MOOAAB04

Quadruple-Bend Achromatic Low Emittance Lattice Studies

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*paper published in **Rev. Sci. Instrm. 78 , 055109 (2007)**

Outline

- Introduction
- Effect of Insertion Devices on Beam Emittance
- Quadruple-Bend Achromat (QBA)
- A QBA Lattice Example
- Conclusion

Introduction

- Main features of new synchrotron light source
  - Intermediate beam energy: ~ 3 GeV
  - Low emittance to provide high brillance
  - Many straight sections for insertion devices

Double-bend cells are often used for lattice design

A simple rule for small emittance:

smaller bending angle ⇨ increase the number of cells and circumference of accelerator

Nonachromatic mode of operation⇨ smaller emittance than DBA mode

Effect of Insertion Devices on Beam Emittance

$$I_2 = \oint \frac{1}{\rho^2} ds$$

$$I_3 = \oint \frac{1}{|\rho|^3} ds$$

$$I_4 = \oint \frac{\eta_x}{\rho} \left(\frac{1}{\rho^2} + 2K \right) ds$$

$$I_5 = \oint \frac{H}{|\rho|^3} ds,$$

(1) Emittance $\varepsilon_x = C_q \gamma^2 \dfrac{I_5}{I_2 - I_4}$

(2) Energy spread $\left(\dfrac{\sigma_E}{E}\right)^2 = C_q \gamma^2 \dfrac{I_3}{2I_2 + I_4}$

$$\boxed{\begin{aligned} \varepsilon_x &= C_q \gamma^2 \langle H \rangle / \rho \\ \langle H \rangle &= \rho \varepsilon_x / C_q \gamma^2 \end{aligned}}$$

$$H = \frac{1}{\beta_x} [D_x^2 + (\beta_x D_x' + \alpha_x D_x)^2]$$

Minimization of emittance is equivalent to the minimization of average H-function in dipoles

WHO is afraid of the dispersion in ID sections?

$$\text{Emittance without ID}: \ \varepsilon_{x0} = C_q \gamma^2 \frac{I_{50}}{I_{20} - I_{40}}$$

$$\text{Emittance with ID}: \ \varepsilon_x = C_q \gamma^2 \frac{I_{50} + I_{5w}}{I_{20} - I_{40} + I_{2w} - I_{4w}}$$

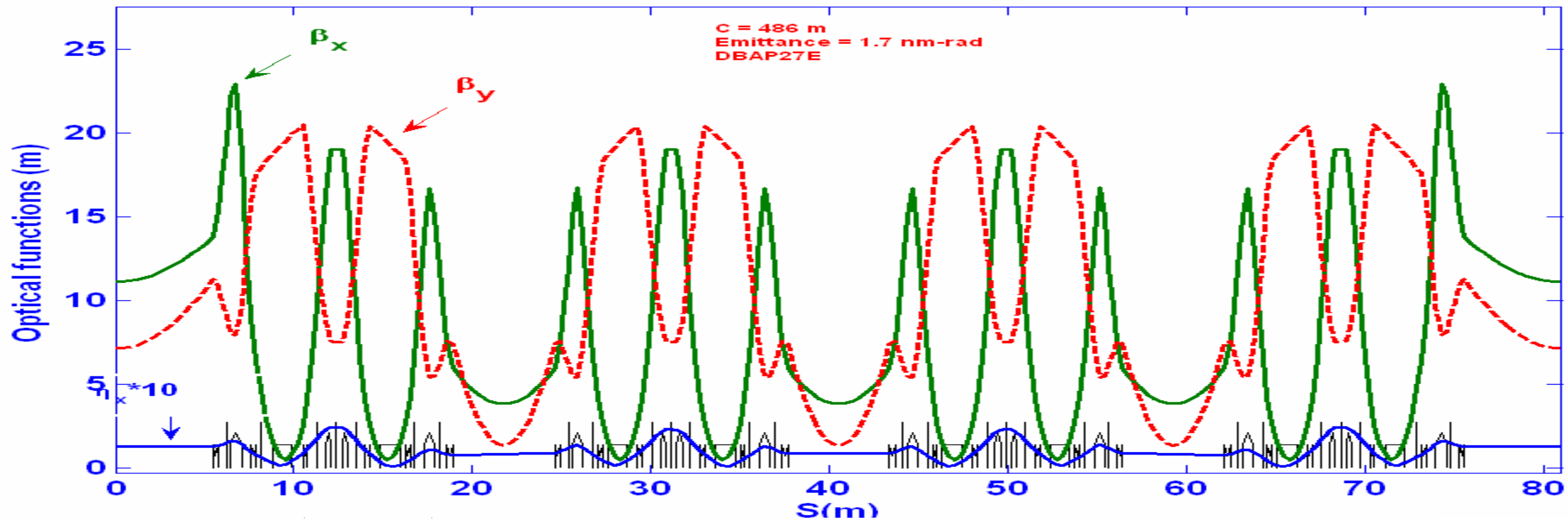
$$f_h \equiv \frac{<H_{dipole}>}{H_{ID}}$$

