DISPERSION TOLERANCE CALCULATION FOR NSLS-II

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Abstract

In this paper we discuss the effect on the emittance of the residual dispersion in the insertion devices. The dispersion in the straights could be generated by the lattice error, trim dipole, and insertion device. The effect on the the emittance is examined, and the dispersion tolerances are given for the NSLS-II.

INTRODUCTION

The NSLS-II is a proposed third-generation light source. The storage ring consists of 30 Double-Bend-Achromatic cells with 15 fold symmetry. The natural emittance will be 1 nm. Damping wigglers are empolyed to achieve this unprecedented value. The damping wigglers, however, excite the emittance if the dispersion is nonzero. The residual dispersion could be generated by the lattice error, the trim dipoles and the insertion devices. In this paper we discuss the tolerance on the residual dispersion in the straight sections of the NSLS-II.

LATTICE FUNCTIONS IN A BARE STRAIGHT

The dispersion function in a ring accelerator can be calculated by

$$D(s_0) = \frac{\sqrt{\beta_x(s_0)}}{2\sin\pi\nu_x}\oint\frac{\sqrt{\beta_x(s)}}{\rho} \times \cos(\pi\nu_x - |\psi_x(s)-\psi_x(s_0)|)ds, \quad (1)$$

where $\rho(s)$ is the bending radius, ν_x is the horizontal tune, and $\psi_x(s)-\psi_x(s_0)$ is the horizontal betatron phase advance from s_0 to s.

In a straight section, the lattice functions can be written as [1]: $\beta(s)=\beta_c[1+u^2]$, $\alpha(s)=-u$, $\gamma(s)=\frac{1}{\beta_c}$, and the betatron phase advance $\psi(s)=\tan^{-1}u$. Here c denotes the center of the straight, $s=0$, and at that point $\alpha_c=0$. Also, we have introduced $u\equiv s/\beta_c$. We note that these functions can be written in the above *u*-presentation in any drift space, except that the center point s_c could be a virtual point.

With the *u*-presentation, the dispersion function can be written in a linear form

$$D(s)=\sqrt{\beta_c}[C_x+\frac{S_x}{\beta_c}s], \quad (2)$$

where

$$C_x = \frac{1}{2\sin\pi\nu_x}\oint_{s_c^+}^{s_c^-}\frac{\sqrt{\beta_x(s)}}{\rho}\cos(\pi\nu_x-\psi_x(s))ds,$$

$$S_x = \frac{1}{2\sin\pi\nu_x}\oint_{s_c^+}^{s_c^-}\frac{\sqrt{\beta_x(s)}}{\rho}\sin(\pi\nu_x-\psi_x(s))ds.$$

Note that C_x and S_x are constants for a given lattice.

DISPERSION GENERATED BY THE INSERTION DEVICES

Effect on the other straights

In this subsection we calculate the dispersion function change in straight 2 due to the ID in straight 1. We assume the beta function is not changed by the insertion devices, or, the beta function perturbation has been corrected. The betatron phase advance between the ID magnet location s and the point of interest s_0 is $\psi(s)-\psi(s_0)=2k\pi\nu_{x,cell}+\tan^{-1}\frac{s-s_{c,1}}{\beta_c}-\tan^{-1}\frac{s_0-s_{c,2}}{\beta_c}$, where $\nu_{x,cell}$ is the betatron tune for one cell, k is the number of cells between these two straights, and $s_{c,1}$ and $s_{c,2}$ are the centers of these straights. From Eq.(1), we found

$$D(s) = \frac{\sqrt{(\beta_c I_{w,1})^2+(\frac{L}{2}I_{w,1}-I_{w,2})^2}}{2\sin\pi\nu_x B_0\rho_0} \times [\cos\psi_{0,w}-\sin\psi_{0,w}\frac{(s-s_{c,2})}{\beta_c}], \quad (3)$$

where L is the length of the ID, $I_{w,1}=\int_{-L/2}^{L/2}B_w(s)ds$ and $I_{w,2}=\int_{-L/2}^{L/2}ds\int_{-L/2}^{s}B_w(s')ds'$ are the first and the second integrals of the ID, respectively, $\psi_{0,w}=\pi\nu_x-2k\pi\nu_{x,cell}-\psi_w$, $\psi_w=\tan^{-1}\frac{I_{w,1}\frac{L}{2}-I_{w,2}}{\beta_c I_{w,1}}$, and $B_0\rho_0$ is the rigidity of the reference particle.

