SIMULATION STUDY OF RESISTIVE-WALL BEAM BREAKUP FOR ERLS

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Abstract

Resistive-wall beam breakup (RWBBU) due to the long-range transverse wake is studied with a developed simulation program. The simulation program is useful and essential for fully understanding the RWBBU behaviour in ERLs. The simulations demonstrate that the position displacement or orbit distortion due to the RW wake is proportional to the square root of time and the wake strength in the early stage of the RWBBU and they are consistent with the RWBBU equations. In case of the planned test ERL in Japan, the maximum position displacements increase up to 3% and 1% of the initial position offset with all the quadrupole magnets off and on at 77 μs. A small-gap insertion device(ID) duct increases the orbit distortion in downstream of the ID section, though a focusing effect due to the ID field may reduce it.

INTRODUCTION

For future ERL-based light sources, average beam current is required to be up to 100 mA. Such a high-current multi-bunch beam may generate and cumulate strong long-range wake-fields by interaction with accelerator components such as superconducting cavities and vacuum ducts, and then strong beam breakup(BBU) may occur. Resistive-wall BBU(RWBBU) due to resistive vacuum ducts has been hardly studied, though the BBU caused by HOMs of superconducting cavities were much investigated. Asymptotic expressions of the transverse RWBBU were derived for a beam that passes through a uniform resistive pipe under uniform external focusing[1]. However the expressions are valid only for limited parameter ranges and initial conditions. Therefore we have developed a computer simulation program to study transverse multi-bunch RWBBU more minutely and generally. In this paper, we will mainly present the simulation results obtained by the simulation program.

RWBBU EQUATION

We consider the basic equations of the transverse RWBBU here. The long-range transverse resistive-wall wake-function for a round pipe with a radius b, length L and electric conductivity σ_c is expressed by

$$W_\perp = -\frac{cL}{\pi b^3 z^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}}\,, \tag{1}$$

where c, Z_0 and z are the velocity of light, the vacuum impedance and the distance from a preceding bunch. Eq. (1) is derived under the following condition:

$$\sqrt[3]{\frac{b^2}{\sigma_c Z_0}} << z << b^2\sigma_c Z_0 \tag{2}$$

The pipe thickness is assumed to be infinite. An electron is transversely kicked by the wake of a preceding bunch with the position y and the distance z and the kick angle is given by

$$\Delta\theta_y = -\frac{e^2N}{E}W_\perp \cdot y = \frac{e^2N}{E}\cdot\frac{cL}{\pi b^3 z^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}}\cdot y\,, \tag{3}$$

where E is the beam energy and N is the electron number in the bunch.

The basic equations of the RWBBU for the initial (M=1) and M-th ($M \geq 2$) bunches are given by

$$y_1''(s) + K_y(s)y_1(s) = 0 \tag{4}$$

$$y_M''(s) + K_y(s)y_M(s) = \sum_{N=1}^{M-1}\frac{a(s)}{\sqrt{M-N}}y_N(s) \quad (M \geq 2) \tag{5}$$

$$a \equiv \frac{e^2N}{E}\cdot\frac{c}{\pi b^3(c\tau_B)^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}} = \frac{eI_B}{E}\cdot\frac{(c\tau_B)^{1/2}}{\pi b^3}\sqrt{\frac{Z_0}{\pi\sigma_c}}$$

where τ_B and K_y are the bunch time separation and the external focusing or defocusing strength, and I_B is the average beam current defined by eN/τ_B. Eq. (5) shows equation of motion for the M-th bunch for the case where an electron beam consisting of a series of identical point-like bunches passes through round pipes with external focusing or defocusing. From these two equations, the following equation is obtained:

$$\xi_M''(s) + K_y(s)\xi_M(s) = \sum_{N=1}^{M-1}\frac{a(s)}{\sqrt{M-N}}\frac{y_N(s)}{y_{00}} \tag{6}$$

where

$$\xi_M(s) \equiv \frac{y_M(s) - y_1(s)}{y_{00}}\,. \tag{7}$$

The symbol ξ_M is the position displacement of the M-th bunch from the initial bunch position normalized by the initial position offset y_{00} and it is caused by the cumulated resistive-wall wake. In the early stage of the RWBBU (ξ_M << 1), this equation is approximately rewritten by

$$\begin{aligned}\xi_M''(s) &\approx \sum_{N=1}^{M-1}\frac{a(s)}{\sqrt{M-N}}\frac{y_1(s)}{y_{00}} \\ &\approx 2\sqrt{M}a(s)\frac{y_1(s)}{y_{00}} \quad (M >> 1)\end{aligned} \tag{8}$$