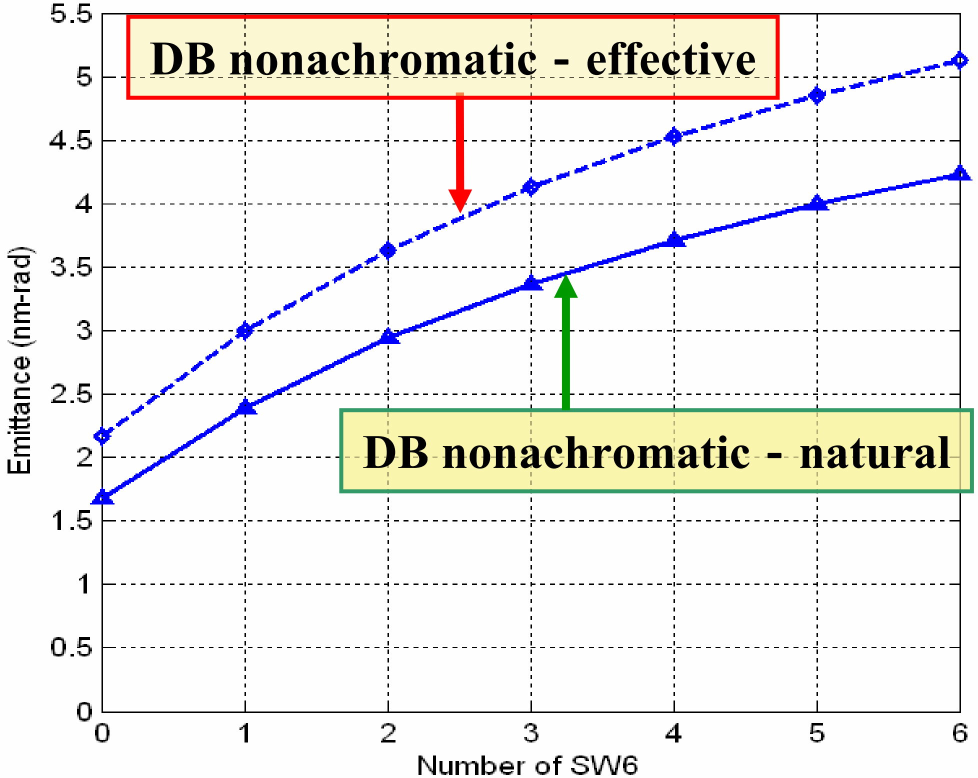
$$U_0 = C_\gamma E^4 / \rho_0$$

$$U_w = \begin{cases} C_\gamma E^4 L_w / (4\pi\rho_w^2), & \text{planar undulator} \\ C_\gamma E^4 L_w / (2\pi\rho_w^2), & \text{helical undulator} \end{cases}$$

$$\frac{\varepsilon_x}{\varepsilon_{x0}} \approx \begin{cases} \dfrac{(1 + \sum_w \frac{8B_w}{3\pi f_h B_0} \frac{U_w}{U_0})}{(1 + \sum_w \frac{U_w}{U_0})}, & \text{planar undulators} \\ \dfrac{(1 + \sum_w \frac{B_w}{f_h B_0} \frac{U_w}{U_0})}{(1 + \sum_w \frac{U_w}{U_0})}, & \text{helical undulators} \end{cases}$$

If $B_w < f_h(3\pi/8)B_0$, the emittance in the dispersive insertion will decrease.
If $B_w > f_h(3\pi/8)B_0$, the emittance in the dispersive insertion will increase.
To minimize emittance growth, one chooses high field solution, i.e. ρ_0 is small!





$$\varepsilon_{x,eff} = \sqrt{\varepsilon_x \varepsilon_{x,1D}} = \sqrt{\varepsilon_x^2 + H\sigma_E^2 \varepsilon_x}$$

$$H = \frac{1}{\beta_x}[D_x^2 + (\beta_x D_x' + \alpha_x D_x)^2]$$

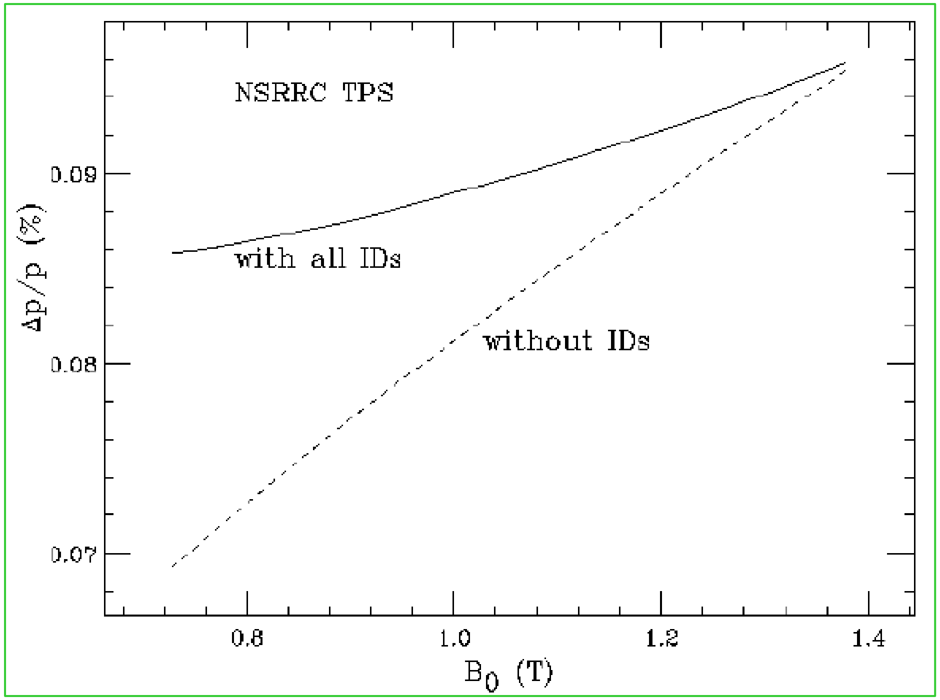
$$\left(\frac{\sigma_E}{E}\right)^2 = C_q \gamma^2 \frac{I_3}{(I_2 + I_4)} = \left(\frac{\sigma_E}{E}\right)_0^2 \frac{1 + I_{3w} / I_{30}}{1 + I_{2w} / I_{20}}$$

