Development of 3D Beam-Beam Simulation for the Tevatron*

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Motivation for beam-beam development

Beam-Beam validation

Impedance model validation

Conclusion

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Motivation: comprehensive simulation

The problem: multiple effects acting together

strong-strong beam-beam interactions (especially longitudinal motion)

36 on 36 bunches cross at 2 head-on + 136 parasitic IPs

impedance and head-tail instabilities (acts longitudinally)

longitudinal motion is the force driving many instabilities

Need one simulation that includes all phenomena to study the combined effects and understand instabilities

Validate each effect separately

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Cornelis and Lamont

Head-tail instabilities
enhanced by the beam-beam
interaction

It was observed in LEP that the horizontal m=-1 mode is enhanced by the presence of a counter rotating beam. A simple model was developed in order to explain the influence of the residual beam-beam interaction on the head-tail modes. The model was successfully used in explaining the results of some beam-beam experiments we performed. The main result is the fact that according to this model the present maximum current for two beams in LEP is not an absolute limit. The model predicts that the two beam limit is a certain fraction of the single beam current limit. Fig. 7 gives an example how the two beam current increases together with the single beam current limit as function of Qs. The "x's" and the "+'s" are bunch currents that could be obtained during different experiments.
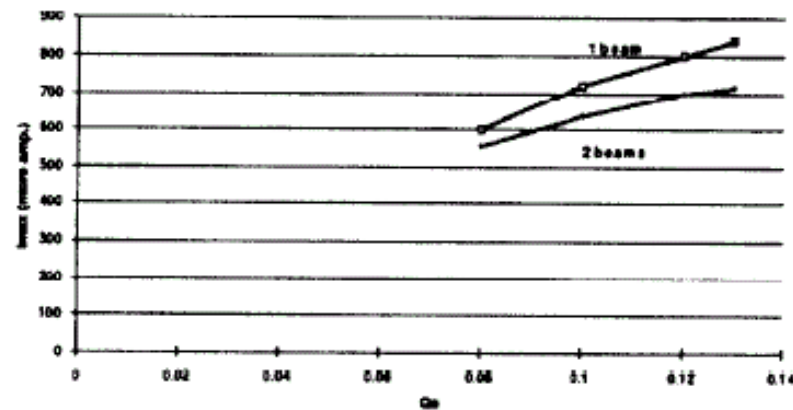

Fig. 7 : *Intensity limits for 1 beam and 2 beams as function of the synchronous frequency.*

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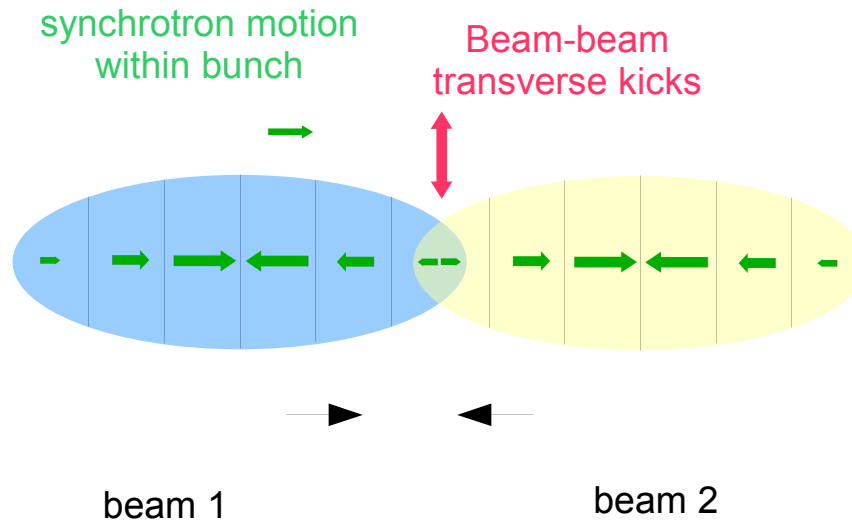
Beam-Beam simulation of synchro-betatron modes

