

## 6 DIMENSIONAL MUON PHASE SPACE COOLING BY USING CURVED LITHIUM LENSES

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### Abstract

A curved Lithium lens ring model can provide the emittance exchange mechanism in obtaining the muon 6 dimensional phase space cooling. With straight Lithium lenses in a muon cooling ring, only transverse phase space cooling has been demonstrated. We demonstrate the 6 dimensional phase space cooling with various parameters of a muon cooling ring in tracking simulation.

### INTRODUCTION

Lithium lens provides strong magnetic focusing field for the muon transportation as well as one of the ideal energy absorbers with the ionization energy loss for the ionization cooling with relatively small amount of multiple scattering through the system. Transverse muon phase space cooling has been demonstrated in a simulation model with straight Lithium lenses, matching solenoid section, and edge focused dipole magnets. [1, 2] Figure 1 shows a schematic diagram of a section of a curved Lithium lens ring, which consists of a curved Lithium lens and a RF cavity gap. A RF cavity gap compensates the longitudinal momentum loss of muons through the Lithium lens. Parameters of this muon cooling ring are muon momentum, the total current in the Lithium lens, RF vacuum gap length, RF frequency, RF gradient, RF offset phase angle, a diameter of the Lithium lens, and the radius of the curved Lithium lens muon cooling ring. A curved Lithium lens ring provides 6 dimensional muon phase space cooling through the emittance exchange mechanism. [3]

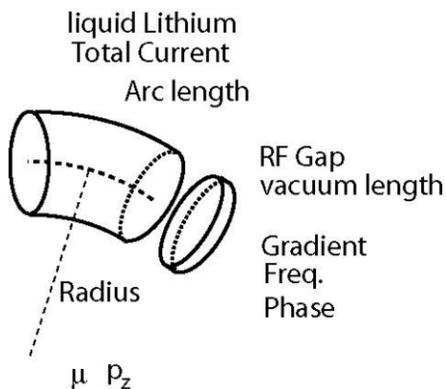


Figure 1: A schematic diagram of a section of a curved Lithium lens ring which consists of a curved Lithium lens(left) and a RF gap(right).

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### 6 DIMENSIONAL MUON PHASE SPACE COOLING

In order to realize the 6 dimensional muon phase space cooling in a muon cooling ring, we must use the emittance exchange mechanism between the transverse phase space and the longitudinal phase space by using wedge energy absorbers in dispersive regions in a muon cooling ring. A muon with higher longitudinal momentum loses more longitudinal momentum through a wedge-shaped energy absorber in a dispersive region, which provides the reduction of the width of muon longitudinal momenta (in the longitudinal phase space) and the increase of the transverse phase space by changing the orbits of muons in a muon cooling ring (in transverse phase space). The cross-relation between the longitudinal emittance and the transverse emittance is the emittance exchange. The function of the wedge energy absorber in dispersive regions is the same in the gas-filled muon cooling ring or in a muon cooling ring made of curved Lithium lenses where the path length of high longitudinal momentum muon is longer than those with lower longitudinal momentum, if the beam transverse size in the ring is relatively large compared to the radius of the muon cooling ring with curved Lithium lenses.

The  $dE/dx$  straggling of muons through an energy absorber and the slope of the  $dE/dx$  as a function of muon longitudinal momentum gives the heating effect in the longitudinal muon phase space, and the multiple scattering through an energy absorber gives the heating effect in the transverse muon phase space. Major muon phase space cooling is done in the ionization cooling in the transverse muon phase space where only the average longitudinal muon momentum loss is compensated by an RF gap after the total muon momentum loss through absorbers, which results in the reduction of the muon angle distribution in the muon transverse phase space. Wedge absorbers cause the longitudinal phase space cooling through the reduction of the width of the muon longitudinal momentum. By selecting the proper absorber material, liquid Lithium in this case, the current strength through the Lithium lens which determines the beam beta, and the radius of the muon cooling ring, we can obtain the 6 dimensional muon phase space cooling through a curved Lithium lens ring.

A tracking simulation code for the muon ionization cooling, ICOOL [4], was used to build a model, and to study the development of the transverse and longitudinal phase space of the muon beam through the muon cooling ring.

### TRACKING SIMULATION

The average muon longitudinal momentum was set at 250 MeV/c. The radius of the muon cooling ring with curved Lithium lenses are in the range of 16 cm and 64 cm. A unit length of the curved Lithium lens was 10 cm and the gap length of the RF was 0.5 mm. The total current through the curved Lithium lenses was adjusted to provide the beam beta at 10 cm, which gives Bp at 0.8 Tesla at a radius at 1 cm from the center of the curved Lithium lens. The RF frequency was 100 MHz, and the RF phase offset was 20 degrees. The RF gradient was adjusted to compensate the average loss of the muon longitudinal momentum through the RF gaps.