$$I_{30} = \frac{2\pi}{\rho_0^2}, \quad I_{3w} = \begin{cases} 4L_w / (3\pi\rho_w^3), & \text{planar undulator} \\ L_w / \rho_w^3, & \text{helical undulator} \end{cases}$$

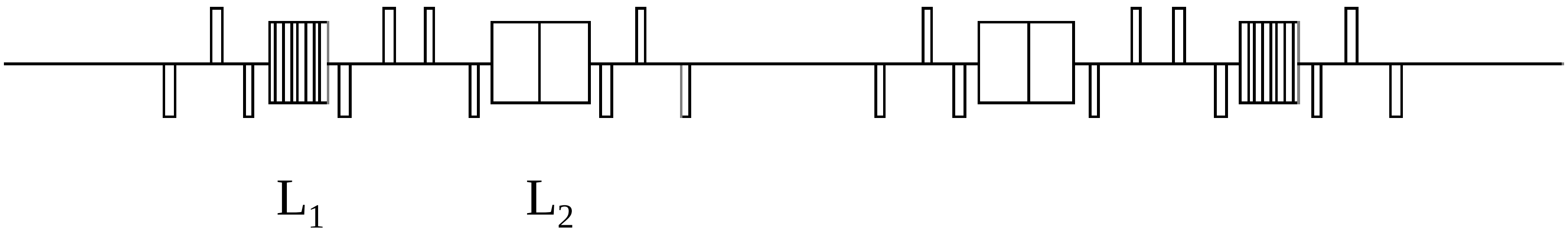
$$\frac{I_{3w}}{I_{30}} \approx \begin{cases} \frac{8\rho_0}{3\pi\rho_w} \frac{U_w}{U_0} = \frac{8B_w}{3\pi B_0} \frac{U_w}{U_0}, & \text{planar undulator} \\ \frac{\rho_0}{f\rho_w} \frac{U_w}{U_0} = \frac{B_w}{B_0} \frac{U_w}{U_0}, & \text{helical undulator} \end{cases}$$

If $B_w < (3\pi/8)B_0$, the momentum spread will decrease.

If $B_w > (3\pi/8)B_0$, the momentum spread will increase.



Quadruple-Bend Achromat (QBA)



A simple calculation with $L_2 = 3^{1/3} L_1$, using the same dipole strength

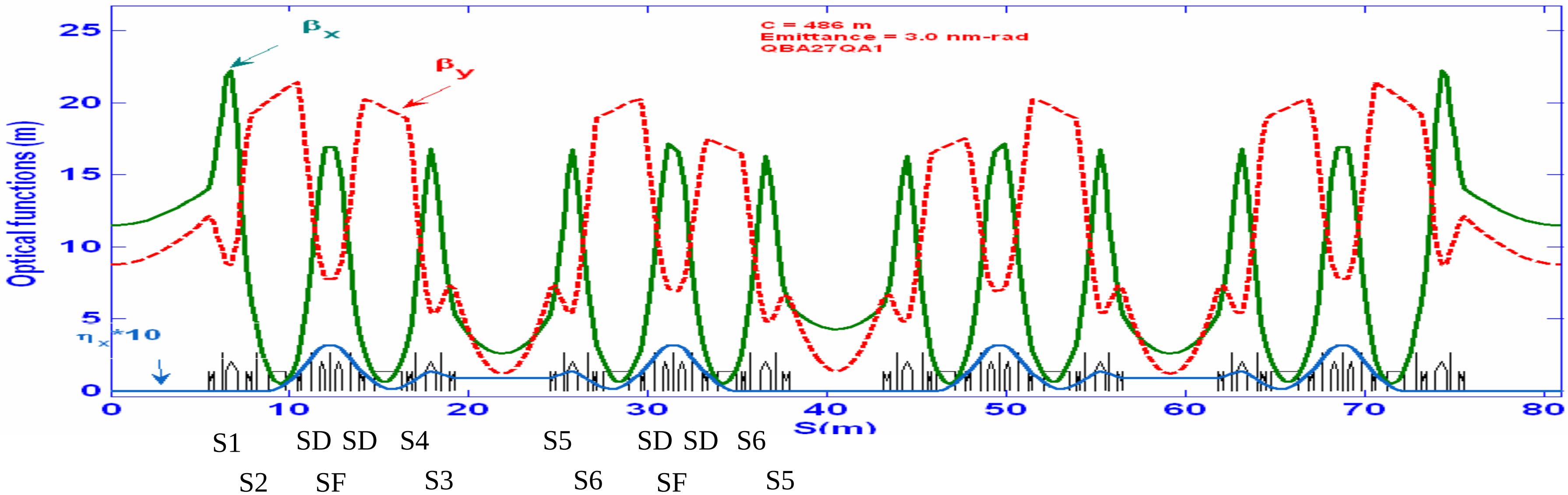
$$\theta_1 + 3^{1/3}\theta_1 = 2\theta_0 \quad \Rightarrow \theta_1 = \frac{2}{1+3^{1/3}}\theta_0$$

