EFFECT OF AMPLIFICATION OF CHERENKOV RADIATION IN AN ACTIVE MEDIUM WITH TWO RESONANT FREQUENCIES*

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Abstract

Some results of analytical and numerical studies of Cherenkov radiation (CR) in an active medium with two resonant frequencies are presented. It is shown that an unbounded bi-resonant medium amplifies CR even in the case of a purely real refractive index if the lower resonance is active, the higher one is passive and some restrictions on charge velocity are met. It is also demonstrated that amplification of CR from an ultra-relativistic beam is possible if the medium is located in a waveguide.

INTRODUCTION

In recent years, the attention of researchers has been attracted to the so-called "PASER" ("Particle Acceleration by Stimulated Emission of Radiation") effect [1-8]. By using the PASER method, an active medium provides the energy that transforms into the energy of Cherenkov radiation (CR) and then is used to accelerate charged particles in a micro-bunch. Initial theoretical and experimental works concerning the PASER focused on acceleration in gaseous CO_2 and ammonia laser media [1-4]. The first proof-of-principle experiment on direct particle acceleration by stimulated emission of radiation in an active medium has been published recently [3,4]. Research is also proceeding in the area of new materials for use for PASER technique [5,6].

It is necessary to emphasize that active media are characterized by resonant type dispersion. Waveguide structures, which are completely or partially filled with an active dispersive medium having a single resonance frequency, were considered in Ref. [1,2]. It was demonstrated that amplification of CR is possible even if the refractive index of the medium is purely real [1,2]. Here we will demonstrate that amplification of CR in an active medium with purely real refractive index is possible without a waveguide [8]. This phenomenon can take place if the medium possesses at least two resonant frequencies. The case of waveguide loaded with bi-resonant active medium will be briefly considered as well.

AMPLIFICATION OF CR IN UNBOUNDED MEDIUM

Let us assume that a point charge q is moving at the constant velocity $\vec{V} = c\beta\vec{e}_z$. The position of the charge at the moment t is determined by the relations $x = y = 0$, $z = Vt$. The expressions for components of the field in a passive medium are well known (see for example [7-9]). In particular the longitudinal component of the electric field is given by the expression

$$E_z = \frac{q}{2c^2}\int_{-\infty}^{+\infty}\frac{1-n^2\beta^2}{\varepsilon\beta^2}\,\omega H_0^{(1)}(s\rho)\exp\left(i\omega\frac{\zeta}{V}\right)d\omega, \quad (1)$$

where $\zeta = z - Vt$, $\beta = V/c$, $H_\nu^{(1)}(\xi)$ is a Hankel function, and

$$s^2(\omega) = \omega^2 V^{-2}(n^2(\omega)\beta^2 - 1). \quad (2)$$

It is noted that this result may be easily modified for the case of a Gaussian bunch with a charge density $\rho = \frac{q}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\zeta^2}{2\sigma^2}\right)\delta(x)\delta(y)$.

We will be considering a medium having two resonant frequencies ω_{r1} and ω_{r2} ($\omega_{r2} > \omega_{r1}$) and characterized by the following refractive index:

$$n^2(\omega) = 1 + \sum_{j=1}^{2}\frac{\omega_{pj}^2}{\omega_{rj}^2 - 2i\omega_{dj}\omega - \omega^2}. \quad (3)$$

In an active medium at least one of the parameters $\omega_{p1,2}^2$ is negative, i.e. the respective "plasma frequency" is imaginary [1].

