BEAM-BEAM EFFECTS IN CRAB CROSSING AND CRAB WAIST SCHEMES

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Abstract

To boost up the luminosity performance in B factories, crab crossing and crab waist schemes are proposed. The crab crossing scheme compensates crossing angle, while the crab waist scheme compensates nonlinear tems induced by crossing angle with sextupole magnets. We discuss which nonlinear terms in the beam-beam map are enhanced by the crossing angle and which terms are compensated by the crab waist sextupole.

INTRODUCTION

We discuss the transfer map of the beam-beam interaction with or without crossing angle. Crab crossing and crab waist schemes have been proposed to improve the beam-beam performance for a finite crossing angle. The crab cavity realizes the collision without crossing angle effectively [1], while the crab waist scheme controls the vertical waist position along x so as to match the longitudinal axis of colliding beam [2]. The crab cavity or sextupole magnets are used for the crab crossing or the crab waist scheme, respectively. The transfer map represented by the Taylar expansion is obtained for the collision with or without crossing angle in this paper. The transfer map with applying the crab waist sextpoles is also obtained. We study which nonlinear terms are varied for the crossing angle, and how the nonlinear terms behave for the crab waist sextpole.

TREATMENT OF THE BEAM-BEAM INTERACTION

The Beam-beam interaction is expressed by a transfer map at the collision point as follows,

$$\boldsymbol{x}(+0) = S\exp\left[-:\int_{-\Delta}^{\Delta} V_0^{-1}(s)H_{bb}V_0(s)ds:\right]\boldsymbol{x}(-0), \tag{1}$$

where S means s ordered product and $\pm\Delta$ is the interaction region of the two beams. [1] V_0 is the transfer map in the drift space,

$$\begin{aligned} V_0(s) &\equiv V_0(s,0) = S\exp\left[-:\int_0^s H_0 ds:\right] \\ &= \exp\left[-:\frac{p_x^2+p_y^2}{2}s:\right], \end{aligned} \tag{2}$$

[1] S ordered product is the same concept as T ordered product popularly used.

where $p_{x(y),\pm}$ is the momentum of positrons/electrons. Note that $:p_x:x = [p_x, x] = -1$ and $:p_x:x = 0$, where [ ] is the Poisson bracket. The drift map V_0 can be replaced by the map in the solenoid magnet as the need arises. H_{bb} is a term which represents the beam-beam interaction. The relativistic beam induces an electro-magnetic field in the transverse plane. The field can be expressed by a two-dimensional static potential. The other beam experiences the electro-magnetic field. We use the weak-strong model for the beam-beam interaction: i.e., the Hamiltonian H_{bb} is expressed by a static potential as $H_{bb} = \phi(\boldsymbol{x})$.

For a Gaussian beam, the transfer map for ϕ as a function of s [3],

$$\begin{aligned} &exp(-:\phi_\pm(x,y,z;s):)(p_y+ip_x) \\ &\quad= \frac{N_\pm r_e}{\gamma}\sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}}\left[w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right. \\ &\qquad\left. -\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right)w\left(\frac{\frac{\sigma_y}{\sigma_x}x+\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right] \end{aligned} \tag{3}$$

Since ϕ is also a function of z, a kick for p_z [4] arises. The beam envelope matrix is deformed due to the beam-beam interaction at each integration step. For an arbitrary s in an integration step, the rms beam sizes or more generally beam envelope matrix is transferred by

$$\langle\boldsymbol{x}(s')\boldsymbol{x}^t(s')\rangle = V_0(s',s)\langle\boldsymbol{x}(s)\boldsymbol{x}^t(s)\rangle V_0^t(s',s), \tag{4}$$

where $V_0(s',s)$ is the transfer matrix for s to s'.

The complex error function is expanded by Tayler polynomial as

$$w(z;\sigma_x,\sigma_y) = e^{-z^2}(1-erf(-iz)) \tag{5}$$

$$erf(z+z_0) = erf(z_0) + \frac{2}{\sqrt{\pi}n!}\sum_{n=1}(-1)^{n-1}H_{n-1}e^{-z_0^2}z^n \tag{6}$$

where $H_n(z)$ is the Hermite polynomial, and $z = x+iy$.

