Short Wavelength SASE FELs: Experiments vs. Theory

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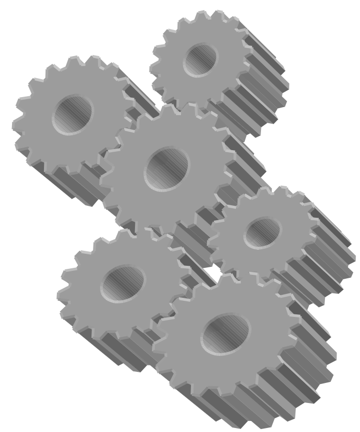
Contents

INPUT (electrons)

- Momentum
- Momentum spread/chirp
- Slice emittance/ phase space distribution
- Total charge
- Long. charge profile
- Peak current
- Orbit control

FEL machinery



OUTPUT (photons)

- Gain length
- Saturation behaviour
- Spectrum
- Harmonics
- Transverse coherence
- Pulse length
- Effective input power
- Fluctuations

Do we understand the machinery ?

FEL Basics

Point-like bunch of electrons radiates coherently $P \propto N_e^2$!

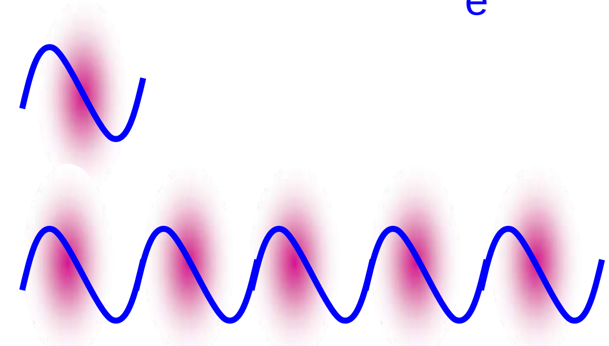
„Point“ means above all: bunch length < $\lambda_{radiation}$

Synchrotron radiation of incoherent electron distribution: $P \propto N_e$

→ desired: bunch length < wavelength

OR (even better)

Density modulation at desired wavelength

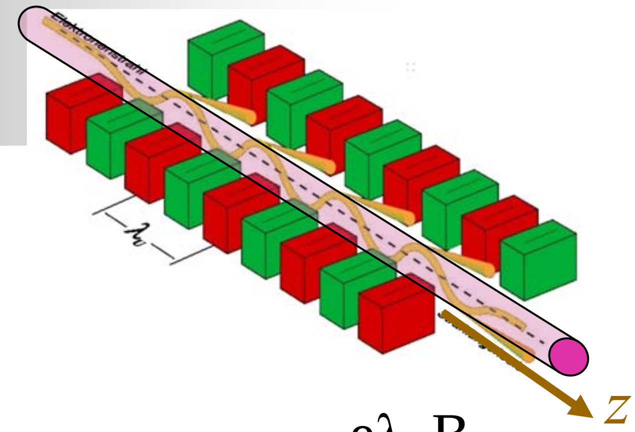


→ Potential gain in power: $N_e \sim 10^6$!!

Idea:

Start with an electron bunch much longer than the desired wavelength and achieve bunching at the optical wavelength automatically

Free-Electron Laser (Motz 1950, Phillips ~1960, Madey 1970)



Step 1: Energy modulation

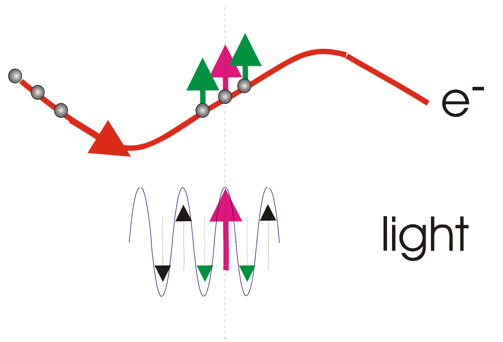
A: Electron travels on sine-like trajectory

$$v_x(z) = c\frac{K}{\gamma}\cos\left(\frac{2\pi}{\lambda_u}z\right)$$, with undulator parameter: $$K = \frac{e\lambda_u B}{2\pi m_e c}$$

B: External electromagnetic wave moving parallel to electron beam:

$$E_x(z,t) = E_0\cos(k_L z - \omega_L t)$$

Change of energy W in presence of electric field:



$$\frac{dW}{dz} = \frac{q}{v_z}\vec{v}\vec{E} = -\frac{qE_0 K}{\gamma\beta_z}\sin\Psi,$$

with the ponderomotive phase: $$\Psi = (k_u + k_L)z - \omega_L t + \varphi_0$$

Basic FEL theory

$$\frac{dW}{dz} = -\frac{qE_0 K}{\gamma\beta_z} \sin\Psi$$

The energy *dW* is taken from or transferred to the radiation field.
For most frequencies, *dW/dz* oscillates very rapidly.

Continuous energy transfer ?

Yes, if Ψ constant.

$$\rightarrow \quad \frac{d\Psi}{dz} = 0 \;!$$

→ Resonance condition: $\lambda_L = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$

Same equation as for wavelength of undulator radiation !

Step 2: Current modulation

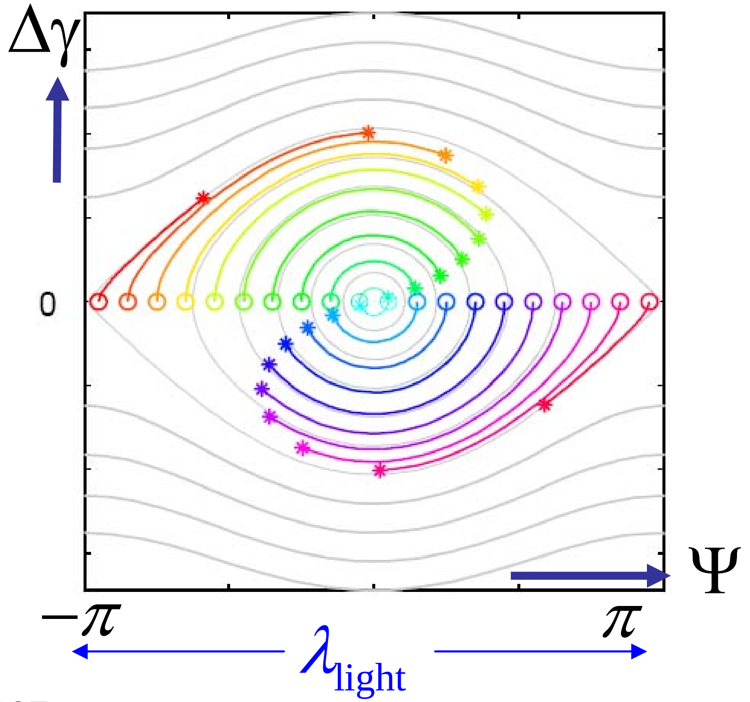
Energy modulation by $\Delta\gamma$ leads to change of Phase Ψ:

$$\frac{d\Psi}{dz} = k_u \frac{2}{\gamma_{res}} \Delta\gamma$$

Combined with Step 1: $\frac{dW}{dz} = -\frac{qE_0 K}{\gamma\beta_z} \sin\Psi$ yields

