USING SMOOTH APPROXIMATION FOR BEAM DYNAMICS INVESTIGATION IN SUPERCONDUCTING LINAC

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Abstract

The superconducting (SC) linac consists of some different classes of the identical cavities. The each cavity based on a superconducting structure with a high accelerating gradient. The distance between the cavities is equal to acceleration structure period L. By specific phasing of the RF cavities one can provide a stable particle motion in the whole accelerator. Three dimensional equation of motion for ion beam in the Hamiltonian form is derived in the smooth approximation for superconducting linac. The nonlinear ion beam dynamics is investigated in such accelerated structure.

INTRODUCTION

Ion superconducting linac is usually based on the SC interdigital cavities. This linac consists of the niobium cavities which can provide typically 1 MV of accelerating potential per cavity. Such structures can be used for ion acceleration with different mass-charge ratio in the low energy region [1] and for proton linac in the high-energy region (SNS, JHF, ESS project). The ions are accelerated and slipping relative to the RF wave in dependence of the ratio between the particle velocity β and the phase velocity of the wave β_G. The period of RF structure in the i-th cavity is equal to $D_i = \beta_G\lambda/2$. The geometrical velocity β_G of the RF wave is constant for cavities, belong to one class. RF field has π mode in all cavities. It is desirable to have a constant geometry of the accelerating cavity in order to simplify manufacturing. Such geometry leads to a non - synchronism but a stable longitudinal particle motion can be provided by proper phasing of the RF cavities. The geometric size of a cavity and a wave velocity β_G must be changed step by step from one class to other class. The optimum number of cavity in each class determines the number of classes in SC linac. The identical cavities operate at the some initial drive phase φ. By controlling the driven phase of the accelerating structure and the distance between the cavities, the beam can be both longitudinally stable and accelerated in the whole system.

Superconducting cavities provide high accelerating gradient in linear accelerating. Together with the higher accelerating rate in SC linac the defocusing factor is much higher in comparison to the normal conducting linear accelerator. The beam focusing can be provided by SC solenoids which follow each the cavity [1]. The conditions of longitudinal and transverse beam stabilities for the structure consisting from the periodic sequence of the cavities and solenoids were studied early using transfer matrix calculation [2]. In SC linac design, it is very important to know the bucket size since it relates to the longitudinal RF focusing. But the linac longitudinal acceptance cannot be obtained by matrix method because of the assumption that the particles have small longitudinal oscillation amplitude. In order to investigate the nonlinear ion beam dynamics in such accelerated structure and to calculate the longitudinal and transverse acceptances it can be used smooth approximation [3,4]. In this paper, three dimensional equation of motion for ion beam in the Hamiltonian form is derived in the smooth approximation for superconducting linac.

PARTICLE MOTION IN SC LINAC

A schematic plot of one period of the accelerator structure is shown in Fig. 1. It consists of a superconducting solenoid for transverse focusing and superconducting RF cavity for acceleration and longitudinal focusing. The distance between the cavities is equal to acceleration structure period L. The general axisymmetric equations of motion for ion moving inside an accelerator can be written as

$$\begin{aligned} \frac{d}{dt}\left(m\gamma\frac{dz}{dt}\right) &= qE_z(\vec{r},t) - \frac{q^2}{2m\gamma}\frac{\partial}{\partial z}A_\varphi^2 \\ \frac{d}{dt}\left(m\gamma\frac{dr}{dt}\right) &= qE_r(\vec{r},t)(1-\beta\beta_G) - \frac{q^2}{2m\gamma}\frac{\partial}{\partial r}A_\varphi^2 \end{aligned} \quad (1)$$

Here $\boldsymbol{E}$ is the acceleration RF field in every cavity and A_φ is the azimuthal vector-potential of the magnetic field in every solenoid ($\boldsymbol{B}$ = rot$\boldsymbol{A}$).

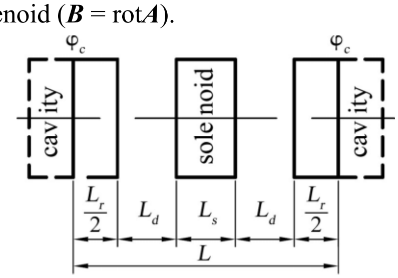

Figure 1: Layout of structure period.