For $y_M'(0) = y_1'(0)$, $y_M(0) = y_1(0)$, $a(s)=a$ and $t=M\tau_B$, the following relation is derived from Eq. (8):

$$\begin{aligned}\xi_M(s) &\propto at^{1/2} \\ &\propto \sigma_c^{-1/2}b^{-3}I_B E^{-1}\tau_B^{1/2}t^{1/2}\end{aligned} \tag{9}$$

RWBBU SIMULATION

A simulation program has been developed to study the RWBBU behaviors and to evaluate effects of the RWBBU on ERLs. It is based on Eqs. (4) and (5). Basic time increment is the bunch separation τ_B divided by an integer N_D (division number). Simulations are first compared with the asymptotic expressions for two extreme cases, no focusing(NF) and strong focusing(SF) cases. Parameter values are properly chosen so that they should satisfy the necessary conditions of the asymptotic expressions. The simulation results are in agreement with the asymptotic expressions. The details of comparisons between simulations and analytic solutions including the asymptotic expressions are described elsewhere[2].

Next the simulation program was applied to the planned test ERL in Japan[3]. Figure 1 shows layout of the test ERL. Simulation conditions are as follows:

- E=60 MeV, I_B=100 mA for uniform bunch filling, and τ_B=0.769 ns (f_{RF}=1.3 GHz).
- All the vacuum ducts are assumed to be Al pipes with b=25 mm and σ_c=3.5x10^7 Ω^{-1} m^{-1}.
- The simulation start and end points are the exit and entrance of the superconducting cavity section as shown in Fig. 1 and the total length L is 56.44 m.
- All the bunches have the same initial position offset y_{00} at the simulation start point.
- 1-D tracking in the vertical direction for $t \leq 77$ μs ($M \leq 100000$), which satisfy the condition of (2).
- Focusing and defocusing of 37 quadrupole(Q) magnets in the beam path are considered. Sextupole magnets, magnet and duct alignment errors, orbit correction by correctors are neglected.
- When the insertion device(ID) is considered, its vacuum duct is assumed to be a stainless-steel(SS) pipe with b=10 mm, σ_c=1.4x10^6 Ω^{-1} m^{-1} and $L_{\rm ID}$=10 m.

First we consider a situation with all the Q-magnets off and without the ID. In this situation the test ERL just equals an Al pipe with b=25 mm and L=56.44 m and without external focusing and defocusing. Figure 2 shows the position displacement $\{y_M(L)-y_1(L)\}/y_{00}$ (=$\xi_M(L)$) due to the RW wake at the simulation end point (s=L) as function of time t or bunch number M. The position displacement obtained from the simulation is almost proportional to $t^{1/2}$ except very small times and increases up to 3% of the initial position offset at 77 μs. It is consistent with the relation of (9). The asymptotic expression of NF case shown in Fig. 2 decreases with the time and disagrees with the simulation result. This means that, in the parameter range, the asymptotic expression is invalid, though the simulation obtains correct results.

Figures 3 (a) and (b) show the orbit distortions $\{y_M(s)-y_1(s)\}/y_{00}$ (=$\xi_M(s)$) due to the RW wake along the beam pipe with all the Q-magnets off and on, together with normalized initial orbits $y_1(s)/y_{00}$. Since every bunch except the first bunch is continuously kicked by the RW wake in the same direction, the orbit distortion monotonously increases with the longitudinal distance s in Fig. 3(a). On the other hand, in Fig. 3(b), the orbit distortion is oscillatory along the beam pipe because the initial orbit is also oscillated around the pipe center by the Q-magnet focusing. As a result, the maximum orbit distortion is reduced down to 1% of the initial position offset from 3%.