$$\frac{d^2\Psi}{dz^2} = -\Omega^2 \sin\Psi$$

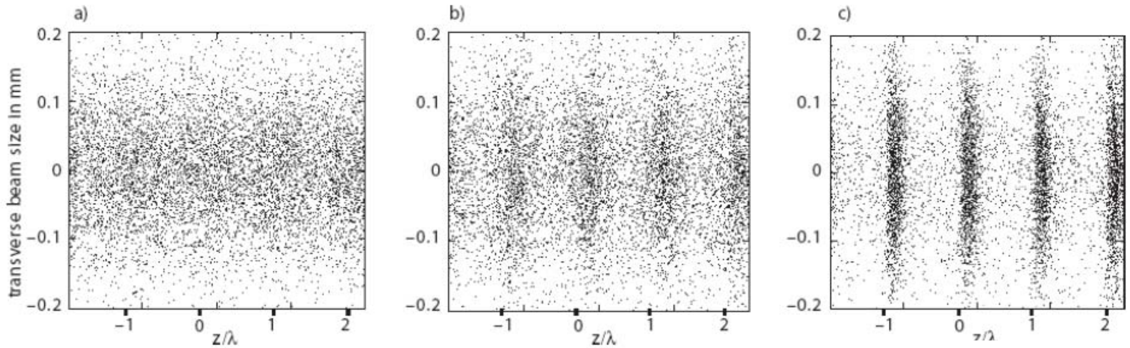
with $\Omega^2 = \frac{q}{m_0 c^2} \frac{E_0 K k_u}{\gamma_{res}^2 \beta_z}$



like synchrotron oscillation

-- but at spatial period λ_{light}

→ current modulation !!



Step 3: Radiation

Current modulation j_{Light} drives radiation of light:

$$\frac{dE_{Light}}{dz} = \mathrm{const} \cdot j_{Light}$$

System of Diff. Eqs. defines **High Gain FEL:**
(Kondratenko, Saldin 1980)
(Bonifacio, Pellegrini 1984)

$$\frac{d}{dz}\begin{pmatrix} \Delta\gamma \\ \psi \\ E_{Light} \end{pmatrix} = \begin{pmatrix} -\frac{qE_{Light}K}{m_e c^2 \gamma_{res}^2}\sin\Psi \\ k_u \frac{2}{\gamma_{res}}\Delta\gamma \\ \mathrm{const} \cdot j_{Light} \end{pmatrix}$$

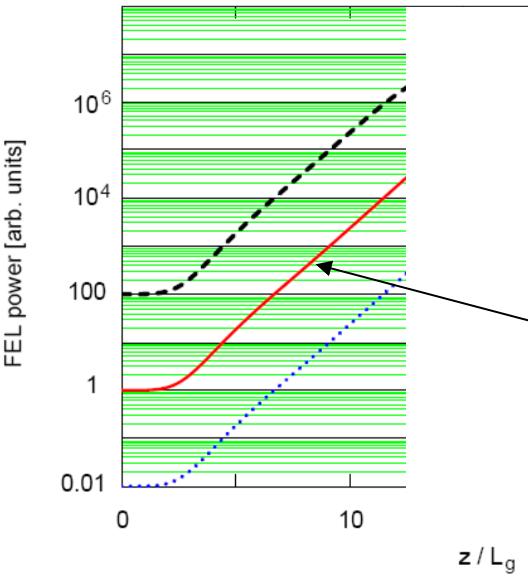
Energy modulation

Density modulation

Radiation

Theory: High-gain FEL

Most simple case:



$$\frac{d^3\mathbf{E}}{dz^3} = i\Gamma^3\mathbf{E}$$. Ansatz: $\mathbf{E} = A\,\mathbf{exp}(\Lambda z)$

$$\Lambda^3 = i\Gamma^3 \Rightarrow \Lambda_1 = -i\Gamma; \Lambda_2 = \frac{i+\sqrt{3}}{2}\Gamma; \Lambda_3 = \frac{i-\sqrt{3}}{2}\Gamma$$

$z >> \Gamma^{-1}$: exponential growth:

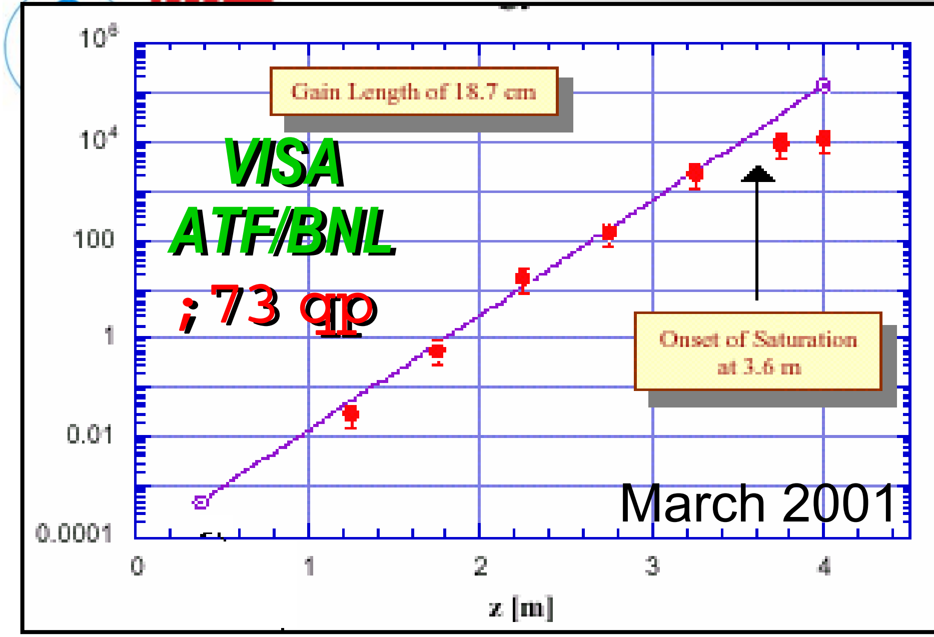
$$P_{rad} = \frac{1}{9} P_{in} \exp\left(\frac{z}{L_G}\right) \qquad L_G = \frac{1}{\sqrt{3}}\left(\frac{I_A \gamma^3 \sigma_r^2 \lambda_u}{4\pi \cdot \hat{I} \cdot K^2}\right)^{1/3}$$

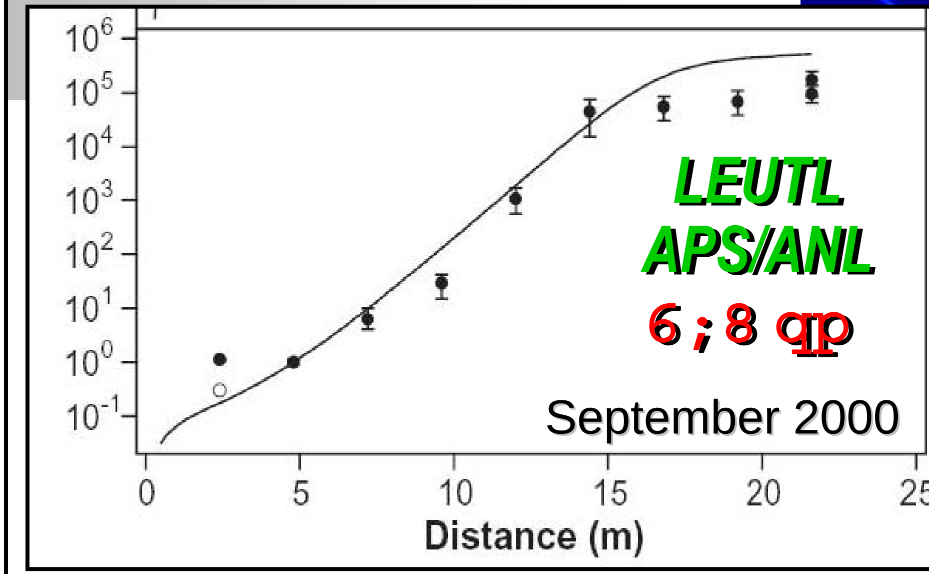
$$L_G \propto (\text{current density})^{-1/3} \qquad \rho_{FEL} = \frac{1}{4\pi\sqrt{3}} \frac{\lambda_u}{L_G} \approx 10^{-4} \ldots 10^{-2}$$