The acceleration RF field of periodic H-cavity is represented as an expansion in spatial harmonics

$$\left.\begin{aligned} E_z &= E_0\sum \mathrm{I}_0(h_n r)\cos(h_n(z-z_i))\cos(\omega t) \\ E_r &= E_0\sum \mathrm{I}_1(h_n r)\sin(h_n(z-z_i))\cos(\omega t) \end{aligned}\right\} \quad (2)$$

where E_0 is amplitude of RF field at the axis ($E_0 \neq 0$ if $-L_r/2 < z - z_i < L_r/2$), $h_n = \pi/D + 2\pi n/D$, n = 0, 1, 2, …, D is the period length of the cavity, L_r is the cavity length, z_i is the coordinate of the i-th cavity center. I_0, I_1 are modified Bessel function. In our case the reference particle velocity β_c and the geometrical velocity β_G are closely in each class of the identical cavities. Retaining in (2) only zeroth harmonic we can use the traveling wave system. In this system ωt can be replaced by $h_0(z-z_i) + \varphi_{0i}$, where φ_{0i} is the RF phase when the reference particle traverses the cavity center.

We consider a dynamics under the assumption that the change of difference $\beta_c - \beta_G$ in one cavity is small enough. The distance between the cavities is equal to acceleration structure period L. If $\varphi_{0i} = \varphi_c$ for every cavity, RF field can be expanded into a Fourier series as

$$E_z = \frac{U}{L} I_0(k_0 r)\left\{ f_{z,0} + \sum_1^\infty f_{z,n}^c \cos k_n z + f_{z,n}^s \sin k_n z \right\}$$
$$E_r = \frac{U}{L} I_1(k_0 r)\left\{ f_{r,0} + \sum_1^\infty f_{r,n}^c \cos k_n z + f_{r,n}^s \sin k_n z \right\} \quad (3)$$

Here $f_{z,0} = S_0 \cos(\varphi_c + \psi)$, $f_{r,0} = -S_0 \sin(\varphi_c + \psi)$,

$f_{z,n}^c = (-1)^n T_n^+ \cos(\varphi_c + \psi)$, $f_{z,n}^s = (-1)^{n+1} T_n^- \sin(\varphi_c + \psi)$,

$f_{r,n}^c = (-1)^{n+1} T_n^+ \sin(\varphi_c + \psi)$, $f_{r,n}^s = (-1)^n T_n^- \cos(\varphi_c + \psi)$,

$T_n^\pm = S_n^+ \pm S_n^-$, $S_n^\pm = \sin(Y_n^\pm)/Y_n^\pm$, $Y_n^\pm = (k_c \pm k_n)L_r/2$.

In this expressions: $E = 2U/L_r$, U is the cavity voltage amplitude; $k_n = 2\pi n/L$, $n = 0, 1, 2, \ldots$; k_c is slipping factor, $k_c = (2\pi/\lambda)(1/\beta_c - 1/\beta_G)$. In the coefficients $f_n^{c,s}$ the phase relative to the reference particle ψ defined by $\psi = \omega(t - t_c)$, t_c is the flight time of the reference particle.

In the simple case the vector-potential of the magnetic field $A_\varphi = Br/2$ can be approximated by the step function at every solenoid. If L_s is effective solenoid length and L is a lattice period, the external solenoid magnetic field can be represented as an expansion into spatial harmonics too.

ANALYSIS OF THE EFFECTIVE POTENTIAL FUNCTION

Let us consider particle acceleration in the polyharmonic fields of the cavities (3) and solenoids. In general, individual particle motion is complicated but can be represented as the sum of a slow smooth motion (ψ and $\rho = h_0 r$) and a fast oscillation ($\tilde{\psi}$ and $\tilde{\rho}$). The force driving the ion motion can be separated into two parts corresponding to the fast and slow motion. Following Ref. [4] one can apply averaging over the fast oscillations and obtain the phase and radial motion equations in smooth approximation.