The coefficient of the polynomial during the whole interaction 1 is integrated along z, with the result that Taylar expansion of the beam-beam map is obtained.

The crossing angle is approximately represented by a transformation before and after the beam-beam interaction,

$$exp(\mp:\theta p_x z:)(x,p_z) = (x+\pm\theta z, p_z\mp\theta x). \tag{7}$$

The crab cavity gives the same transformation as Eq.(7). The transformations of Eq.(7) and crab cavity can be an approximately identical transformation by choosing the voltage as

$$V = \frac{cE\tan\theta}{\omega_{RF}\sqrt{\beta_{x,crab}\beta_x^*}}. \qquad (8)$$

where ω_{RF} is frequency of the crab cavity.

The map is factorized as [5]

$$\exp(- : F_2 :)\exp(- : F_{\leq 3} :), \qquad (9)$$

where $\exp(- : F_2 :)$ and $\exp(- : F_{\geq 3} :)$ are the linear and nonlinear maps, respectively. The linear map, which is represented by a matrix transformation, includes the tune shift of the beam-beam interaction. The nonlinear map $F_{\geq 3}$ is a polynominal higher than 3-rd order for dynamical vaiables. The terms contributes resonance, for example $x^n y^m z^k$ term $n\nu_x + m\nu_y + k\nu_s$ =integer.

TRANSFER MAP FOR CROSSING ANGLE AND CRAB WAIST

Low beta option for KEKB is evaluated. The parameters are the energy $E = 3.5$ GeV, the beta functions at collision point $\beta_{x/y/z} = 0.2/0.003/5.7$ m, and the emittances $\varepsilon_{x/y/z} = 1.8\times 10^{-8}/1.8\times 10^{-10}/2.8\times 10^{-6}$. The bunch poplation of the interacting beam and the half crossing angle are varied $N = 1 - 10\times 10^{10}$ and $\theta = 0 - 22$ mrad, respectively. Piwinsky angle, the ratio of the crossing angle and beam aspect ratio in $x - z$ plane, $\theta\sigma_z/\sigma_x = 0 - 1.5$.

Figure 1 shows the coefficients of nonlinear terms of $F_{\geq 3}$. The number with 6 digit "abcdef" is that of $X^a P_X^b Y^c P_Y^d Z^e P_Z^f$, where the varables are normalized by β function: i.e., $X = x/\sqrt{\beta_x}$ and so on. The five lines correspond to the different bunch poplation for the interacting beam, $N_b = 1, 2, 3, 4$, and 5.5×10^{10}. The nominal beam-beam parameter is $\xi = 0.1N_b/10^{10}$. Plots (a) depict the coefficient of X^4 term, which is one of the dominant term for the head-on collision. The coefficient decreases as increasing the crossing angle. The terms for $X^2P_X^2$ and P_X^4 is neglisible small compare than the x^4 term, because the beam-beam force is a function of X, but is not that of P_X. These terms give perturbation

$$F_{400} = \frac{a_{400000}}{8}J_x^2(3 + 4\cos 2\phi_x + \cos 4\phi_x) \qquad (10)$$

The three terms of F_{400} give an amplitude dependent tune shift, and contribute 2-nd and 4-th order resonances, respectively. For the present, if we use the tune shift, the resonance width, which characterize its strength Other nonlinear terms give tune shift. In the head-on collision, only symmetric terms, which are even order for x and y, appear in the transfer map. Only a_{400000} in x^4 terms is significant, while all terms a_{004000}, a_{002200} and a_{000400} in y^4 are significant. It is due to that the betatron phase variation is meaningful for the vertical motion ($\beta_y \approx \sigma_z$).

For increasing the crossing angle, asymmetric terms, which are odd order, appear and increase. To compare the values of each coefficient, they have to be normalized by the emittances: i.e., $a_{abcdef}\varepsilon_x^{-(a+b)/2}\varepsilon_y^{-(c+d)/2}\varepsilon_z^{-(e+f)/2}$.

