BEAM-BEAM EFFECT WITH AN EXTERNAL NOISE IN LHC

K. Ohmi[1] *, R. Calaga, R. Tomas, W. Hofle, F. Zimmermann
[1]KEK, Tsukuba, Japan
CERN, Geneva, Switzerland

Abstract

Proton beam do not have any damping mechanism for an incoherent betatron motion. A noise, which kicks beam particles in the transverse plane, gives a coherent betatron amplitude. If the system is linear, the coherent motion remains in an amplitude range. Nonlinear force, beam-beam and beam-electron cloud interactions, causes a decoherence for the betatron motion with keeping an amplitude of each beam particle, with the result that an emittance growth arises. We focus only a fast noise, the correlation time is 1-100 turns. Slower noise is less serious, because it is regarded as an adiabatic change like closed orbit change. As sources of the noise, we consider the bunch by bunch feedback system and phase jitter of cavities which turns to transverse noise via Crab cavity.

Table 1: Basic parameters of LHC

variable	symbol	nominal	upgrade
circumference	L	26,658 m	
beam energy	E	7 TeV	
bunch population	N_b	1.15×10^{11}	1.7×10^{11}
half crossing angle	θ	0.14 mrad	0.22 mrad
beta function at IP	$\beta^*_{x,y}$	0.55 m	0.25 m
emittance	ε_r	5.07×10^{-10} m	
beam-beam tune shift	ξ	0.0033	
bunch length	σ_z	7 cm	3.78 cm
synchrotron tune	ν_s	0.0019	
betatron tune	$\nu_{x(y)}$	63.31/59.32	
revolution frequency	f_0	10^9/day	

INTRODUCTION

An external noise, which kicks beam in transverse, induces an offset on the beam-beam collision. When the centroids between two colliding beams deviate an amplitude δx at the collision point, the luminosity degrades geometrically as

$$L(\delta x) = L_0 \exp\left(-\frac{\delta x^2}{2\sigma_x^2}\right) \quad (1)$$

where L_0 and σ_x are the luminosity without deviation and the beam size. When δx fluctuates with a rms value, $\langle \delta x^2 \rangle$, an averaged luminosity is given by

$$\langle L \rangle = L_0 \left(1 - \frac{\langle \delta x^2 \rangle}{2\sigma_x^2} \rangle\right) \quad (2)$$

The degradation is negligible for $\delta x/\sigma \ll 1$.

As everybody know, the beam-beam interaction is strongly nonlinear. The collision offset caused by the noise gives an diffusion of particle motion[1], and induces a coherent oscillation between two beams. The coherent motion is transferred to emittance growth due to its smear out[2]. We treat an emittance growth and luminosity degradation induced by an external noise in the beam-beam interaction. The emittance growth is analyzed by the weak-strong and strong-strong models, in which beam particles moves in a potential given by colliding beam as a fixed charged distribution, and two beams moves with interacting each other, respectively.

Parameter of LHC is shown in Table 1.

NOISE SOURCES

The crab cavity can be a source of diffusion. Since the crab cavity is operated by a transverse mode, the deviation and jitter of RF phase give a dipole kick to the beam, with the result that transverse offset at the collision point is generated. Both of phases of main RF and crab cavity can be source of the transverse offset.

Jitters of RF phase of main cavity causes a deviation of timing of beam arrival at the crab cavity. The transverse offset, which arise from the jitter of main RF system, is expressed by

$$\delta x = \frac{c \tan\phi}{\omega_{RF}} \delta\psi_{RF}. \quad (3)$$

where $\delta\psi_{RF}$ is the phase error of the main RF system.

The crab cavity gives a transverse kick due to its jitters of RF phase, with the result that the offset given by the kick is expressed as follows,

$$\delta x = \frac{c \tan\phi}{\omega_{RF}} \frac{\cos[\pi\nu_x - \Delta\Psi(s^*, s_c)]}{2\sin\pi\nu_x} \delta\psi_{crab}, \quad (4)$$

where $\Delta\Psi(s^*, s_c)$ and $\delta\psi_{RF}$ are the betatron phase difference between the collision point and the crab cavity and the deviation of the RF phase of the crab cavity, respectively. In the both cases, the jitter of transverse offset is given by $\delta x \approx c \tan\phi \delta\psi/\omega_{RF}$.

Transverse bunch by bunch feedback system damps a transverse dipole motion of the beam. Read error of the position monitor and kicker noise give a transverse kick on the beam. Betatron amplitude is transferred by the feedback system as follows,

$$\begin{aligned} X_{i+1} &= X_i - G(X_i + \delta X_{mon}) + \delta X_{kick} \\ &= (1-G)X_i - G + \delta X_{mon} + \delta X_{kick} \quad (5) \end{aligned}$$

where G, δX_{mon} and δX_{mon} are the feedback damping rate, kicker noise and monitor noise, respectively. The fluctuation of the betatron amplitude is given by

$$\langle X^2 \rangle = \frac{1}{2G}\left(G^2\langle X^2_{mon}\rangle + \langle X^2_{kick}\rangle\right). \quad (6)$$

These fluctuations induce a diffusion in betatron amplitude and an excitation of a coherent beam-beam mode.