The emittance should scale like: $\boxed{\left(\frac{2}{1+3^{1/3}}\right)^3 = 0.54}$

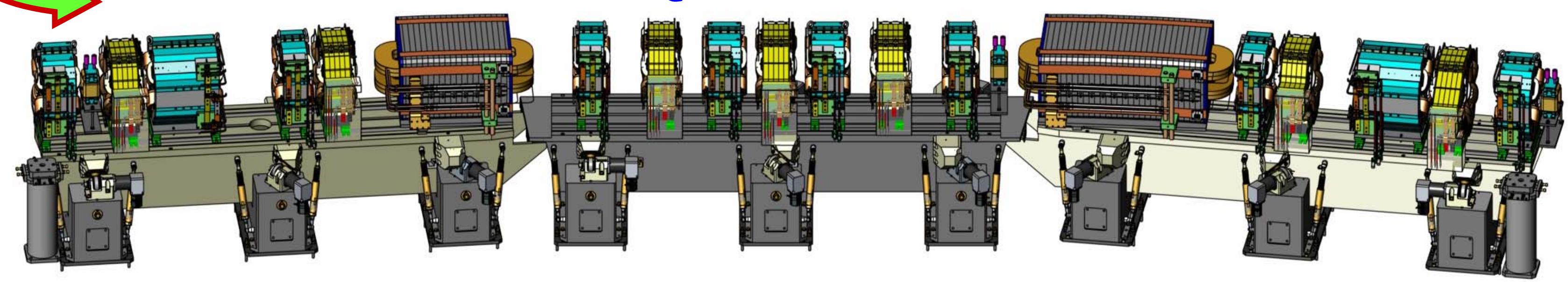
It is easy to find the following facts:

- Matching is relatively easy (as compared to TBA lattices)
- The chromatic properties is as good as the DB lattice!
- The emittance obeys the scaling law: $\gamma^2\theta^3$

A QBA Lattice Example

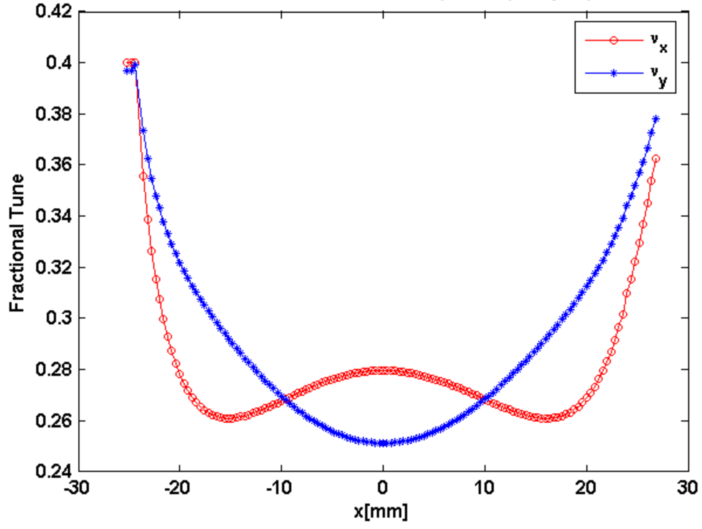


QBA 1/24

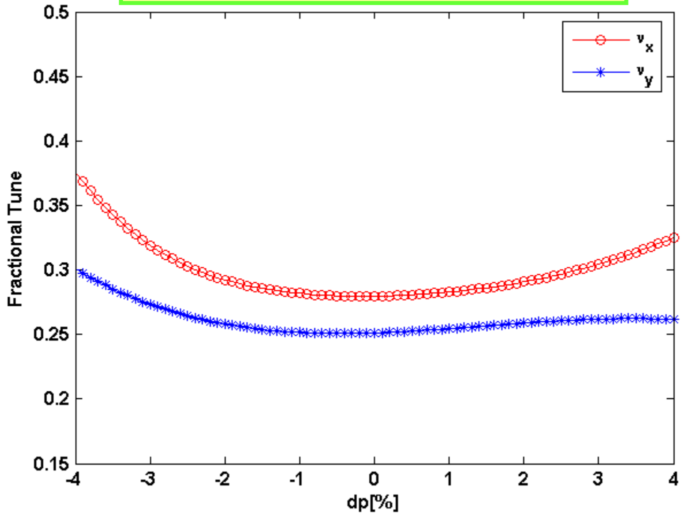


- Dipole length 1 m and 1.5 m, dipole field 1.047 T
- 486 m, 12 QBA cells, 6-fold symmetry
- Natural emittance: 3.0 nm-rad
- Natural chromaticity (x/y): -60/ -30