In the case of $\omega_{d1} = \omega_{d2} = 0$ one can obtain the following expression:

$$s^2(\omega) = -\frac{1-\beta^2}{V^2}\frac{\omega^2(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}{(\omega^2-\omega_{r1}^2)(\omega^2-\omega_{r2}^2)}, \quad (4)$$

$$\omega_{1,2}^2 = \frac{1}{2}\left[\omega_{r1}^2 + \omega_{r2}^2 - \alpha(\omega_{p1}^2 + \omega_{p2}^2) \mp \sqrt{D}\right], \quad (5)$$

$$D = \left[\omega_{r1}^2 + \omega_{r2}^2 - \alpha(\omega_{p1}^2 + \omega_{p2}^2)\right]^2 - 4\omega_{r1}^2\omega_{r2}^2 + 4\alpha(\omega_{p1}^2\omega_{r2}^2 + \omega_{p2}^2\omega_{r1}^2), \quad (6)$$

$\alpha = \beta^2(1-\beta^2)^{-1}$. If $D > 0$, the values $\omega_{1,2}^2$ are real, and the roots $\omega_1^\pm = \pm\sqrt{\omega_1^2}$, $\omega_2^\pm = \pm\sqrt{\omega_2^2}$ are either purely real or purely imaginary. If $D < 0$, the zeroes have both real and imaginary parts. In this situation the effect of amplification of the electromagnetic field takes place. It can be shown that $D < 0$ only under the following conditions:

$$\omega_{p1}^2 < 0, \quad \omega_{p2}^2 > 0, \quad (7)$$

*Work supported by the US Department of Energy (grant DE-FG02-05ER84355), and the Russian Foundation for Basic Research (grant 06-02-16442-a).
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$$\beta_{\min} < \beta < \beta_{\max}\,, \tag{8}$$

$$\beta_{\substack{\min\\ \max}} = \sqrt{\frac{\omega_{r2}^2 - \omega_{r1}^2}{\left(\omega_{p2} \pm |\omega_{p1}|\right)^2 + \omega_{r2}^2 - \omega_{r1}^2}}\,. \tag{10}$$

We will be considering the case when these conditions are met. In this situation, the dependence of the squared refractive index on frequency is shown in Fig.1. Frequencies of the radiation lie in the range $\omega_{r1} < \omega < \omega_{r2}$. The view of cuts and the integration contour in the right half-plane are shown in Fig. 2 (in the left half-plane the picture is symmetrical with respect to the imaginary axis). The cuts placed on the imaginary axis determine a quasi-static field. Other

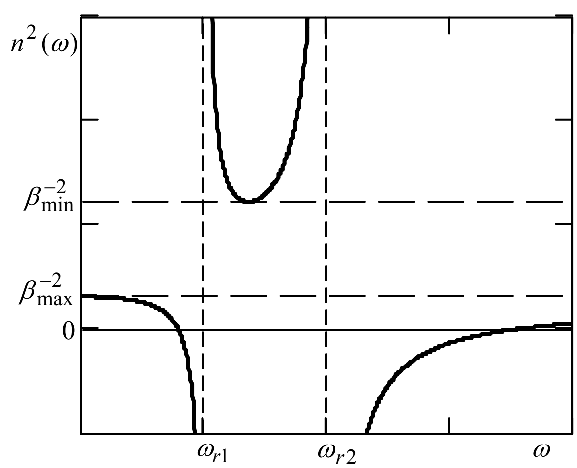

Figure 1: Dependence of the squared refractive index of an active bi-resonant medium on frequency for the case $\omega_{p1}^2 < 0$, $\omega_{p2}^2 > 0$.

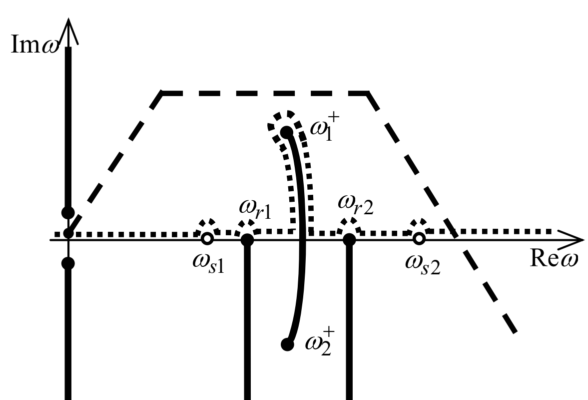

Figure 2: General view of branch cuts (bold continuous lines) and integration contours for the amplification regime ($\beta_{\min} < \beta < \beta_{\max}$). The dotted line shows the initial contour of integration, the dashed line shows the contour for numerical calculation in the domain behind the charge ($\zeta < 0$), and $\omega_{s1,2}$ are zeros of the function $n(\omega)$.