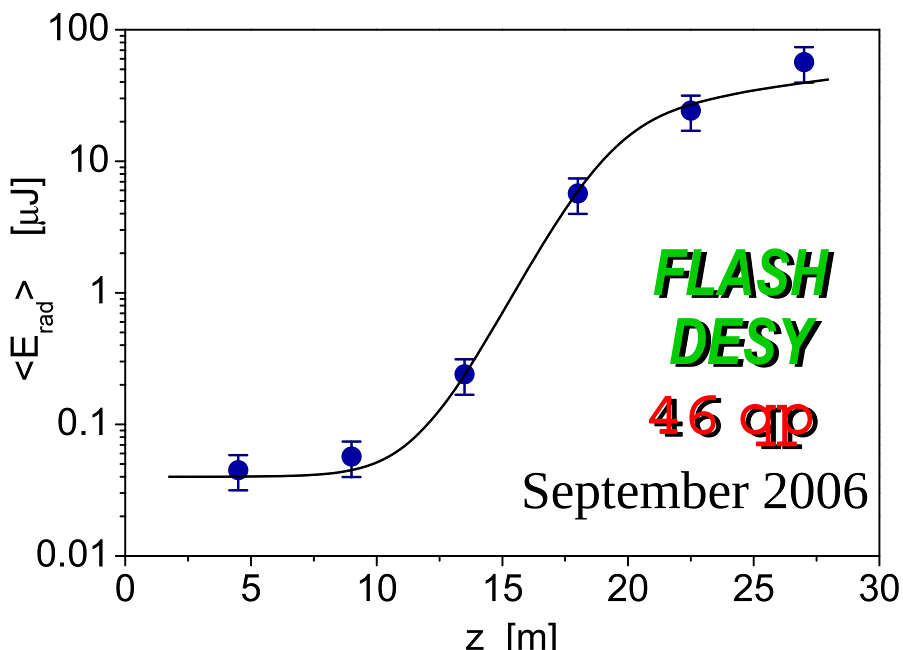
1. Expect exponential gain with e-folding length L_G
   Major additional assumption: Orbit is perfectly straight

2. Gain should saturate when modulation is complete

What do we observe ?







For all experiments there exists a "reasonable" electron beam parameter set such that gain length and saturation level agree with theoretical expectations.

Exponential growth ? ✔

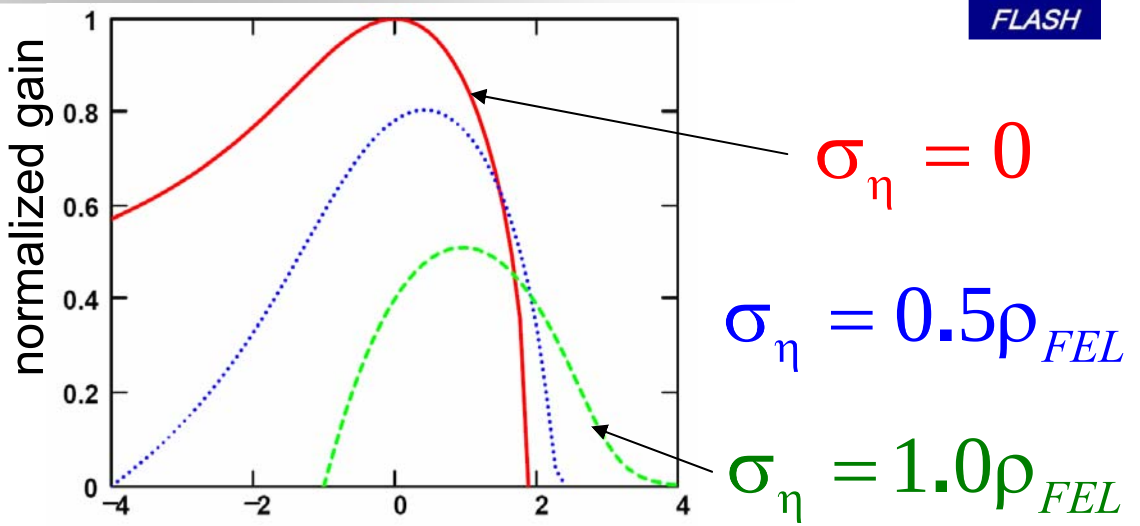
Reasonable gain length ? ✔

Achieve full density modulation ? ✔

But: measurement of relevant beam parameters is not precise enough to just predict gain length with reasonable precision.

Bandwidth

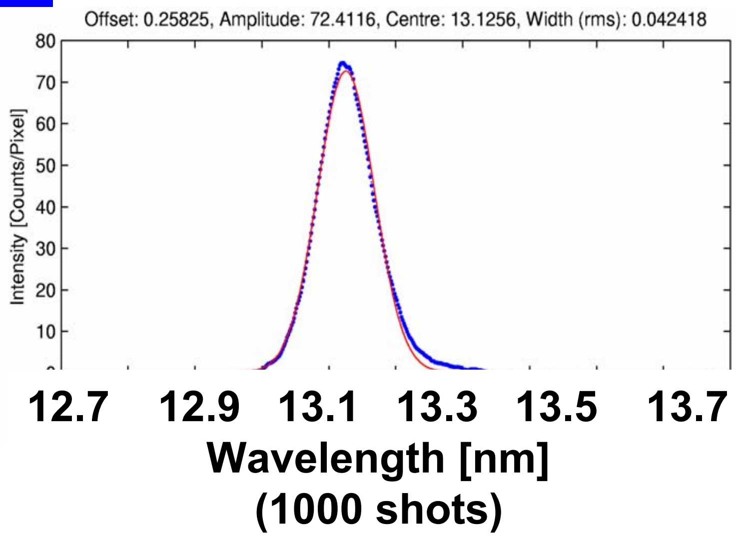
Gain vs.
momentum error η=dp/p
(momentum spread σ_η)
Note:
Emittance effect similar



FEL is a narrow band amplifier
Note: Cannot produce few-cycle pulses !