Two type of implementation for noise are installed in simulation codes. First type of noise is expressed by a transformation as follows,

$$T_1(-\delta)\exp(-U_{col})T_1(\delta)M_0 \quad (7)$$

where

$$T_1(\delta)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+\delta_x \\ y+\delta_y \end{pmatrix}. \quad (8)$$

δ's are random variables, which are common values for every macro-particles. RF feedback system of accelerating and crab cavities is closed and is little influenced by the beam-beam interaction. We use this transformation for studying a crab cavity type of noise.

Bunch by bunch feedback system kicks the beam to reduce its coherent betatron amplitude. Beam-beam interaction can influence the feedback system. The transformation of the bunch by bunch feedback system is characterized by two variables, damping rate and fluctuation, as follows,

$$\exp(-U_{col})T_2(\delta,\tau)M_0 \quad (9)$$

where

$$T_2(\delta,G)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-G\langle x\rangle+\delta_x \\ y-G\langle y\rangle+\delta_y \end{pmatrix}. \quad (10)$$

Actually there is no difference between the two type of noises. The crab cavity also equips a feedback loop to stabilize the phase.

SIMULATION OF BEAM-BEAM INTERACTIONS WITH THE NOISES

We execute a weak-strong and strong-strong simulations to study the noise effects. The strong-strong simulation contains a numerical noise due to the statistics of macro-particles. Indeed the dipole moment fluctuates $\sigma/\sqrt{N}$ turn by turn. The simulation gives an artificial emittance growth due to the numerical noise; the external noise less than the numerical noise is not visible. The number of particles should be increased according to the noise level and emittance growth rate to be studied.

Crab cavity type of noise

We first discuss the beam-beam effect with the noise given by Eq.(8). Figure 1 shows the evolution of emittance and luminosity for various noise amplitude given by the weak-strong simulation. The emittance growth rate and luminosity decrement, which are estimated in Figure 1. In the weak-strong simulation, macro-particles moves in a static potential. Nonlinear beam-beam force (potential) can cause emittance growth even if no noise. Since the beam-beam tune shift is rather small ($\xi = 0.0033$), the motion is near solvable, therefore emittance growth is very weak, $< 10^{-10}$. For increasing noise amplitude, the emittance growth and luminosity decrement become visible in the simulation. Since the revolution frequency is 10^9 par day, the luminosity decrement 10^{-9} corresponds to 1 day life time. The noise level, δx =0.1-0.2%, is limit for 1 day luminosity life time.

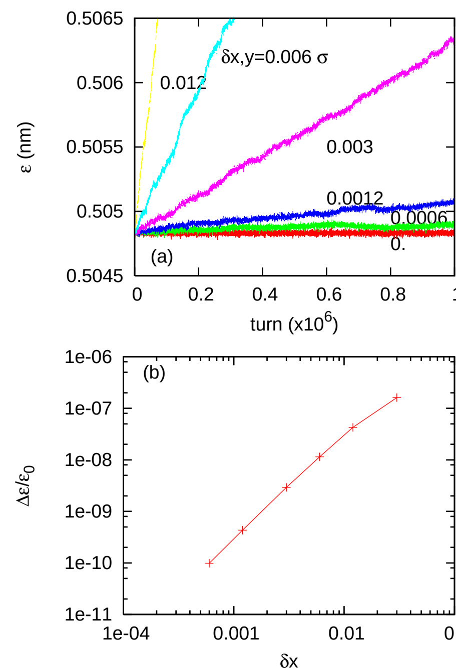

Figure 1: Emittance growth due to noise given by a weak-strong simulation. Plots (a) and (b) depict the evolutions of emittance for various noise amplitude and their emittance growth rate, respectively.

In the weak-strong simulation, the noise does not induce a coherent beam-beam mode. The effect due to excitation of a coherent motion is estimated by the strong-strong simulation. The simulations were done with 1,000,000 macro-particles. The simulation contains the intrinsic error due to the statistics of the number of macro-particles [3]. Indeed the dipole moment fluctuates 0.1% due to this statistics. The strong-strong simulation gives an emittance growth rate 0.8×10^{-9} without fluctuation. The weak-strong simulation gives the emittance growth rate 0.4×10^{-9} for the fluctuation of 0.12% as shown in Figure 1. The emittance growth is coincide within the factor 2. This means the emittance growth in the strong-strong simulation is caused by the numerical noise of 0.1%. Strong-strong simulation with less macro-particle does not have an ability of the prediction for the luminosity decrement 10^{-9}.