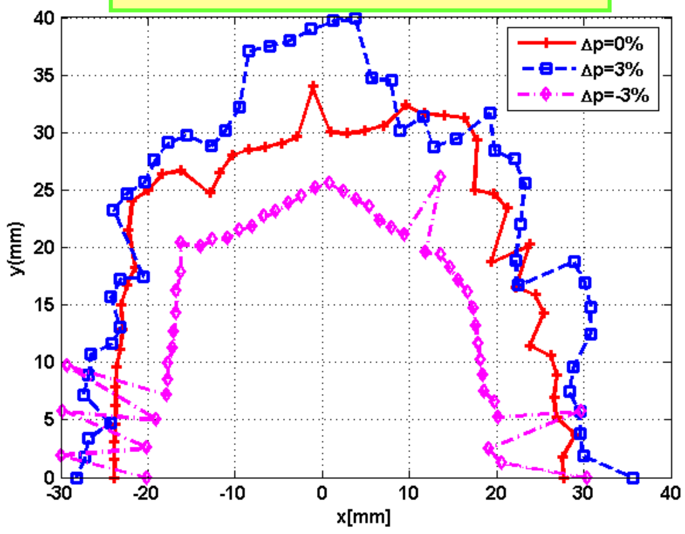
Tune shift with amplitude



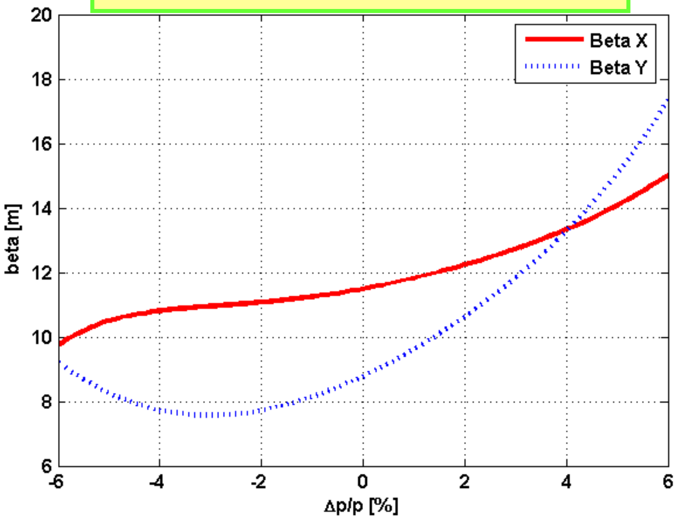
Tune shift with energy



Dynamic aperture

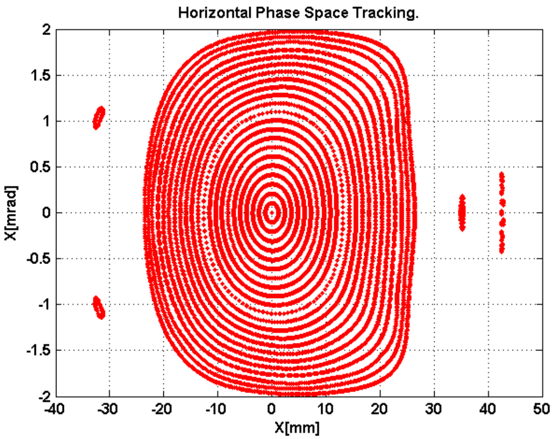


Beta function vs. energy

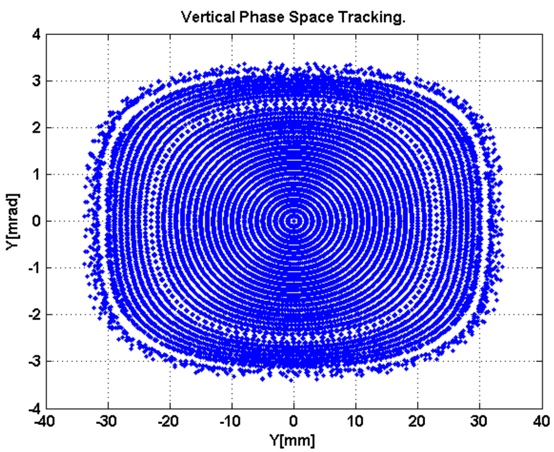


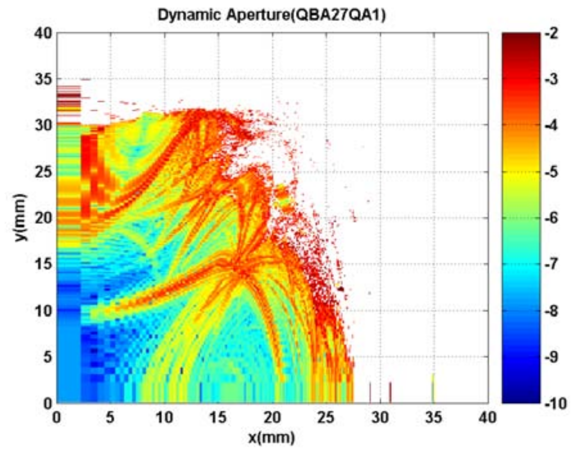
Phase space tracking
Frequency map analysis

x-x’



y-y’