FLASH experiment:

Bandwidth ? ✔



Start-up from noise

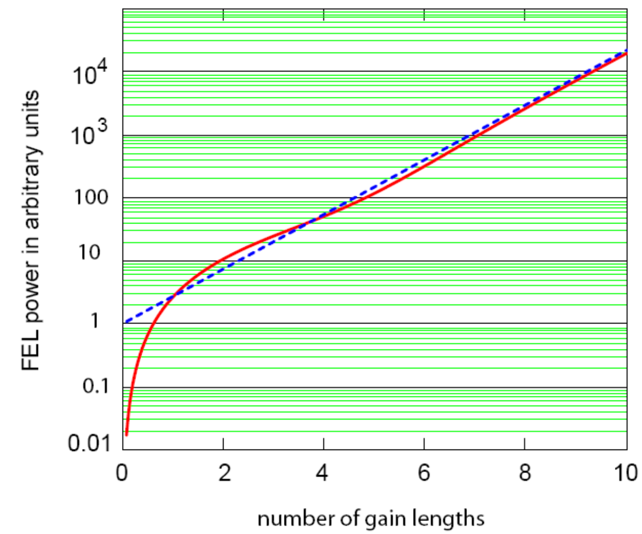
FEL can also start from initial density modulation given by noise.

Equivalent: starting from spontaneous undulator radiation.

Self-Amplified Spontaneous Radiation

SASE

Very robust mode of operation !



Theory must model shot noise.

Predicts effectiv “initial conditions”

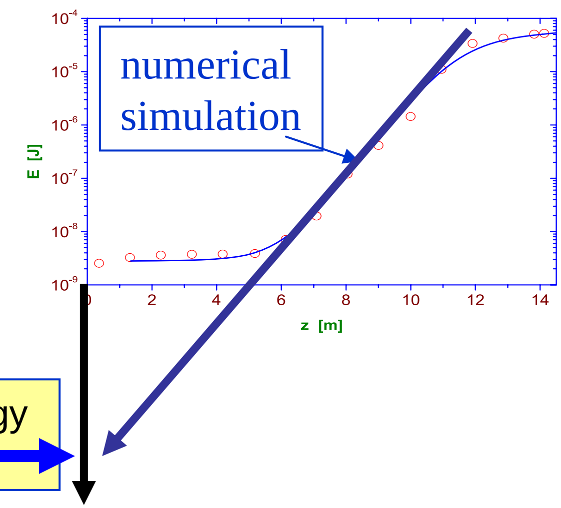
Critical bench mark test for numerical FEL codes, e.g.

GENESIS (Reiche)

GINGER (Fawley)

SIMPLEX (Tanaka)

FAST (Yurkov)



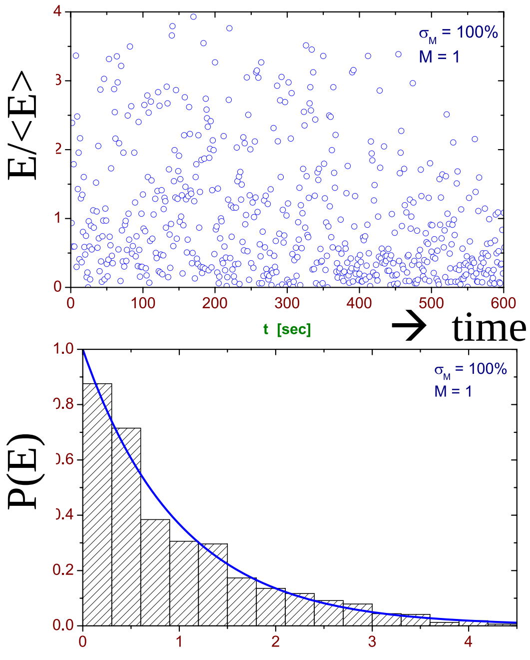
Equivalent input energy by shot noise:0.3 pJ

Jörg Rossbach, Univ HH

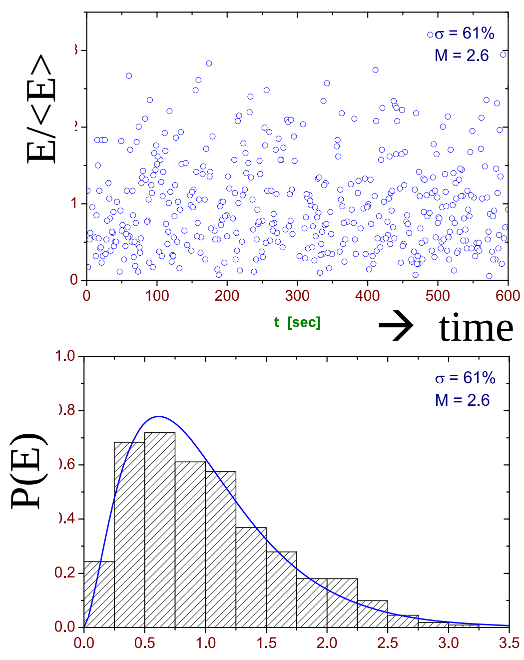
Start-up from noise

SASE output will fluctuate from pulse to pulse,
-- just as ANY part of spontaneous synchrotron radiation does !
Remember: FEL is just an amplifier !

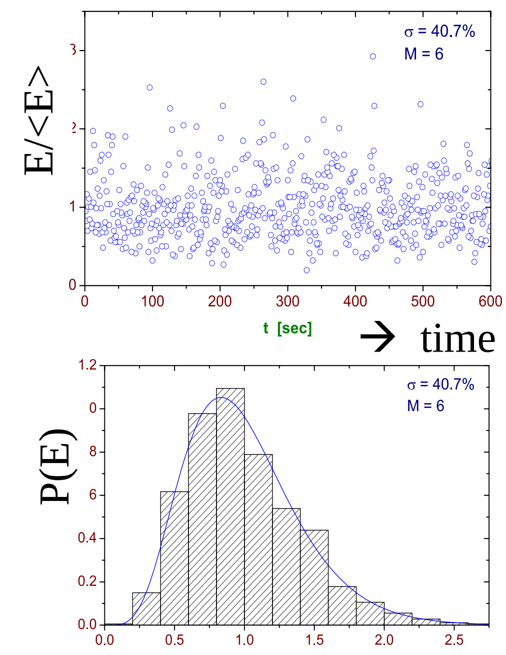
single Mode
(after monochromator slit)



short pulses
M=2.6 modes



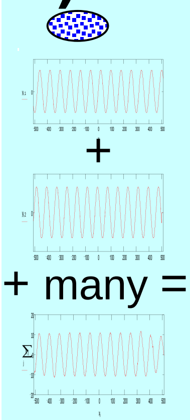
long pulses
M=6 modes



Start-up from noise

Simple 1D model: Superposition of many wavetrains with random phases

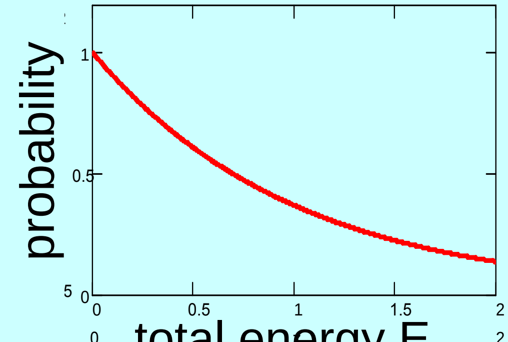
A) Short bunch << wavetrain



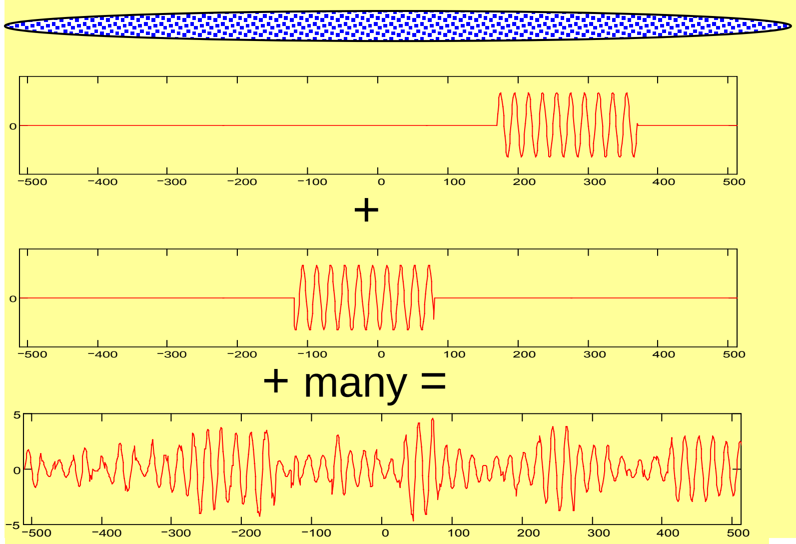
Large probability of destructive interference