Figure 2 shows the emittance growth given by the strong-strong simulation. Plot (a) informs an oscillation and growth of the emittance which indicate an excitation of a coherent motion. The growth rate is summarized in Plot (b), where τ_{cor} is the correlation time (turn) of the fluctuation ($\tau_{cor} = 1$ is default in this paper). The growth rate at $\delta x/\sigma = 0.12\%$ is comparable with that without fluctuation. The fluctuation is close to the numerical noise level of the macro-particle. These results inform the fluctuation with 1% of the beam size is critical for one day luminosity life time. The emittance growth is somewhat higher than that given by the weak-strong simulation. Coherent motion may affect the growth. The growth for fluctuation with 100 correlation time is also plotted. The tolerance of the fluctuation looses with $\sqrt{\tau_{cor}}$.

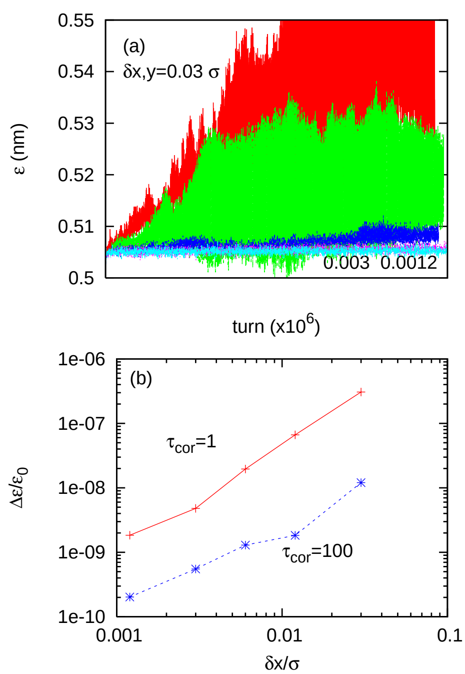

Figure 2: Emittance growth due to the fluctuation given by the strong-strong simulation. t_{cor} is the correlation time (turn) of the fluctuation.

Noise of bunch by bunch feedback system

A simulation has been performed for the second type of noise. An excitation of the beam-beam mode is the source of the emittance growth, therefore strong-strong simulation is essential for this study.

Figure 3 shows the emittance growth for the noise. Plot (a) depicts the evolutions of emittance for various feedback gain with the kick noise $\delta x = 0.02\mu$m (0.12% of σ). Emittance growth is seen, but coherent motion seems to be suppressed by the feedback system. The simulations were performed for several higher δx, and the growth rates are summerized in Plot (b). The growth rate well agree with an analytical estimate [2].

Resolution of position monitors also affect the beam fluctuation in Eq.(6). Emittance growth is slower at a higher gain for given kicker noise, while it is faster for given monitor resolution.

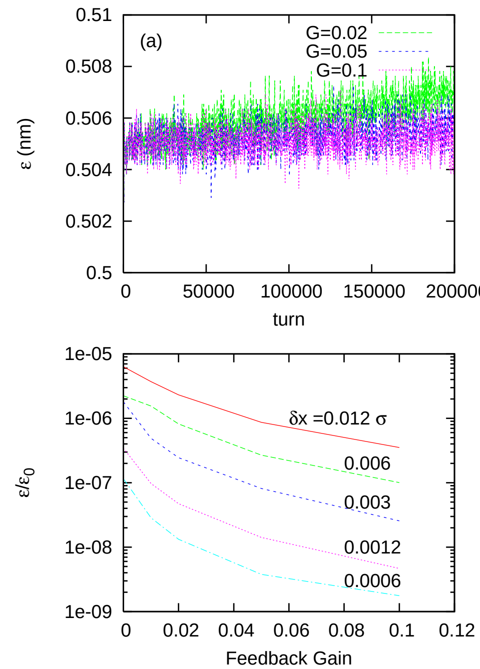

Figure 3: Evolutions of Dipole moment (a) and emittance (b) for various feedback gain with the kick noise $\delta x = 0.02\mu$m. (0.12% of σ)

CONCLUSION

Emittance growth and luminosity decrement due to external noise in beam-beam collision system have been studied. To achieve 1 day luminosity life time, the noise ($\delta x/\sigma_x$) should be 0.1% for turn by turn noise ($t_{cor} = 1$ turn). If the correlation time of the noise is 100 turn, the tolerance is 1%. The tolerance roughly scale the correlation time as $\sqrt{t_{cor}}$. The noise level 0.1% correspond to phase fluctuation 0.6 mrad using $\Psi_{RF} = 10^{-3}\omega_{RF}\sigma_x/c\tan\phi$, where $\omega_{RF} = 2\pi \times 400$ MHz, $\phi = 0.22$ mrad.

For transverse bunch by bunch feedback, the noise level, $\delta x_{kick}/\sigma = 0.0006$ and G=0.1, is about the limit of the luminosity decrement 10^{-9}. The corresponding monitor resolution is $\delta x_{mon} = 0.0006/G = 0.006 = 0.6\%$, the resulting fluctuation in Eq.6 is 0.1%. For feedback system with lower gain, the monitor resolution is looser.

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