still see various pulses before wave breaking, but when η is so large that resonances are no longer present, wave breaking can be instantaneous. We finally note the following relevant fact. For $V_o \to 1$, it is known that the amplitude of the electromagnetic pulses are small [9]. But as one goes beyond the adiabatic regime, our discussion on diffusion allows to conclude that even small initial pulses eventually reach very high amplitude values for the plasma waves, which provides the condition for formation of strong electric fields with the corresponding implications on particle acceleration.

We read all these features as it follows. For small enough η's there are locked solutions representing isolated pulses coexisting with surrounding quasiperiodic solutions where ϕ does not quite vanish when ψ does. As η increases past the mentioned tangent bifurcation but prior to full destruction of isolating KAM surfaces, one reaches a regime of periodical returns to $\psi = 0$, although in the presence of a slightly chaotic ϕ motion. Those cases where $\psi = 0$ but $\phi \neq 0$, correspond to quasineutral ψ pulses accompanied by trails of ϕ activity as described in [10] and [11]. We see that trails can be regular or chaotic. Finally, for large enough η's, KAM surfaces no longer arrest diffusion and wave breaking does occur as $r_e \to 0$, as we have checked. At this point adiabatic motion is lost and this is likely to correspond to that point where small amplitude solitary solutions are entirely destroyed as commented in [9] and [6].

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