# SIMULATIONS OF PARAMETRIC-RESONANCE IONIZATION COOLING*

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*Abstract*

Parametric-resonance ionization cooling (PIC) is a muon-cooling technique that is useful for low-emittance muon colliders. This method requires a well-tuned focusing channel that is free of chromatic and spherical aberrations. In order to be of practical use in a muon collider, it also necessary that the focusing channel be as short as possible to minimize muon loss due to decay. G4Beamline numerical simulations are presented of a compact PIC focusing channel in which spherical aberrations are minimized by using design symmetry.

## INTRODUCTION

PIC [1] takes advantage of an induced half-integer resonance in a ring or beam line where the normal elliptical motion of the particles in $x - x'$ phase space becomes hyperbolic. Thin absorbers placed at the focal points of the channel will then cool the angular divergence of the beam by the usual ionization cooling mechanism. The purpose of the resonance is to cause the particles to move to smaller $x$ and larger $x'$ at the absorber plates as they travel longitudinally down the lattice. (This is almost identical to the technique used for half-integer extraction from a synchrotron where the hyperbolic trajectories go to small $x'$ and larger $x$ to pass the wires of an extraction septum.) With the absorbers properly designed based on the dispersive characteristics of the beamline, the longitudinal emittance can be maintained by emittance exchange, while the transverse emittance is reduced. RF cavities will be used for energy recovery, either after each absorber, or after a series of cells.

The basic theory of PIC is being developed to include aberrations and higher order effects, and an understanding of the basic optical features of an appropriate beamline is required prior to implementing aberration control. Both OPTIM [2] and G4beamline (G4BL) [3] were used to simulate linear channels of alternating dipoles, quadrupoles, and solenoids with a schematic layout shown in Figure 1. Understanding of the non-linear and resonant dynamics will only come from a clear understanding of the linear dynamics of the lattice.

## OPTIM SIMULATIONS

Initially, OPTIM [2] was used to simulate the beamline to determine a basic design and set of starting parameters.

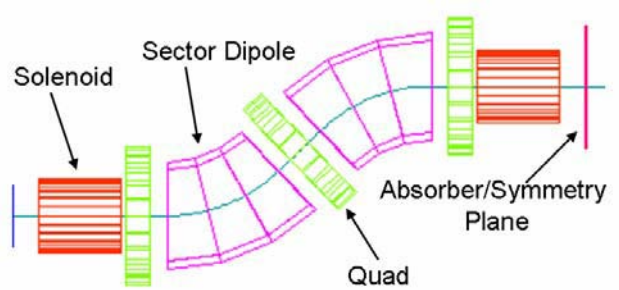


Figure 1:..One cell of the G4BL implementation of a PIC lattice.

Since OPTIM is a linear, transverse matrix model, it is not well suited to understand the lattice beyond the parameter space where each of the elements is linear, compact and non-interacting. Two tracks for the initial simulations were taken. The first used solenoid and quadrupole field values that gave betatron oscillations away from any resonances (including the half-integer resonance). The transverse beta-functions and dispersion are shown in Figure 2. The other line of study focused on field values that tuned the lattice to near the half-integer resonance.

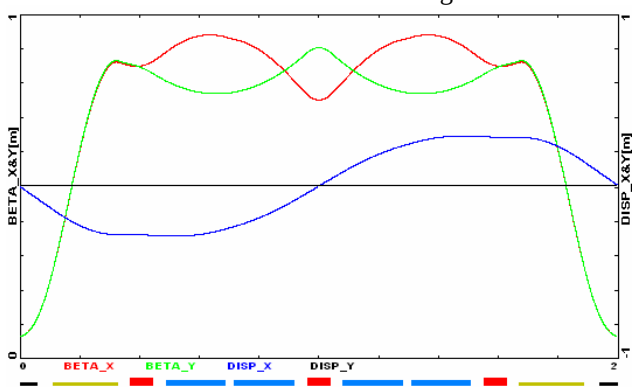


Figure 2: OPTIM simulation result for a single 2 m long "detuned" PIC Cell. The red and green lines are the x and y beta functions, and the blue line is the dispersion in the x direction. The dispersion in the y direction has a value of zero at all positions along the reference orbit.