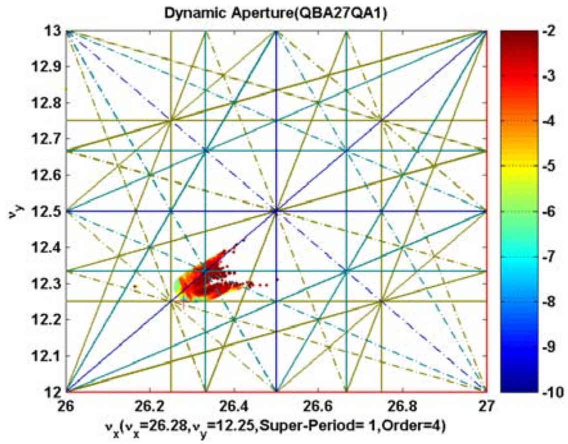


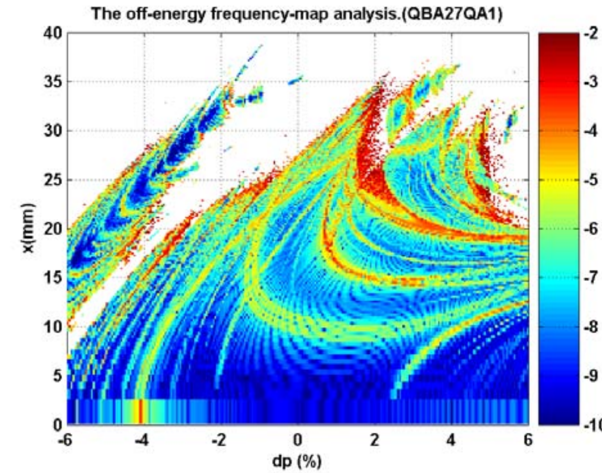


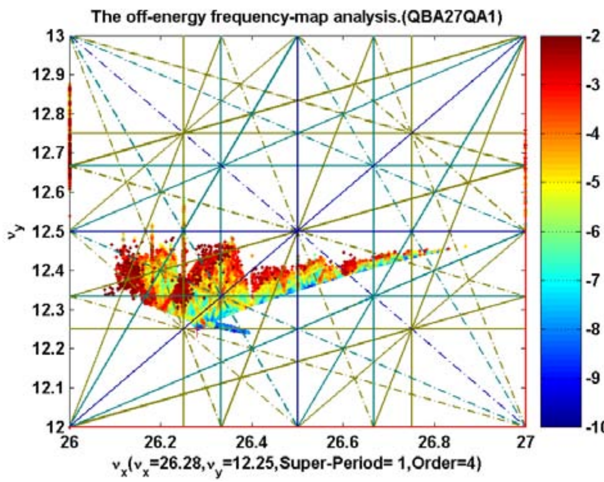
Diffusion D = $\log_{10}(||\Delta\nu||)$

$||\Delta\nu|| < 10^{-10}$ low diffusion

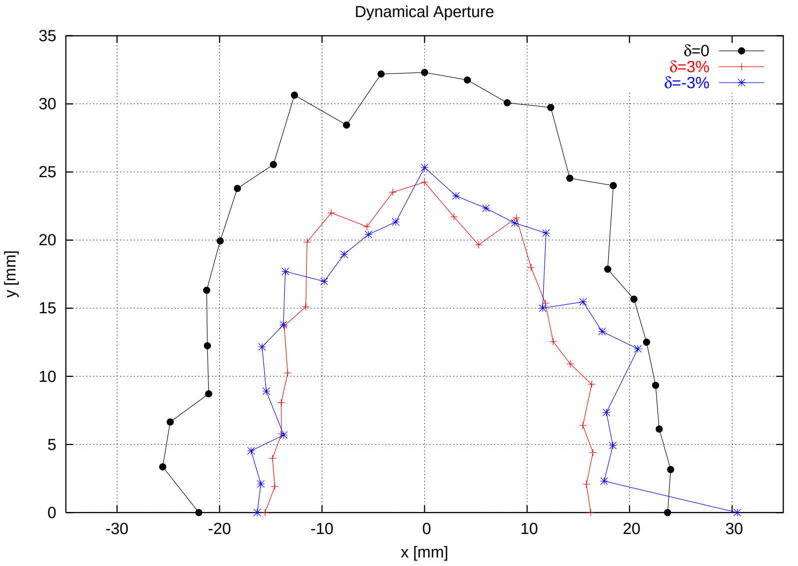
$||\Delta\nu|| > 10^{-2}$ high diffusion