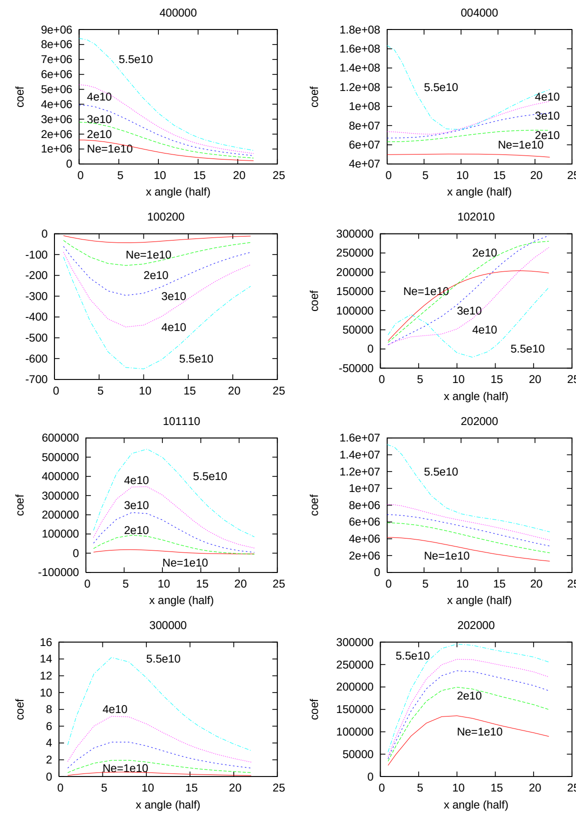

Figure 1: Coefficients of nonlinear terms as functions of the crossing angle. The nonlinear term is printed in each plot.

The crab waist scheme is

$$exp(\mp : \frac{K_w}{2}xp_y^2 :)(p_x, y) = (p_x \pm \frac{K_w}{2}p_y^2, y \mp K_w x p_y) \qquad (11)$$

The transformation is realized by putting sextupole magnets an integer or a half integer betatron phase difference in horizontal, and a qurter integer (1/4 or 3/4) in vertical. This transformation shifts the waist position of the vertical beta function as $K_w x$. The essentials of the crab waist scheme is to choose $K_w \approx 1/2\theta$.

Figure 2 shows the coefficients as functions of the strength of the crab waist sextupole magnets. The terms of X^4, X^3 and X^3Z are not affected by the crab waist sextupole, while the coupling terms XY^2 and XY^2Z clearly depend on the sextupole strength.

CONCLUSION

Coefficient of the nonlinear map for the beam-beam ineteraction was obtained with or without crossing angle.

Odd order terms for x and y are appear and enhanced by the crossing angle. These terms seem to degrade the lumi-

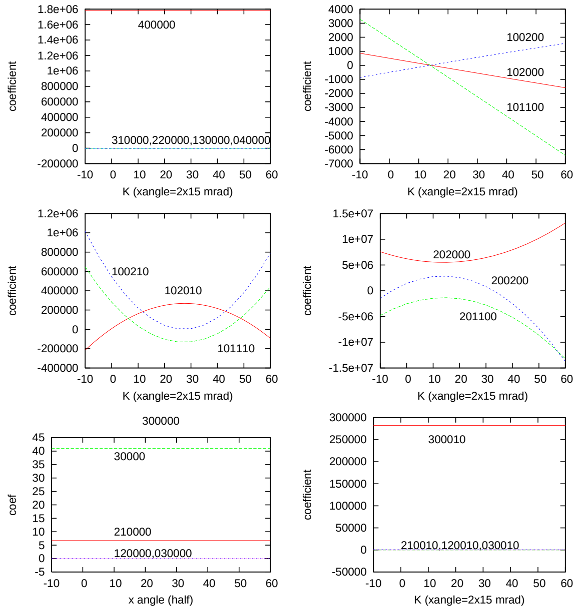

Figure 2: Coefficients of nonlinear terms as functions of the strength of the sextpole strengths.

nosity performance for the collision with a finite crossing angle.

Behaviors of the nonlinear terms are investigated for the strength of the crab waist sextupole. The crab waist sextupoles cancel the nonlinear terms related to xy^2 as is expected, while remain the terms related to the synchrobeta resonances.

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