## G4BEAMLINE SIMULATIONS

Initial studies in G4BL were performed on the detuned lattice as designed with OPTIM. The automatic dipole tuning in G4BL greatly simplified the process of finding the periodic reference particle orbit. Figure 3 shows the results of a G4BL simulation of the "detuned" lattice based on the OPTIM design. The G4BL lattice had dramatically different values for the solenoid and quadrupole fields. These differences are partially because

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G4BL does not yet include end fields on either the sector dipole magnets or on the "generic multipole" magnets that were used to create the quads. Despite the differences in the detailed field values of the beamline elements, the general appearance of the G4BL results look remarkably similar in shape and relative amplitude to the results from the OPTIM simulation.

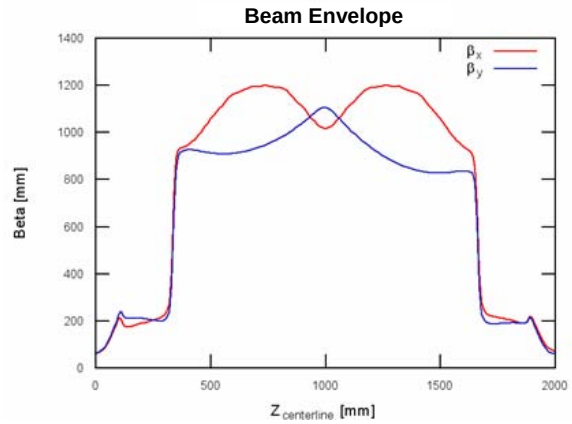


Figure 3: Transverse beam envelopes (expressed as beta functions) of the "detuned" PIC cell as calculated from G4BL simulations.

## *Solenoid Fields*

Initial simulations of the snake lattice lead to two fundamental issues resulting from the solenoid magnets. The use of solenoidal focusing intimately coupled the two transverse directions and the implementation of the solenoids in G4BL lead to long end fields if a coil of any acceptable aperture was used. The coupling problem was removed by using two identical bucked solenoids in a "counterwound" configuration. Basically, the second solenoid reverses any mixing in the first solenoid.

The initial simulations used the "solenoid" element in G4BL that constructs a field map based on adding the field values from many infinitely thin current sheets. This results in a simple to implement magnet that satisfies Maxwell's Equations. Unfortunately, the long end fields also interfere with the vertical dipole fields causing a kick in the particle motion that lead to rapid (within a few cells) ejection from the lattice. In order to better understand the effect of each element, the solenoid was replaced with a field map (see Figure 4) with no radial dependence of the axial field and an extremely short fringe field. While this field profile still satisfied Maxwell's Equations to first order in radius, it was extremely non-physical in the sense that it would be extremely difficult, if not impossible to construct. Since the point of this initial study was to determine the linear lattice and then proceed to investigate aberration control, the idealized solenoid field was not seen as an important issue at this point.

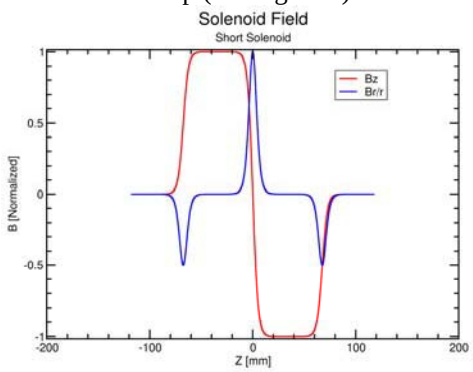


Figure 4: Field map of the "short" solenoid magnet.

## *Monochromatic Transverse Motion*

The modification to the solenoid magnet model increased the allowable size of the magnet, resulting in the ability to model more realistic beam sizes (at the expense of unrealistic magnets). Figure 5 shows the transport line for a "detuned" structure for transverse emittances near the anticipated value. The introduction of absorbers was not performed at this stage because without longitudinal studies, no real understanding of the cooling will be achieved. These cells were tuned to transmit a 1 mm mrad beam while maintaining the spot size, but are displayed with a 200 mm mrad beam. In addition to demonstrating the stability of the lattice, Figure 5 also shows the two anticipated layouts. The "snake" lattice meanders with each cell being a translated version of the previous cell. The "chicane" lattice is named because it appears similar to a beamline chicane, although its purpose is different. These design layouts each have different features, but at the basic level, they are the same. The major differences arise based on the symmetry of the cell layout.