Figure 2 shows the horizontal(top) and vertical(bottom) normalized emittance as a function of z along the center of the curved Lithium lens ring with various radii. At z=0 the muon beam is cold, which means that the phase space is zero in the 6 dimension. The normalized emittance developed to reach 600 mm\*mrad in both horizontal, and vertical phase space at z around 5 m with the ring radius at 32 cm and at 64 cm. Due to the loss of muon beam in the curved Lithium lens ring with the radius at 16 cm, both the transverse and vertical normalized phase space go beyond 600 mm\*mrad at z after 3 m.

Figure 3 shows the transmission (top) and longitudinal normalized emittance (bottom) as a function of z. Transmission without decay is as high as around 99% which is expected with the simple toy model of the curved Lithium lenses with the ring model with radii at 32 cm and at 64 cm. At the ring radius at 32 cm, the normalized longitudinal phase space reach to the equilibrium phase space with z at around 5 m, which shows the effect of the emittance exchange between the transverse phase space and the longitudinal phase space.

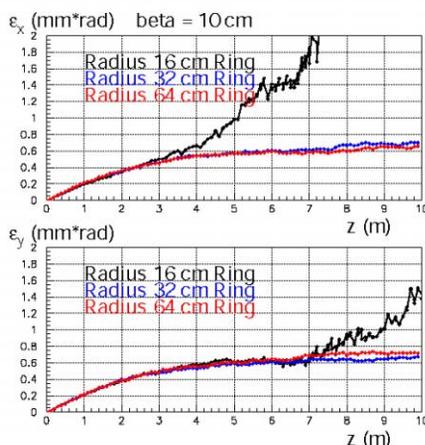


Figure 2: Horizontal(top) and vertical(bottom) normalized emittance as a function of z.

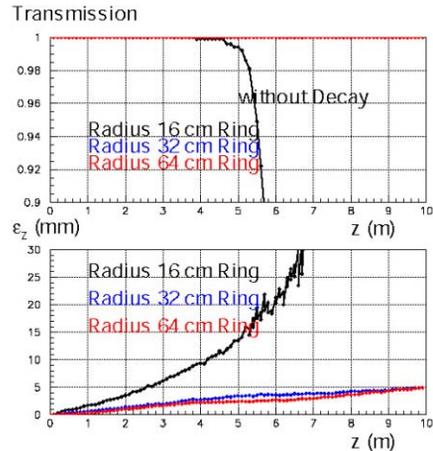


Figure 3: Transmission (top) and longitudinal normalized emittance(bottom) as a function of z.

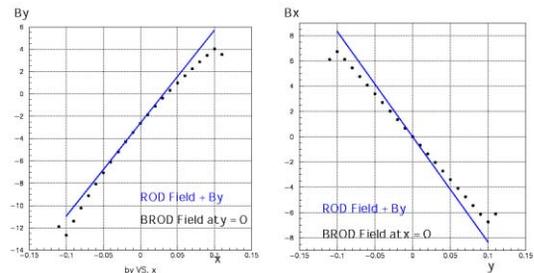


Figure 4: Vertical magnetic field at the horizontalPlane (left) and horizontal magnetic field at thevertical plane (right) through the center of the curvedLithium lenses

Figure 4 shows the vertical magnetic field at the horizontal plane (left) and horizontal magnetic field at the vertical plane (right) through the center of the curved Lithium lenses. A superimposed magnetic bending field was used in this simulation, but more accurate magnetic field for the bent current rod is available in this simulation code.

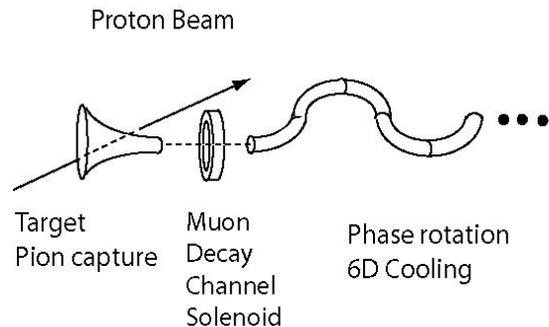


Figure 5: A schematic diagram of the muon front end channel starting with a tapered Lithium lens target, a muon decay channel, and a single path muon cooling channel with curved Lithium lenses.

Figure 5 shows a schematic diagram of a muon front end channel starting with a tapered Lithium lens target, a muon decay vacuum channel with a matching solenoid coil, and a single path muon cooling channel with continuous half circle of the curved Lithium lenses which eliminates the injection and extraction section which must be used in the curved Lithium muon cooling ring.

### CONCLUSION

With the muon beam beta at 10 cm and the with the radius of the curved Lithium lens ring at 32 cm, we demonstrated that both the transverse and longitudinal normalized phase space reached to equilibrium phase space values through a muon cooling ring. This indicates that the longitudinal phase space cooling is working through the emittance exchange mechanism which strength is larger than the heating of the longitudinal phase space due to the straggling of the muon ionization energy loss through curved Lithium lenses and the slope

of the  $dE/dx$  as a function of the longitudinal muon momentum at 250 MeV/c.

### REFERENCES

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