cuts determine a wave field (corresponding to Cherenkov radiation). Furthermore, the integrands may have poles at the points $\pm\omega_{s1,2}$ where $\varepsilon(\omega) = 0$ (for example, in the case when $\mu = 1$, $n^2 = \varepsilon$). These poles determine a so-called "plasma trace" of the source.

The fact that the contour is located above the singularity at ω_1^+ predetermines the effect of amplification of Cherenkov radiation. For computation of the field, it is convenient to replace the initial contour with some broken line placed above all poles and branch points and parallel to the asymptote of the fastest descent contour to infinity (Fig.2).

Some results of the computations are shown in Fig.3.

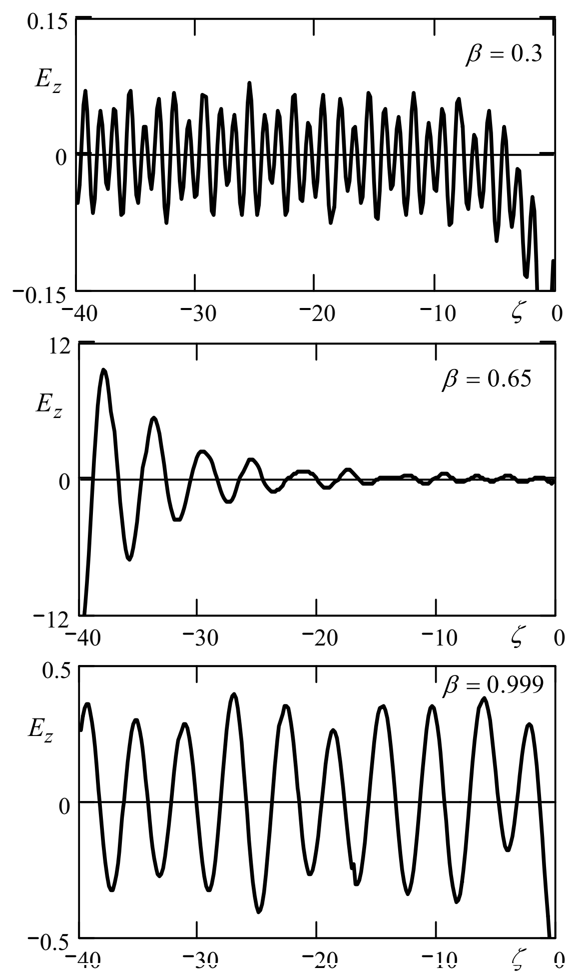

Figure 3: Longitudinal component of the electric field E_z (in units of $q\omega_{r1}^2 c^{-2}$) depending on distance ζ (in units of $c\omega_{r1}^{-1}$) in an unbounded medium. Medium parameters (in units of ω_{r1}) are the following: $\omega_{p1} = 0.5i$, $\omega_{r2} = 1.2$, $\omega_{p2} = 1$, $\omega_{d1} = \omega_{d2} = 10^{-10}$. The beam half-length is $\sigma = 0.1c\omega_{r1}^{-1}$. The distance from the z-axis is $\rho = c\,\omega_{r1}^{-1}$.

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It is assumed that $n^2 = \varepsilon$. It is noted that non-zero values of parameters ω_{d1} and ω_{d2} were taken into account (these values were considered to be very small to demonstrate that their role is not important for amplification effect in the situation under consideration). Using parameters indicated in the caption of Fig.3 we obtain that $\beta_{\min} \approx 0.404$ and $\beta_{\max} \approx 0.799$. In fact, one can see in Fig.3 that the amplification does not take place for $\beta = 0.3$ and $\beta = 0.999$. If $\beta = 0.65$ so we have the maximum amplification. Another example of the CR amplification effect in a bi-resonant medium may be found in Ref. [8].