6D tracking with nonlinear synchrotron oscillation



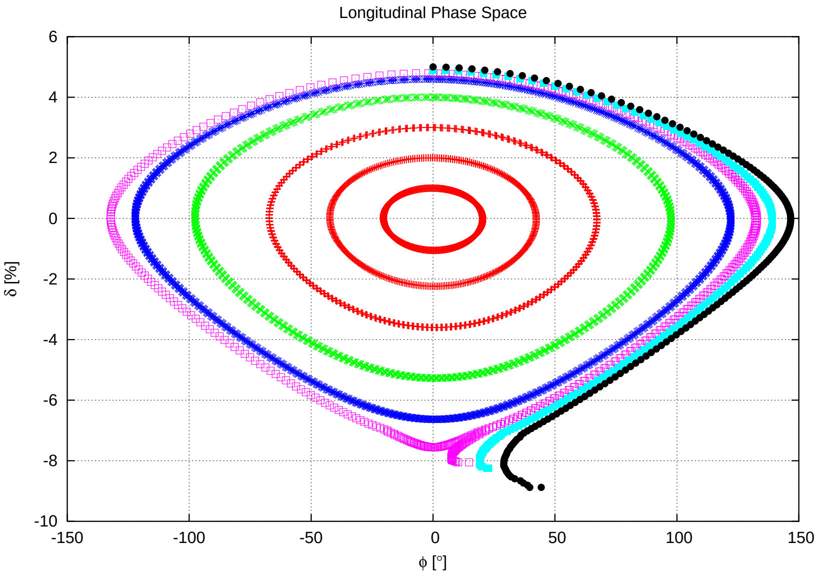
Momentum compaction factor:

$$\alpha_1 = 2.7\times10^{-4}, \alpha_2 = 2.1\times10^{-3}$$

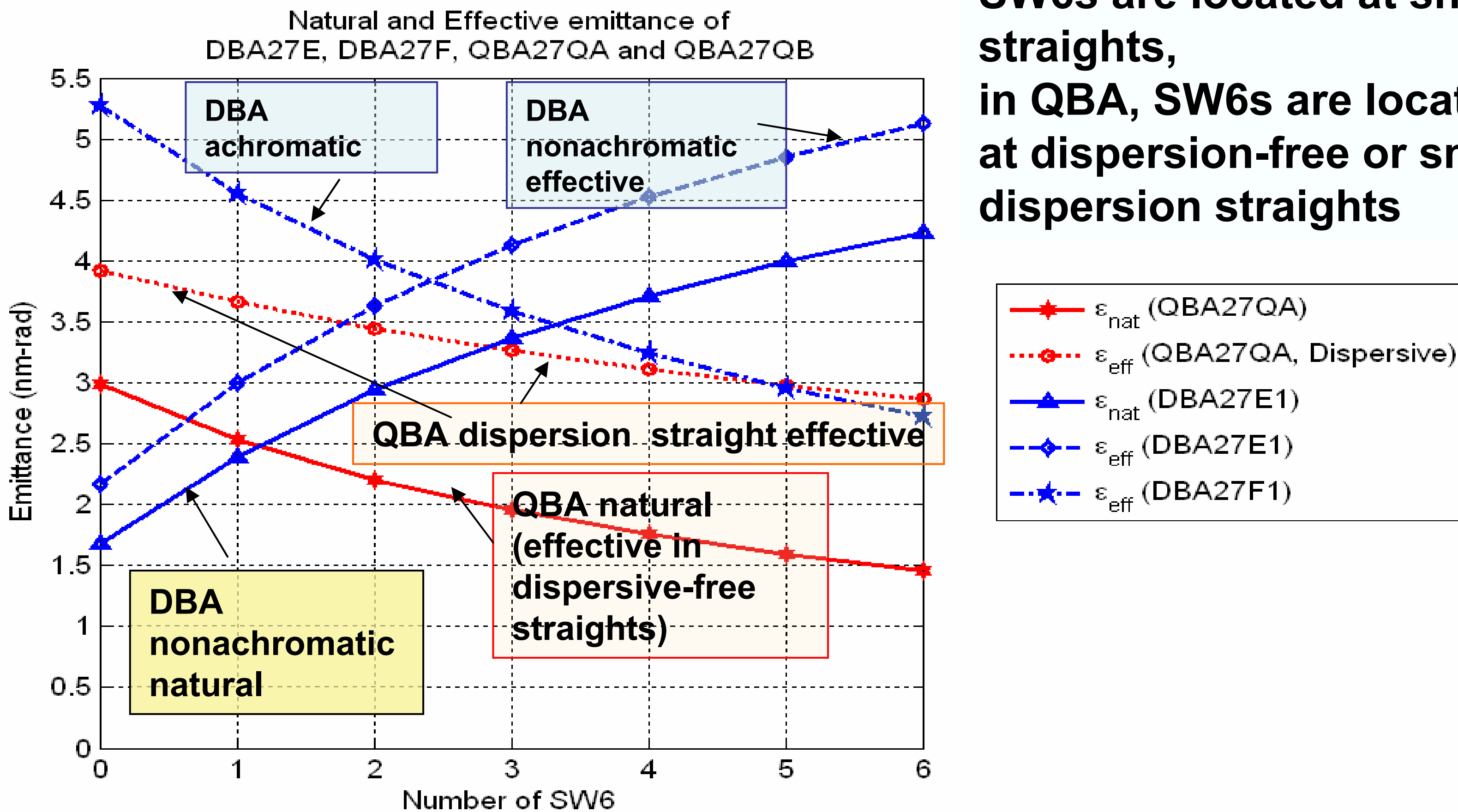
$$\alpha_1 = \frac{1}{L_0}\oint\frac{\eta_0}{\rho}ds$$