Therefore, the maximum amplitude of the dispersion is

$$D(s)=\frac{\sqrt{(\beta_c I_{w,1})^2+(\frac{L}{2}I_{w,1}-I_{w,2})^2}}{2\sin\pi\nu_x B_0\rho_0}\sqrt{1+(\frac{L}{2\beta_c})^2}. \quad (4)$$

Note this is the result for one ID. In the case of more than one ID, the total dispersion is the linearly superposition. However, note that ψ_0 changes for each straight, therefore there is cancellation between the amplitudes. Here we estimate the dispersion generated by one ID.

Take the NSLS-II damping wiggler as an example. In the high-β straight, $\beta_c=18$ m, and using the APS ID field tolerances (i.e., $I_{w,1}=20\ G\cdot cm$ and $I_{w,2}=20000\ G\cdot cm^2$)[2] and scale to the length of 7 m, one gets $D_{max}\sim 50\ \mu m$. The number is even smaller for the low-β straight insertion devices. Therefore, the dispersion leaking from the IDs is not a concern due to the tight ID field tolerances.

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1-4244-0917-9/07/$25.00

Dispersion inside the ID

Following Ref. [3], we use the expression for the dispersion function inside an ID

$$D(s) = \frac{1}{k_w^2 \rho_w}[1 - \sin k_w(s - s_c)], \quad (5)$$

where $k_w = 2\pi/\lambda_w$ and λ_w is the wiggler period, and ρ_w is the maximum bending radius in the wiggler. Using the NSLS-II damping wiggler parameters (i.e., $k_w = 62.8\ \mathrm{m}^{-1}$ and $\rho_w = 5.6$ m), we get the magnitude of the dispersion to be $\frac{1}{k_w^2 \rho_w} \approx 45\ \mu$m, which is very small.

DISPERSION GENERATED BY CANTING

At NSLS-II we are considering canting the insertion devices [4], especially the damping wigglers because they are about 7 m long. However, the trim dipoles that are used to create an orbit bump also generate dispersion. The closed orbit change due to trim dipoles can be described by

$$x_e(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\sum_i \sqrt{\beta_i}\theta_i \cos(\pi\nu - |\psi(s) - \psi(i)|), \quad (6)$$

where the sum is over the kickers, and $\theta_i = B_i \Delta L_i / B_0 \rho_0$ is the kick angle. This is the same as the expression for the dispersion function given in Eq. (1); therefore, the dispersion function generated by the trims equals the closed orbit change. If we arrange three kickers as $\theta\ L\ (-2\theta)\ L\ \theta$, namely, the kickers are L apart, and the center kicker is a two times reverse kick, then the dispersion inside the chicane is given by

$$D(s) = (L - |s - s_c|)\theta. \quad (7)$$

The radiation opening angle of the NSLS-II damping wiggler is about 3 mrad, hence the canting angle will be in the order of milliradian. Assume a canting angle $\theta = 1$ mr. For a 7 m damping wiggler, the maximum dispersion amplitude will be about 3.5 mm, therefore is more of a concern. A limit on the maximum canting angle will be given after we discuss the effect on the emittance.

EFFECT ON THE EMITTANCE

So far we have discussed the dispersion generated by the lattice, the insertion device, and the trim dipole. In general, the dispersion functions in the straight can be expressed as follows:

$$D(s) = c_0 + c_1 s - c_2 \sin k_w s \quad (8)$$
$$D'(s) = c_1 - c_2 k_w \cos k_w s. \quad (9)$$