Figure 5: G4BL simulation of a large, monochromatic beam in both the "snake" and "chicane" design layouts (solenoid fields are present as shown in Figure 1, but invisible).

## *Dispersion Effects*

The first place dispersion and the derivative of dispersion (dispersion prime) must be considered is with the inclusion of a momentum spread. An examination of the cell entrance and exit in Figure 2 shows that while there is no dispersion at these symmetry points, there is a definite slope in the dispersion curve. In practice, an entrance/exit cell must be designed to match this lattice to the dispersion-free beamline. In terms of simulations, the input beam must be constructed so that it is matched to this dispersive behavior.

Figure 6 shows the simulation results of 100 particles, all starting with $x = y = x' = y' = 0$ and a momentum spread of ±5%. For this detuned betatron advance, the snake lattice appears more unstable to a mismatch in the dispersion prime as compared to the chicane lattice. As additional cells were added, the loss of particles in the snake lattice disappeared while losses in the chicane lattice began to grow significantly. Although not shown, when the initial value of $x'$ matched the value at the edges of the lattice cell, the snake lattice showed significantly improved performance. Detailed studies of the chicane lattice have yet to be performed using G4BL.

Figure 6: G4BL simulation of the snake and chicane layouts with unmatched dispersion prime.

## SNAKE VS CHICANE LAYOUT

The difference between a snake layout and a chicane layout only appears after the first cell, before then, the two layouts are identical. While this might appear an obvious statement, it is critical to understanding the differences in behavior. The nature of the chicane layout is that of a 2-cell structure, thus, it will have dispersion solutions that are different than the periodic solution in Figure 2. In addition, for the same values of the lattice elements, there are several possible dispersion solutions as more cells are added. Because of a difference in symmetry, one feature of the chicane layout is that the geometric spherical aberration should be mitigated. This aberration will unavoidably arise from the use of dipole magnets, but in a beamline with total reflection symmetry, an exact cancellation can occur. The chicane layout displays this symmetry, while the snake does not.

Figure 7 shows the results of an OPTIM simulation to test this feature. The initial beam (a) is an OPTIM-determined matched beam with a transverse emittance of 100 mm mrad and almost no momentum spread. (b) and (c) show the output from pushing the initial beam through two cells of a snake and chicane (respectively) layout beamline. The output from the snake channel (b) shows a slight bulge on the positive *x* side and several particles straggling out to much larger *x* values. The most notable change is in the *x* phase space where the particles with large *x*-momentum appear to be primarily responsible for the position excursion. This is expected since the aberration is dipole dependent and the bend is only in the *x*-plane. Additionally, the *y*-phase space shows no appreciable change as would be expected with all of the dispersive elements only in the *x*-direction.

The chicane design, however, appears to have significantly canceled the effect of this spherical aberration. While notable deviations between the input and output beams exist in the chicane layout, these would be expected since the effect must be added in the first cell (since it is identical to the first cell of the snake layout), then subtracted in the second cell, the overall affect on the beam profile is significantly reduced.

Figure 7: OPTIM simulation of a matched muon beam through both a snake and a chicane layout of identical lattices. a) Input beam b) output from snake layout and c) output from chicane layout.

## TRANSVERSE PHASE SPACE

Initial investigations into the transverse phase space mapping of this lattice indicate strange behavior. Preliminary analysis shows apparent unstable fixed points at positive and negative values of *x'* and $x = 0$. The phase space map in the *y*-phase space shows identical behavior. This behavior is currently under continued study.

## SUMMARY

Simulations of the PIC lattice in both G4Beamline and OPTIM are progressing. Much of the linear optics of the snake layout have been investigated and are expected to apply directly to the chicane layout. The inclusion of the dispersive effects in the initial longitudinal profile are an important first step in understanding the full 6D motion of these lattices and their ultimate ability to realize the needed cooling factors. Strange transverse phase space behavior requires additional study to if aberration control is to succeed.

## REFERENCES

[1] Yaroslav Derbenev et al., COOL05.
[2] OPTIM, http://www-bdnew.fnal.gov/pbar/organizationalchart/lebedev/OptiM/optim.htm
[3] G4beamline, http://g4beamline.muonsinc.com