"single Mode"

$$P(E)dE = \exp(-E)\,dE$$



B) Bunch length >> wavetrain: ***"many Modes"***

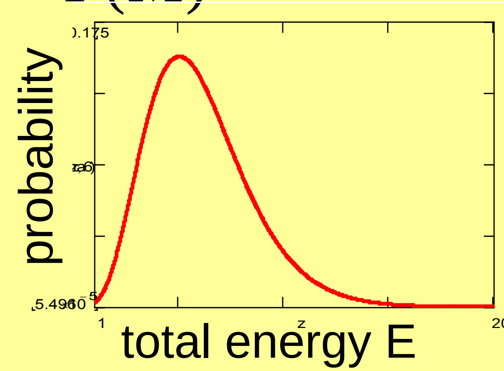


$$g_M(E)\cdot dE = \frac{1}{\Gamma(M)}(E)^{M-1}e^{-E}\cdot dE$$

Extract M from

histrogram

→ pulse length



Fluctuation properties ? ✔

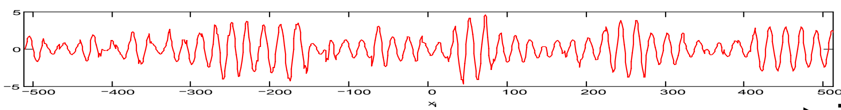
Pulse length

Time-domain measurement of pulse length:

not (yet) available for X-ray (established in the visible, FROG etc.)

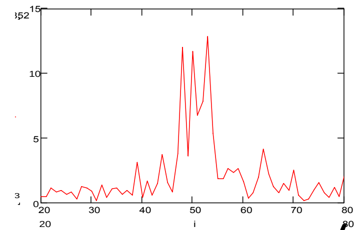
Alternative: intensity fluctuation translates into spectral fluctuation:

Width of frequency spikes ↔ length of pulse



→ T

Fourier →



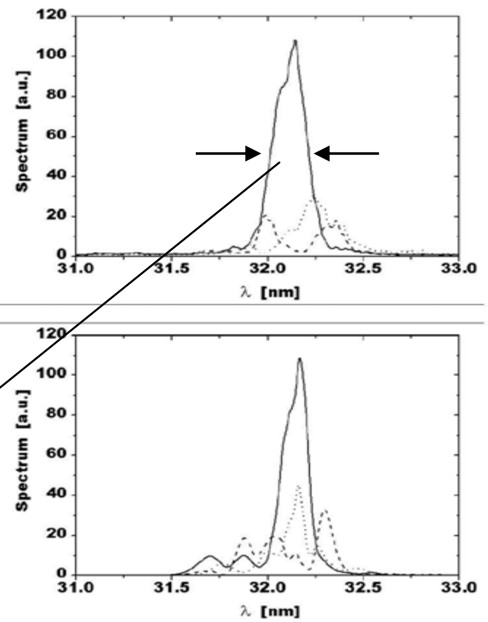
→ ω

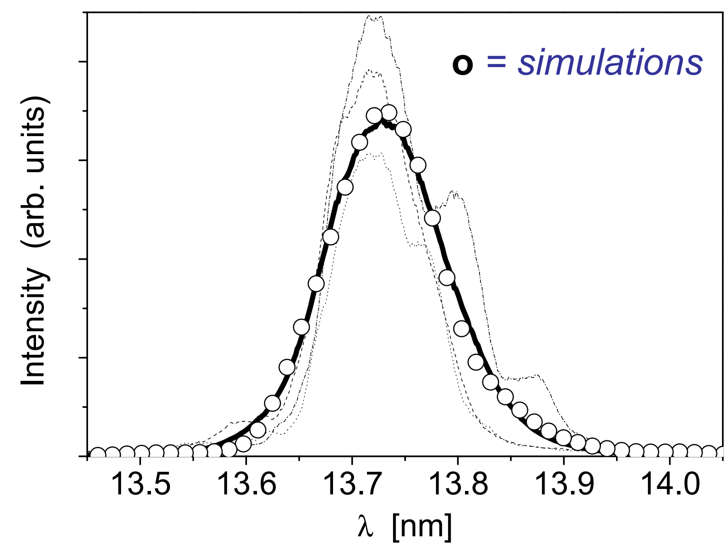
new_fund_harmonic_w.avi

3 single pulse Spectra @FLASH:

measured

predicted





~0.4% → ~25 fs pulse duration @ 32 nm

Pulse length ?

Transverse Coherence

Emittance of a perfectly coherent ("gaussian") light beam emittance:

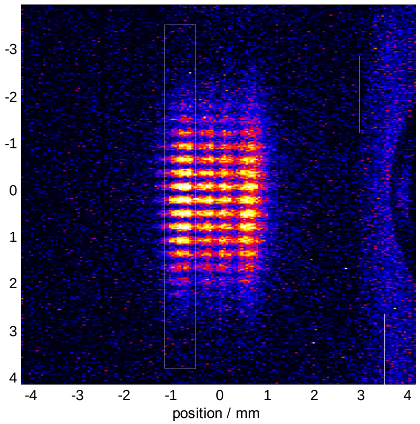
$$\varepsilon_{Light} = \sigma_r \cdot \sigma_\theta = \frac{\lambda_{Light}}{4\pi}$$

→ FEL theory predicts high transverse coherence of photon beam, if electron beam emittance:

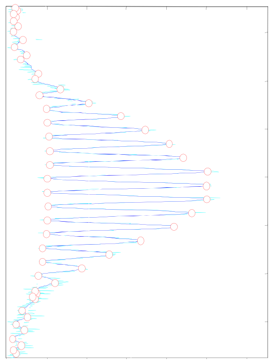
$$\varepsilon_{electrons} < \approx \frac{\lambda_{Light}}{4\pi}$$

Observation of interference pattern at FLASH:

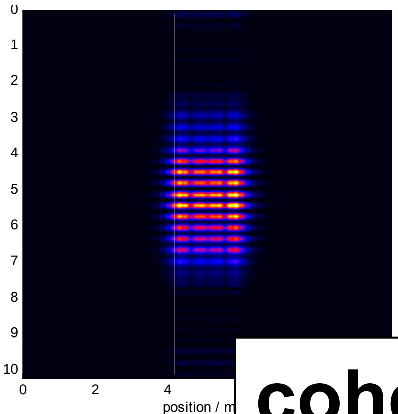
double slit



intensity modulation



FEL simulation



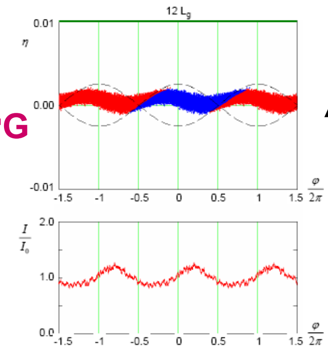
Verified @ 32nm + 13nm

coherence ? ✔

Higher Harmonics

Density modulation becomes anharmonic at high gain:

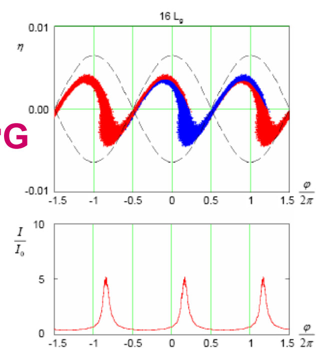
$12\ L_G$



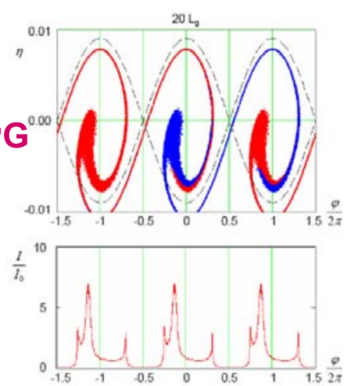
$\Delta\gamma / \Psi$

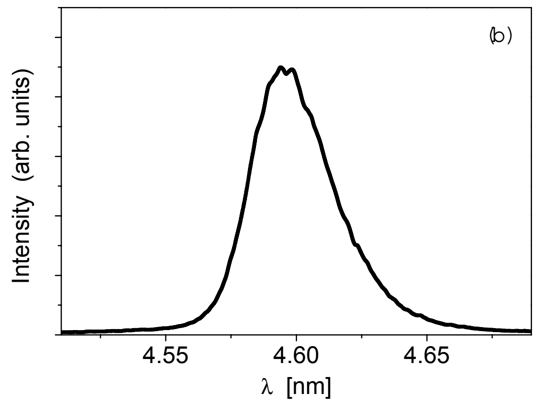
current

$16\ L_G$

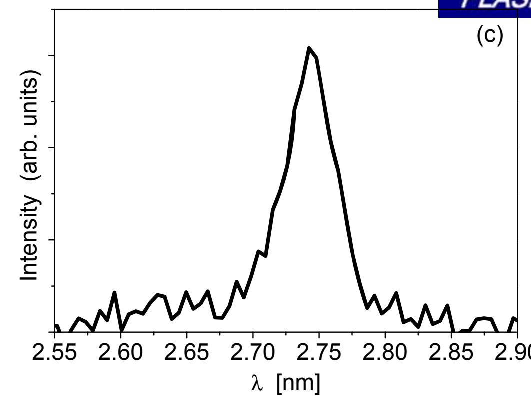


$18\ L_G$





3rd harmonics
@ 4.8 nm



5th harmonics
@ 2.75 nm

new third harmonic

FLASH typical pulse energies (avg.):

Fundamental (13.8 nm):	40 μJ
3rd harmonics (4.6 nm):	(0.25 ± 0.1) μJ
5th harmonics (2.75 nm):	(10 ± 4) nJ

Harmonics ? ✔

Electron Beam on the fs Scale

Most electron beam parameters relevant within slices < coherence length ~1 … 10 fs

- relaxes requirements on beam specs
- complicates measurements and beam dynamics

Emittance:	$\varepsilon \leq \lambda/4\pi$
Short Pulse length	σ_s = 10 – 100 fs
Peak current inside bunch:	Î > 1 kA
Energy width:	σ_E/E ≤~10-3
Straight trajectory in undulator	< 10 μm

Increasingly difficult for shorter wavelength:

longer undulator, smaller emittance, larger peak current

E. Prat:

THAN 026

Beam dynamics simulation tools

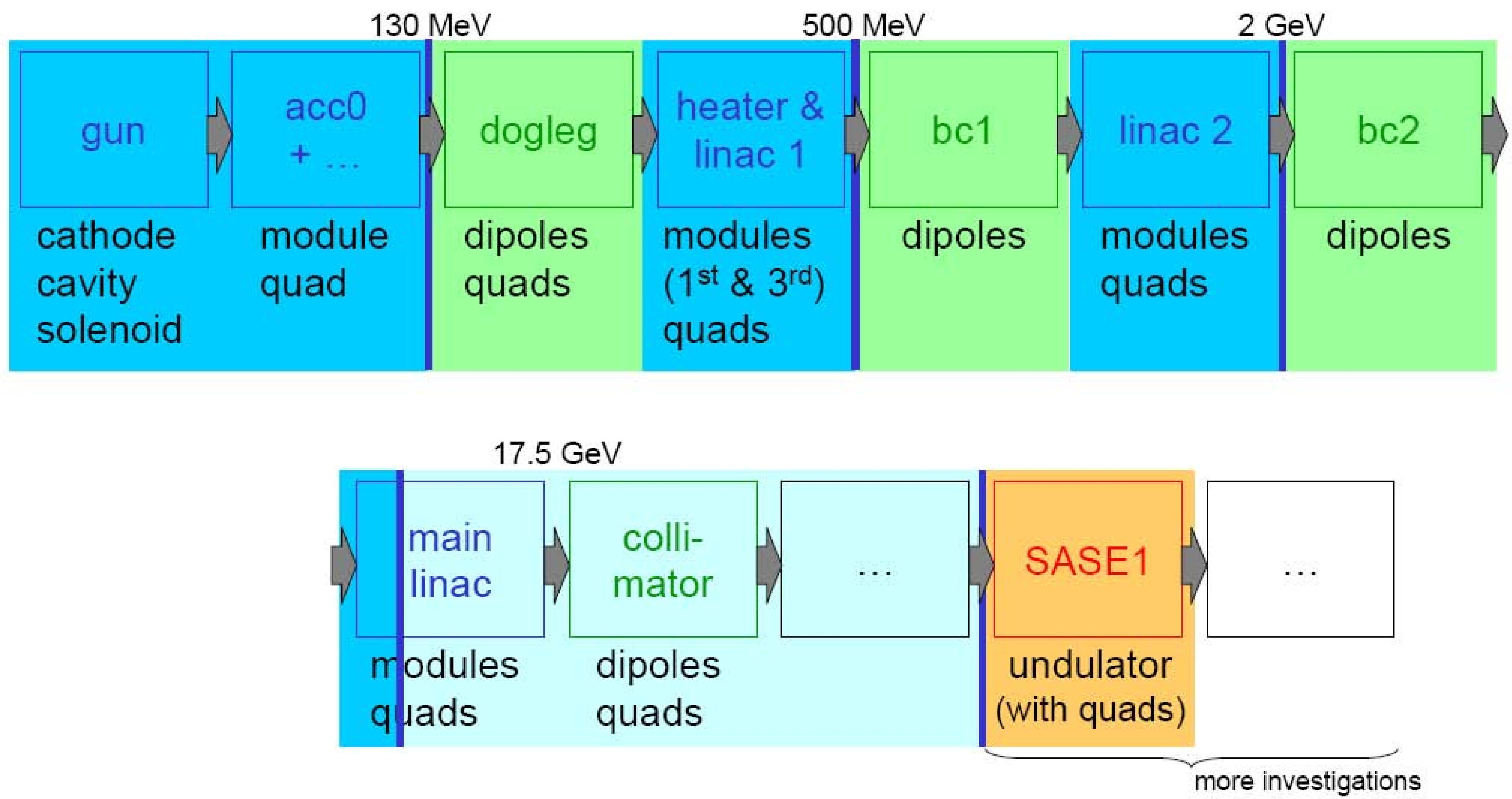
ASTRA — Flöttmann

CSRtrack sub-bunch — Dohlus

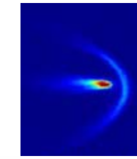
ELEGANT — Borland

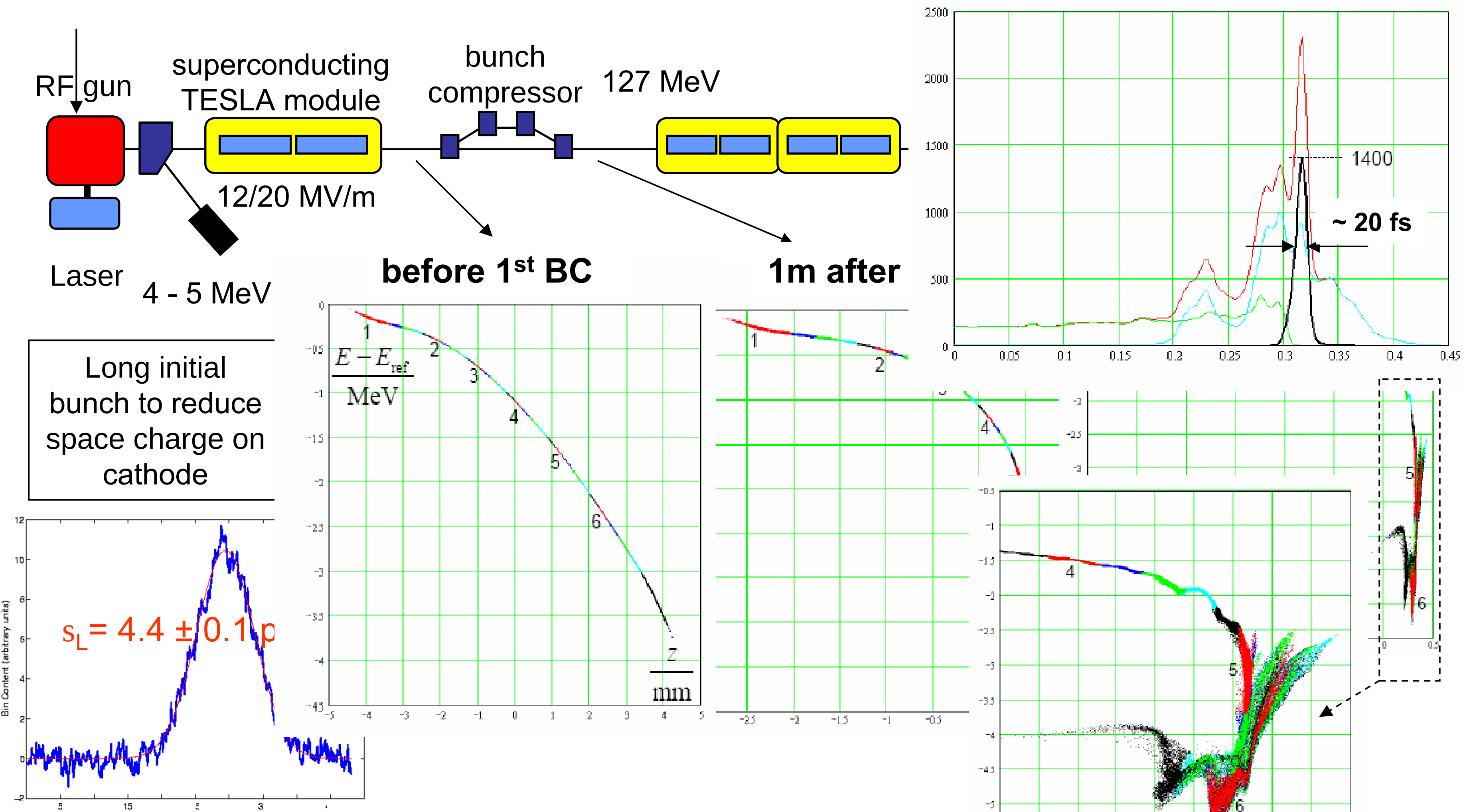
rf-field
cavity wake & s.c. field

GENESIS — Reiche



Longitudinal bunch compression



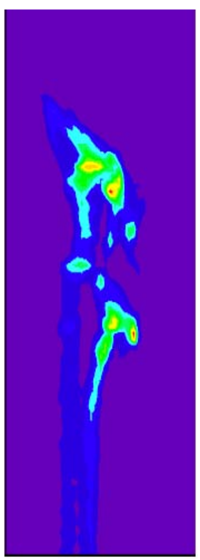


- Very complicated beam dynamics due to coherent synchrotron radiation
- Difficult access to relevant parameters
- Ultra-short photon pulses created ~20fs FWHM

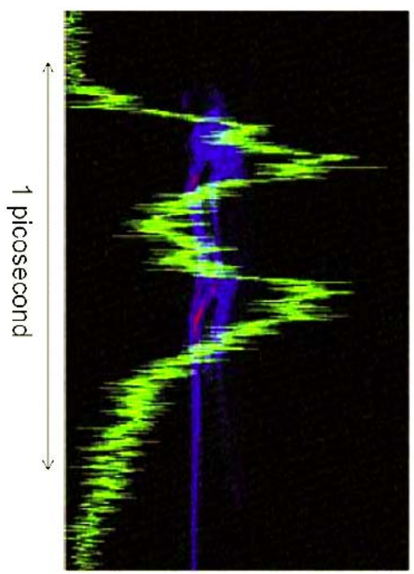
Resolving 20 fs with LOLA

Three examples for different compressor settings:

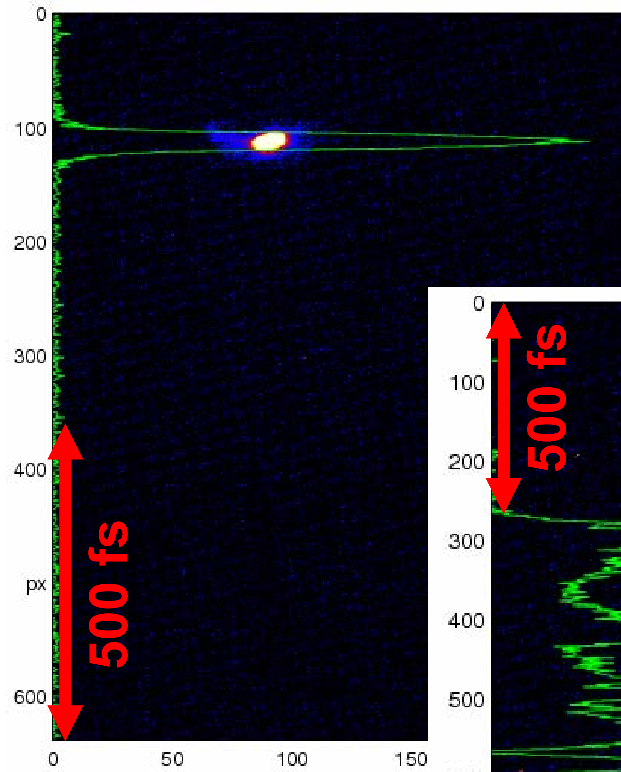
Resolution ~20 fs

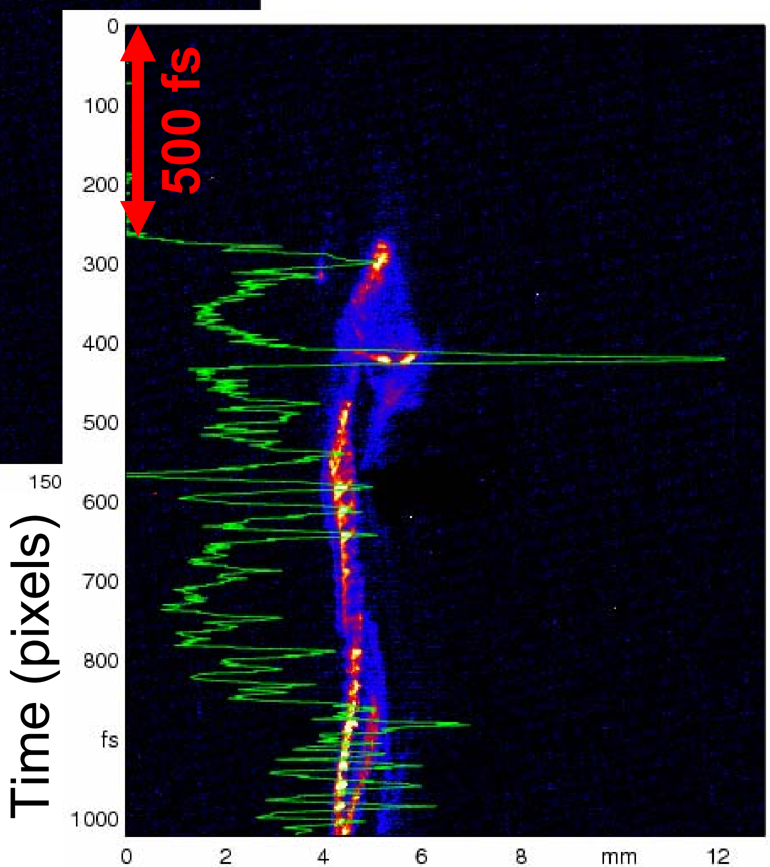


simulation



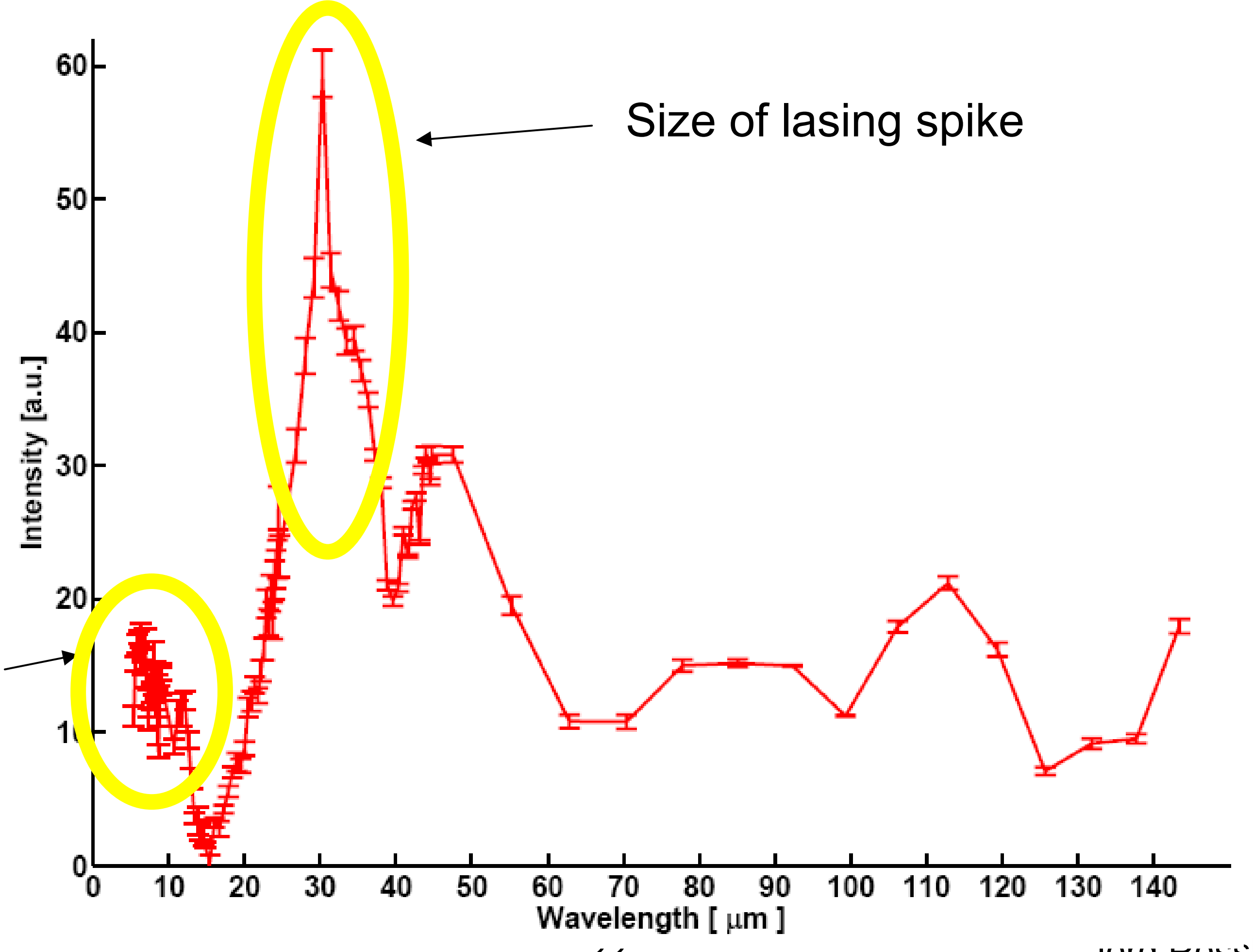
LOLA





fs diagnostics with THz radiation

Single shot spectrum of coherent infrared radiation exhibits structure in the longitudinal density modulation < 5 µm

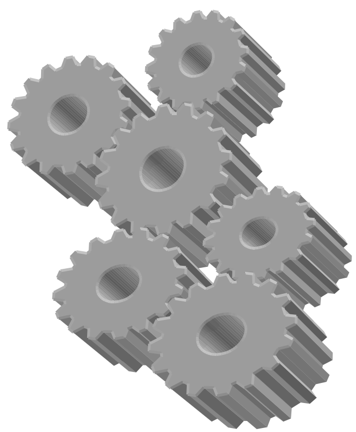


Do we understand the machinery ?

INPUT (electrons)

- Momentum
- Momentum spread/chirp
- Slice emittance/ phase space distribution
- Total charge
- Long. charge profile
- Peak current
- Orbit control

FEL machinery



OUTPUT (photons)

- Gain length
- Saturation behaviour
- Spectrum
- Harmonics
- Transverse coherence
- Pulse length
- Effective input power
- Fluctuations

Most probably yes, but we should know more details about the operator (electron beam).