Therefore,

$$\begin{aligned}\mathcal{H} &= \beta D'^2 + 2\alpha D D' + \gamma D^2 \\ &= \beta_c c_1^2 + \frac{c_0^2}{\beta_c} + \beta_c c_2^2 k_w^2 \cos^2 k_w s \\ &+ \frac{c_2^2}{\beta_c} k_w^2 s^2 \cos^2 k_w s - 2\beta_c c_1 c_2 k_w \cos k_w s \\ &+ \frac{2c_0 c_2}{\beta_c} k_w s \cos k_w s - \frac{2c_2^2}{\beta_c} k_w s \cos k_w s \sin k_w s \\ &+ \frac{c_2^2}{\beta_c}\sin^2 k_w s - \frac{2c_0 c_2}{\beta_c}\sin k_w s. \quad (10)\end{aligned}$$

The emittance integral

$$\int_{-N_w\pi}^{N_w\pi} \frac{\mathcal{H}}{|\rho^3|} ds \approx \frac{2N_w}{k_w \rho_w^3}\{\frac{4}{3}(\beta_c c_1^2 + \frac{c_0^2}{\beta_c}) + \frac{4}{15}\beta_c c_2^2 k_w^2 + (0.986 + \frac{1}{45}k_w^2 L^2)\frac{c_2^2}{\beta_c}\}, \quad (11)$$

where $L = 2\pi N_w / k_w$ is the total length of the insertion device.

The natural emittance is given by

$$\frac{\epsilon_{x,w}}{\epsilon_{x,0}} = \frac{1 + f_1/f_2}{1 + f_3/f_4}, \quad (12)$$

where $f_1 = \frac{N_w}{k_w \rho_w^3}\left[\frac{4}{3}(\beta_c c_1^2 + \frac{c_0^2}{\beta_c}) + \frac{4}{15}\beta_c c_2^2 k_w^2 + (0.986 + \frac{1}{45}k_w^2 L^2)\frac{c_2^2}{\beta_c}\right]$, $f_2 = \int \frac{\mathcal{H}_0}{\rho_0^3} ds$, $f_3 = \frac{N_w \pi}{k_w \rho_w^2}$ and $f_4 = \frac{2\pi}{\rho}$. $\mathcal{H}_0$ is the dispersion action in the bending dipoles. Note this is the complete result, while in reference [5] only the major terms are kept, for simplicity. Also, if the external dispersion is zero and the ID length $L \ll \beta_c$, then factor f_1 is dominated by the $\frac{4}{15}\beta_c c_2^2 k_w^2$ term, and the natural emittance goes back to the expression given in reference [3].

A large portion of the vertical emittance, however, is contributed from the damping wiggler. This is because $\rho_0 \gg \rho_w$, and the vertical residual dispersion is of the same order around the ring. For the NSLS-II 1-nm lattice, the energy loss in the damping wiggler is about the same as that in all the bending magnets; for simplicity we assume the vertical emittance is 4 pm, half of the final emittance, if dispersion is zero in the wiggler.

DISPERSION TOLERANCES

The current proposal of IDs for the NSLS-II 1-nm lattice is shown in Table 1. Including all the listed insertion devices, we calculated the emittance increase ratio as a function of the external dispersion, and the results are shown in Fig. 1 and Fig. 2. When the dispersion is nonzero, the beam size is also enlarged due to momentum spread; therefore the effective emittance increase is also plotted. Note the effective emittance is given by $\epsilon_{x,eff} = \sqrt{\epsilon_{x,0}(\epsilon_{x,0} + \mathcal{H}_x \sigma_\delta^2)}$. In the calculation, we use the dispersion given by Eq. (8), and the emittance growth ratio is calculated using Eq. (12). The tolerance on the maximum dispersion and the corresponding emittance growth are given in Table 2. For a 10% emittance increase, the horizontal dispersion must be less than 1.2 cm and the vertical dispersion must be less than 3.1 mm.

Using the same method, we calculated the emittances as functions of the canting angle of the damping wigglers.

Table 1: Parameters of NSLS II candidate insertion devices.