$$\frac{d^2\psi}{d\xi^2} + 3\left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\psi}{d\xi} = -\frac{\partial \overline{U}_{eff}}{\partial\psi}$$
$$\frac{d^2\rho}{d\xi^2} + \left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\rho}{d\xi} = -\frac{\partial \overline{U}_{eff}}{\partial\rho} \quad (4)$$

where $U_{eff} = U_0 + U_1 + U_2$ is effective potential function. We use the following designations:

$$U_0 = 4\alpha[I_0(\rho)\sin(\varphi_c + \psi) - \psi\cos\varphi_c - \sin\varphi_c]S_0 + \frac{1}{2} b \frac{L_c}{L}\rho^2$$

$$U_1 = \alpha^2 \sum_1^\infty \left[\frac{I_0^2(\rho)}{(2\pi n)^2}\left({f_{z,n}^c}^2 + {f_{z,n}^s}^2 \right) + \frac{I_1^2(\rho)}{(2\pi n)^2}\left({f_{r,n}^c}^2 + {f_{r,n}^s}^2 \right) \right],$$

$$U_2 = -4\alpha b\rho I_1(\rho)\frac{L_s}{L}\sum \frac{f_{r,n}^c}{(2\pi n)^2}\frac{\sin X_n}{X_n} + b^2 \sum \frac{1}{(2\pi n)^2}\left(\frac{\sin X_n}{X_n}\right)^2 \rho^2$$

Where $\alpha = LU_c/4L_V$, $U_c = qU/mc^2$, $L_V = \lambda\beta_c^3\gamma_c^3/2\pi$, $b = (qBL/2mc\beta_c\gamma_c)^2$, $X_n = \pi n L_s/L$.

Equations (4) have the damping terms from the first derivatives of phase ψ and ρ. The effective potential U_{eff} provides the full description of the ion dynamics in the smooth one-particle approximation. In Ref. [5] the longitudinal smooth approximation with acceleration in SC linac was investigated by the numerical simulation. In our case the analysis of the effective potential (5) makes it possible to study the condition at which the radial and phase stability of the beam is achieved, as well as to formulate applicability of a 3D smooth approximation to given electrodynamic problem. We begin our analysis with expanding U_{eff} in the vicinity of its minimum ($\psi = 0$, $\rho = 0$):

$$U_{eff} = U_{eff}(0,0) + \frac{1}{2}\Omega_z^2\psi^2 + \frac{1}{2}\Omega_r^2\rho^2 + \ldots \quad (5)$$

The expansion coefficients here depend on the parameter of interaction α, the values of L_r/L, L_s/L and the slipping factor k_c. The radial and phase stability of the beam will be provided when $\Omega_z^2 > 0$, $\Omega_r^2 > 0$, where Ω_z, Ω_r are frequencies of small longitudinal and transverse oscillations.

In the simplest case when the phase velocity β_G changes from cavity to cavity and $k_c = 0$

$$\Omega_z = 2\sqrt{\alpha\sin(-\varphi_c) - \chi\alpha^2\cos(2\varphi_c)}$$
$$\Omega_r = \sqrt{-2\alpha\sin(-\varphi_c) + \chi\alpha^2\left(1 + \cos^2(\varphi_c)\right) + b\frac{L_s}{L}} \quad (6)$$

Here the value of χ depends on the ratio of L_r/L. For some of L_r/L the value of χ is listed in Table 1.

Table 1. The factor of the period fillup, χ.

χ	1/3	3/16	1/12	0
L_r/L	0	1/4	½	1

In single wave approximation when $L_r = L$ and a fast oscillation terms are absent, the value of $\chi = 0$. In this case the expression (5) coincides with the longitudinal and transverse phase advances per period ($\mu_z = \Omega_z$, $\mu_r = \Omega_r$) which were founded by transfer matrix calculation in [2]. But the conditions of focusing are changed if the parameter α is large and the fast oscillations are considered. The functions $\Omega_z(\alpha)$ and $\Omega_r(\alpha)$ for different values of φ_c are shown in Fig 2. The phase advances should be less than about $\pi/2$ for stable beam motion [6]. In this case, the smooth approximation is expected to be almost accurate.

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As it can see from Fig. 2, the longitudinal oscillation will be stability for $-\pi/4 < \varphi_c < 0$ if the interaction parameter α is limited, $\alpha < \alpha_{max} = \sin(-\varphi_c)/\chi\cos(2\varphi_c)$. In this case the frequency of longitudinal oscillation has maximum value in $\alpha = \alpha_{max}/2$. If the magnetic field is absent the radial stability of the beam is achieved, if the interaction parameter $\alpha > \alpha_{min} = 2\sin(-\varphi_c)/\chi(1+\cos^2\varphi_c)$.