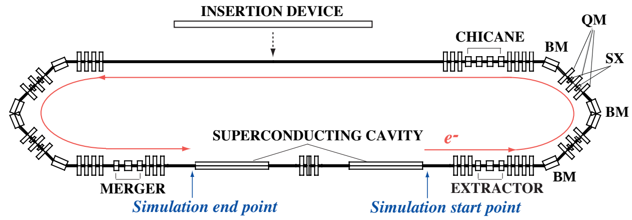

Figure 1: Layout of the planned test ERL in Japan

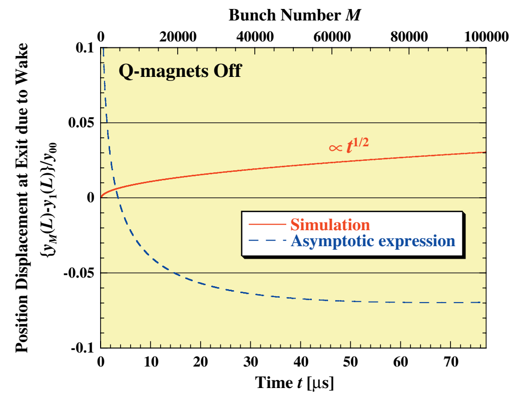

Figure 2: Position displacement at exit due to RW wake with all the Q-magnets off as a function of time and bunch number.

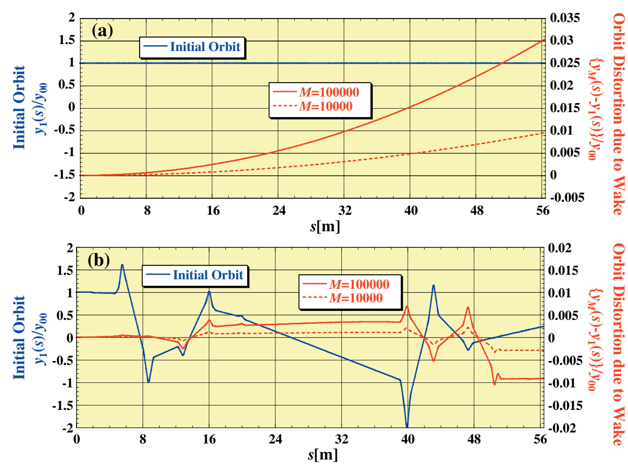

Figure 3: Initial orbits and orbit distortions due to the RW wake for M = 10000 and 100000 (t = 7.7 and 77 μs) with all the Q-magnets (a) off and (b) on in the test ERL.

Figures 4 (a) and (b) show the effects of electric conductivity and average beam current on the RWBBU in case with all the Q-magnets on. In Fig. 4(a), the ratio between position displacements at the exit for two cases of SS and Al pipes is almost constant and equal to the square root of the ratio between electric conductivities of Al and SS. The ratio between position displacements for average currents of 100mA and 10mA is also constant, as shown in Fig. 4(b), and it is equal to the ratio of the average currents. These results are consistent with the relation of (9) and suggest that the position displacement due to the wake is almost proportional to the wake strength a in this range.

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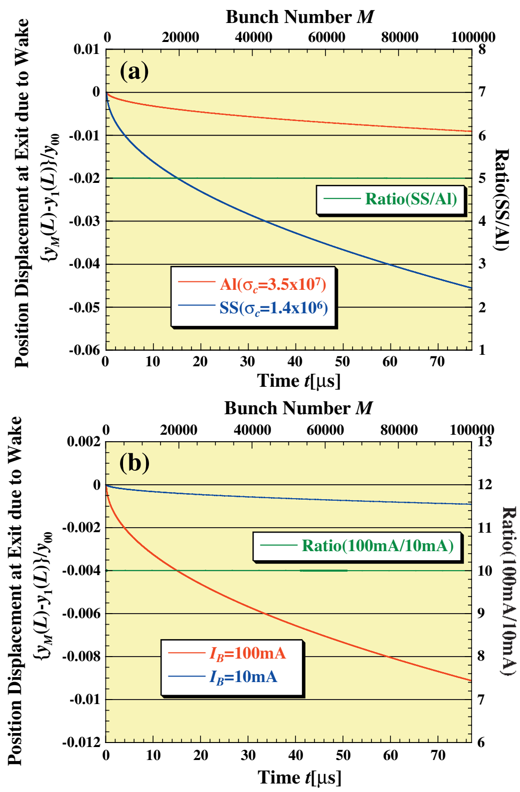