Name	U14	U19	U45	U100	DW	SCW
Type	SCU	CPMU	EPU	EPU	PMW	SCW
λ_u (mm)	14	19	45	100	100	60
L (m)	2.0	3.0	4.0	4.0	7.0	1.0
B_w (T)	1.68	1.14	1.03	1.5	1.8	3.5

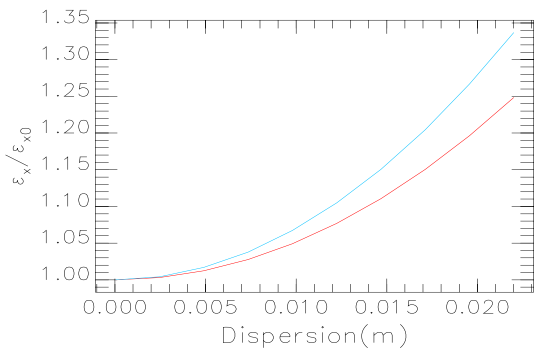

Figure 1: The horizontal emittance growth ratio as a function of external dispersion in the straights. The blue line is for the effective emittance, and the red line is for the horizontal emittance $\epsilon_{x,0}$.

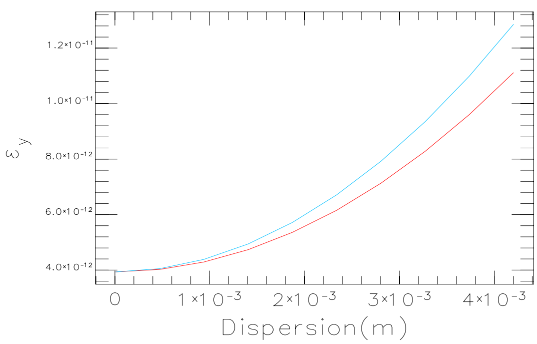

Figure 2: The vertical emittance as a function of the external dispersion. The blue line is the effective emittance, and the red line is the vertical emittance $\epsilon_{y,0}$.

Table 2: Dispersion tolerances and emittance growth.

$\epsilon_{x\|y}/\epsilon_{x\|y,0}$	1.1	1.2	1.3
D_y(mm)	3.1	3.4	3.6
D_x(cm)	1.2	1.7	2.1

The results are shown in Fig. 3 and Fig. 4. In the calculation three 7-m damping wigglers were included and all of them were canted. In the horizontal direction, the canting angle has to be less than 1.5 mr in order for the emittance growth to be less than 10%. In the vertical direction, the canting angle must be less than 0.15 mr. However, the initial vertical emittance can be made smaller after linear coupling correction; therefore, the vertical canting angle could be larger.

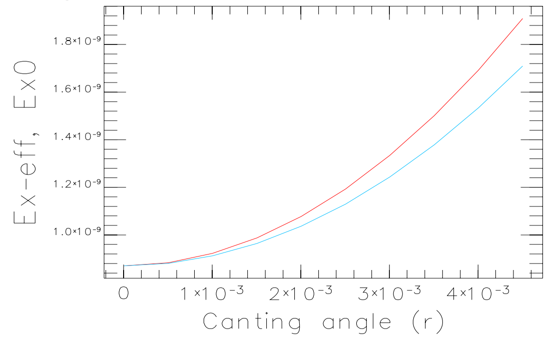

Figure 3: The natural and effective horizontal emittance as functions of the canting angle.

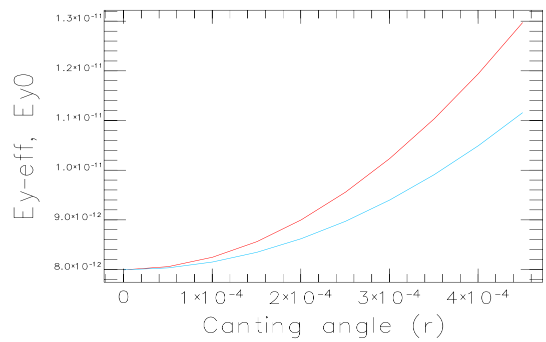

Figure 4: The equilibrium and effective vertical emittance as functions of the canting angle.

CONCLUSION

We discussed the tolerance on the residual dispersion generated by the lattice error, the trim dipole and the insertion device. For NSLS-II we found that the dispersion residue must be less than 1.2 cm horizontally and 3.1 mm vertically to ensure a less than 10% emittance increase. This is well within the capability of the present orbit feedback technology. The dispersion function generated by the insertion devices is about 50 μm, therefore is not a concern. With the same criterion, we found the damping wiggler canting angle must be less than 1.5 mr horizontally and 0.15 mr vertically.

REFERENCES

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