$$\alpha_2 = \frac{1}{L_0}\oint\left[\frac{\eta_0^2}{2}+\frac{\eta_1}{\rho}\right]ds$$

$$\alpha = \alpha_1 + 2\alpha_2\Delta p/p$$

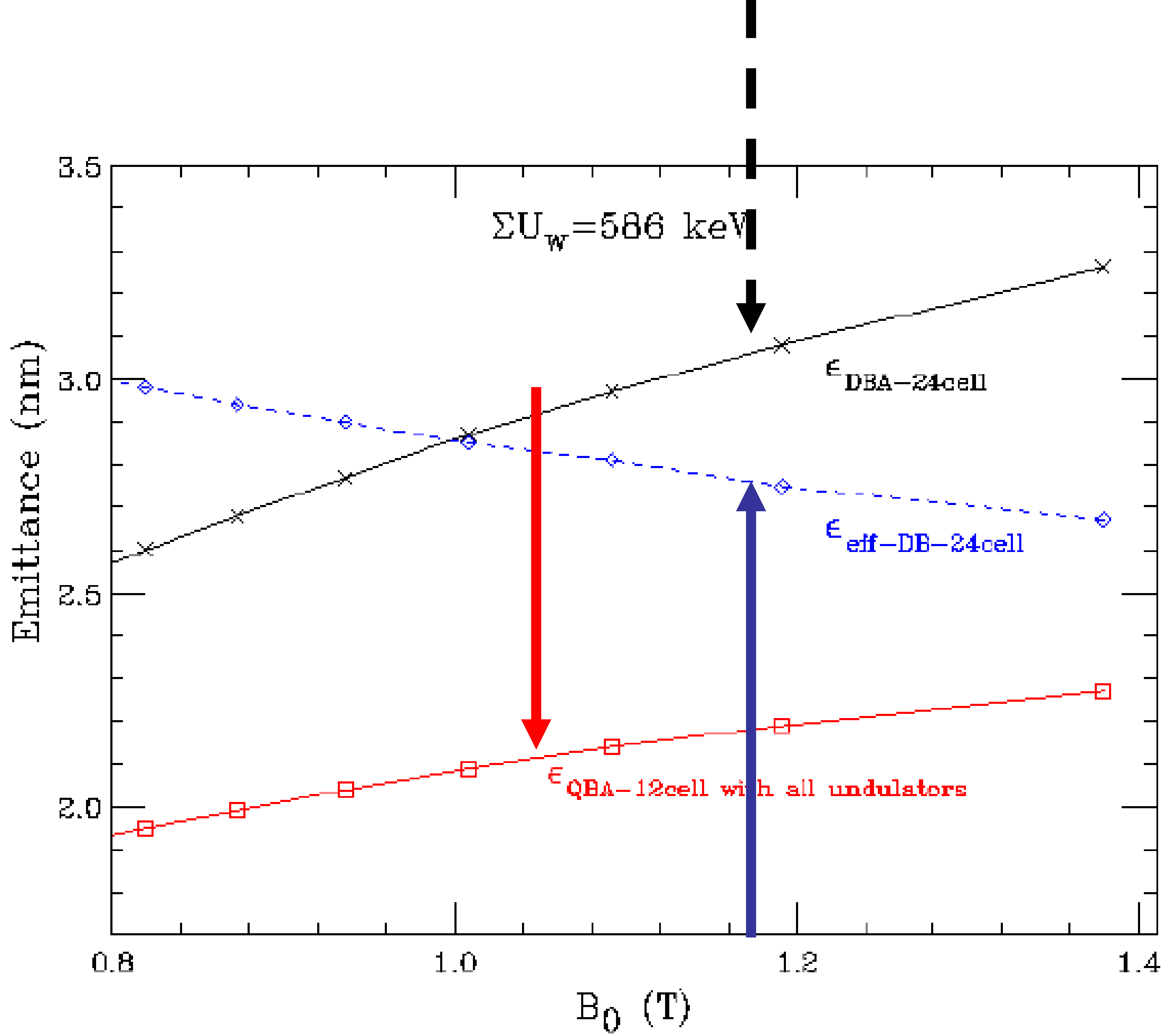


Natural emittance and effective emittance as a function of number of SW6 (2 m, 3.5 T, 6 cm period length



SW6s are located at short straights,
in QBA, SW6s are located at dispersion-free or small dispersion straights

TPS



3.5
3.0
2.5
2.0
Emittance (nm)
ΣU_w=586 keV
ε_DBA−24cell
ε_eff−DB−24cell
ε_QBA−12cell with all undulators
0.8
1.0
1.2
1.4
B_0 (T)

Conclusion

- The QBA lattice is a combination of DBA and nonachromatic DB lattices
- The theoretical minimum emittance is half of the DBA lattice
- The QBA lattice has moderate natural chromaticities, $\xi_{x,y}/Q_{x,y}$~ -2
- The effective emittance of QBA lattice is smaller than that of nonachromatic DB lattice
- The QBA lattice provides some zero-dispersion sections for high field damping wigglers to gain emittance reduction
- The dipole field in QBA should be chosen to gain emittance reduction for wigglers in the dispersion free straight sections, and not too small to allow weaker field undulators in the dispersive straight sections.