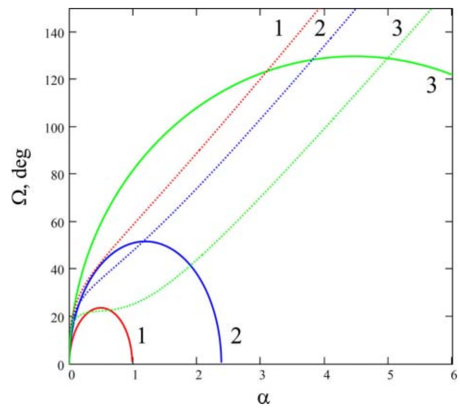

Figure 2: The frequencies of longitudinal (solid lines) and transverse (dot lines) oscillations for $B = 4$ T ($1 - \varphi_c = -10°$, $2 - \varphi_c = -20°$ and $3 - \varphi_c = -35°$).

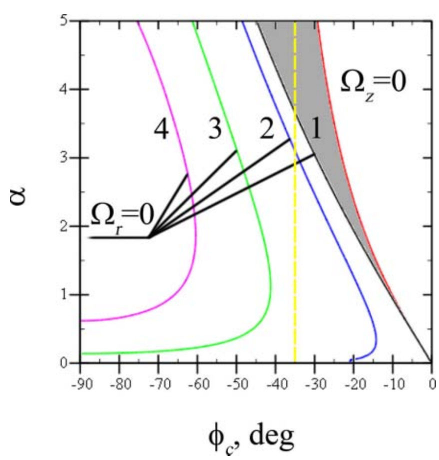

Figure 3: Stability area for different values of the magnetic field: $1 - B = 0$, $2 - B = 1$ T, $3 - B = 2$ T and $4 - B = 2{,}5$ T.

The borders of stability area between $\Omega_z = 0$ and $\Omega_r = 0$ are shown in Fig. 3 for deferent values of magnetic field, B, when $L_r/L = 1/4$. The value α on stability diagram moves down quickly on the $\alpha = 0$ axis as beam energy increases ($\alpha \sim 1/\beta^3$). For $B = 0$, the area of stability tapers abruptly and the radial stability is absent when the beam velocity increases. The area of stability can be extended by a solenoid focusing which will also provide a separate control of transverse and longitudinal beam dynamics. For a proton beam velocity $\beta \geq 0.1$, it is sufficient to increase the solenoid field (see Fig. 2). For example, the reference particle phase $\varphi_c > -40°$ if $B = 2$T.

It interesting to investigate the nonlinear ion beam dynamics in such accelerated structure. By means of the effective potential U_{eff} we can calculate the longitudinal acceptance. In Fig. 4 it is shown the separatrixes for $\varphi_c = -35°$ and $\rho = 0$ when parameter α increases. The energy spread of the separatrix, $\Delta\gamma = \frac{L_V}{L}\frac{d\psi}{d\xi}$, at first increases and then it decreases, but the phase length of separatrix decreases always. If the value of the cavity voltage amplitude U is preset the separatrix area depends on the value of L_r/L ratio. In the case when $\alpha = 1$ the phase acceptance and the energy spread will decrease nearly in two times when the cavity length, L_r, changes from L to zero. Thus, the smooth approximation gives a weaker effective RF bucket, i.e. smaller phase acceptance and potential well depth, compared with the single wave approximation when the fast oscillations are not considered.

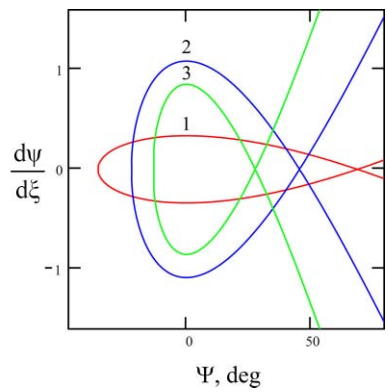

Figure 4: Separatrixes for $\varphi_c = -35°$ and $\rho = 0$ for different parameter α: $1 - \alpha = 0.1$, $2 - \alpha = 3$ and $3 - \alpha = 6$.

CONCLUSION

Three dimensional equation of motion for ion beam in the Hamiltonian form is derived in the smooth approximation for superconducting linac. The borders of stability diagram are found for deferent values of the cavity voltage amplitude, U, the magnetic field, B, and the drift length, $2L_d + L_s$. It was shown the high accelerating gradient will be limited by the advent of the phase instability in SC linac if $-\pi/4 < \varphi_c < 0$. Increasing of the drift length between cavities can decrease the separatrix area in several times.

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