It is interesting that the velocity range $(\beta_{\min}, \beta_{\max})$ may be narrow enough for some medium parameters [8] that the amplification effect can be used for detection of a charged particle's velocity. In principle, it may be possible to design a differential Cherenkov detector based on an active medium.

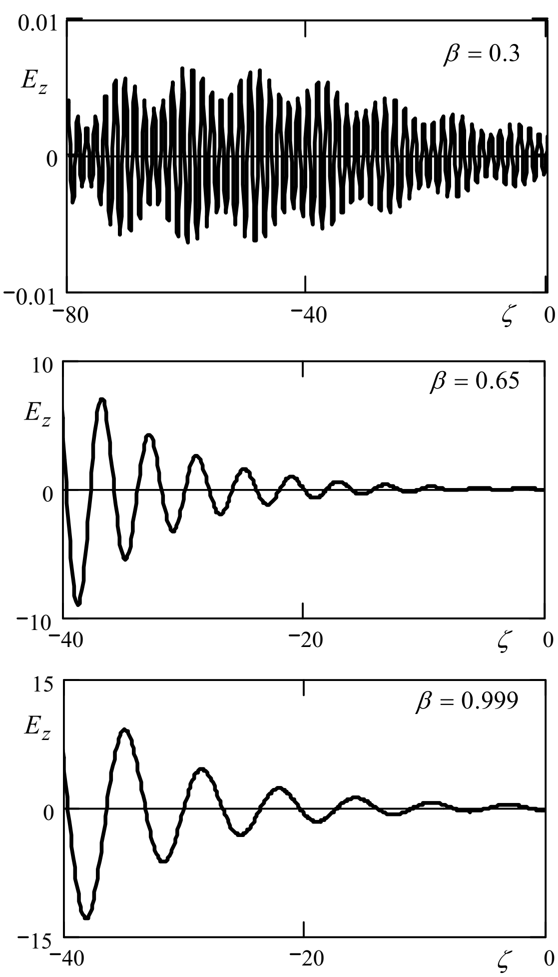

Figure 4: Longitudinal component of the radiated part of electric field E_z (in units of $q\omega_{r1}^2 c^{-2}$) depending on distance ζ (in units of $c\omega_{r1}^{-1}$) in a waveguide with radius $a = 2c\,\omega_{r1}^{-1}$. Other parameters are the same as in Fig. 3.

AMPLIFICATION OF CR IN A WAVEGUIDE

Now we touch very briefly on the case of waveguide with an active bi-resonant medium. In this case a wave field (a field of Cherenkov radiation) can be presented as a sum of modes. We do not taken into account the quasi-static field and the plasma trace, but they are not important for demonstration of CR amplification.

It can be shown that the velocity range for CR amplification in the case of a waveguide is wider than in the case of unbounded medium. In particular, this effect can take place for ultra-relativistic motion of a bunch.

Some examples of wave fields in a circular waveguide with radius a are shown in Fig. 4 (contributions of quasi-static field and plasma trace are not taken into account here). In this situation, the amplification effect takes place in the range from $\beta_{\min} \approx 0.46$ to $\beta_{\max} = 1$. As one can see in Fig. 3, amplification is absent for $\beta = 0.3$, but for $\beta = 0.65$ and $\beta = 0.999$ we have a significant amplification effect. Thus, in contrast to the case of unbounded medium this effect does not vanish for ultrarelativistic motion of the charge and therefore can be used for wakefield acceleration of high-energy charged particles.

The authors would like to thank P. Schoessow, A. Kanareykin, I. Chekhonin, and V. Novikov for useful discussions.

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