Figure 4: Position displacements at exit due to RW wake (a) for electric conductivities of pipes σ_c = 1.4x10^6 Ω^{-1} m^{-1} (SS: stainless steel) and 3.5x10^7 Ω^{-1} m^{-1} (Al: aluminum) and (b) for the average currents I_B = 100 mA and 10 mA with all the Q-magnets on as a function of time. Their ratios are also shown.

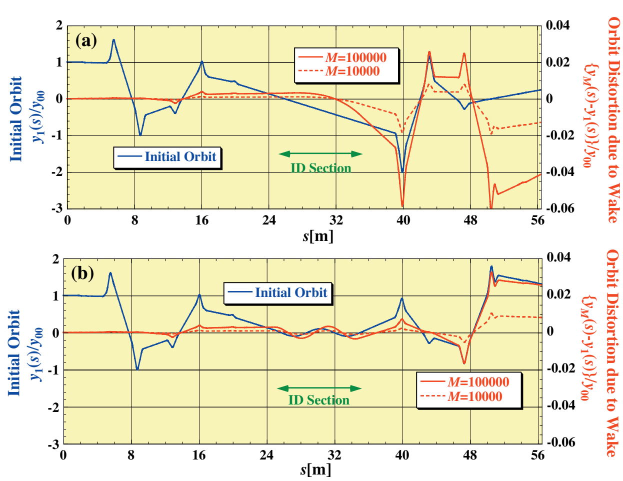

Figure 5: Initial orbits and orbit distortions due to the RW wake for M = 10000 and 100000 (t = 7.7 and 77 μs) with ID focusing strength (a) K_y = 0 m^{-2} and (b) K_y = 1 m^{-2} in a small-gap ID pipe at ID section.

Figures 5 (a) and (b) show the effects of a small-gap ID duct and ID focusing respectively. In Fig. 5(a), the simulated orbit distortion significantly increases in downstream of the ID section, because a strong resistive-wall wake is generated by the small-gap SS duct installed at the ID section. On the other hand, in Fig. 5(b), the orbit distortion is well suppressed because the initial orbit is oscillated around the pipe center by the ID focusing (K_y=1 m^{-2}) in the ID section. However the ID focusing strength is not always strong during the user operation and the ID focusing is not expected for suppression of the RWBBU. A copper-coating on the inner surface of the SS duct can be effective for reducing the wake-field at the ID duct[4].

CONCLUSIONS

A simulation program of the RWBBU has been developed for studying the transverse RWBBU more minutely and generally than the analytic expressions. In the simulations for the planned test ERL in Japan, the position displacements due to the RW wake with all the Q-magnets off and on increase in proportion to the root square of time and reach about 3% and 1% of the initial position offset at 77 μs. The Q-magnet focusing contributes to suppressing the RWBBU growth. A small-gap ID duct significantly increases the orbit distortion, and the ID focusing well suppresses the RWBBU growth, though it is changeable in the user operation.

The RWBBU should be further simulated for longer simulation time to know how large the RWBBU grows. The upper time limits for validity of Eq. (1) are obtained from the condition of (2) to be 27.5 ms, 1.1 ms and 176 μs for the Al pipe with b= 25 mm and the SS pipes with b=25 mm and b=10 mm and all of them are smaller than the maximum simulation time, 77 μs. However the validity of Eq. (1) is not guaranteed if the simulation time much increases and exceeds these upper time limits. Moreover, effects of finite pipe thickness must be considered. The simulation program should be improved to keep validity of the wakefunction for wider time range.

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