Synchro-betatron modes are coupled oscillations where the head of a beam bunch couples to the tail mediated by beam-beam interactions with bunches from another beam.
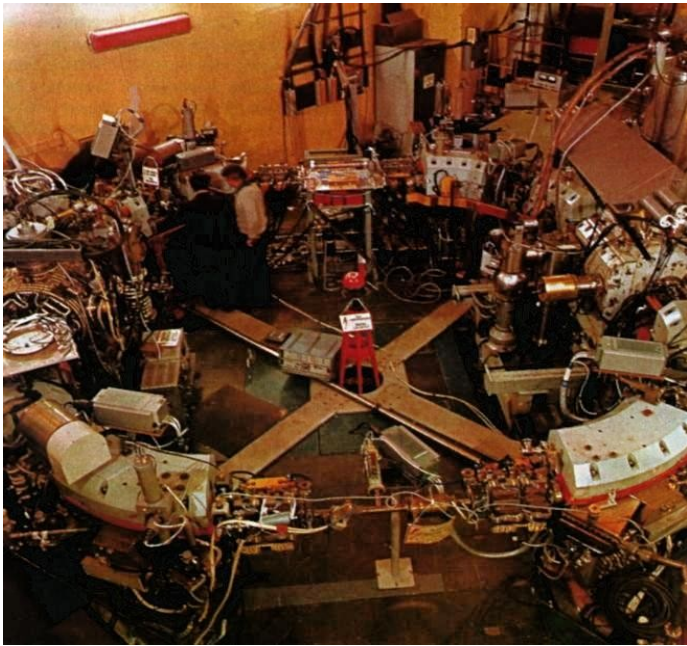
BeamBeam3d, J. Qiang, LBNL

Beam-beam force characterized by beam-beam parameter

$$\xi = \frac{N_e r_e \beta}{4\pi \gamma \sigma^2}$$

synchrotron motion within bunch

Beam-beam transverse kicks

beam 1

beam 2

The VEPP-2M Collider

Particles: $e^+ e^-$

Machine circumference: 17.88 m

Beam Energy: 500 MeV

Horiz. Betatron Tune: 3.065

Vert. Betatron Tune: 3.1011

Energy Spread: 6e-4

Horiz. Emittance: 0.5 mm mr

Vert. Emittance: 0.0045 mm mr (flat beam)

Horiz. Beta*: 0.4 m

Vert. Beta*: 0.06 m

Bunch Length (sigma): 0.04 m

Synchrotron Tune: 0.007

Beam spot 0.44 mm(x) .016 mm(y)
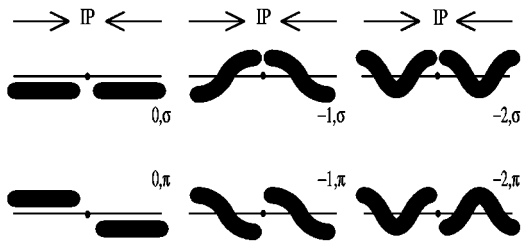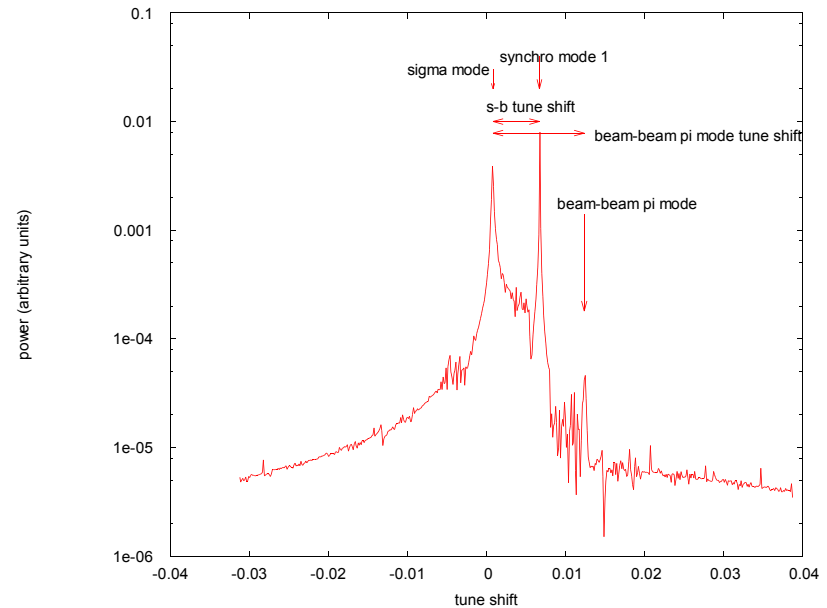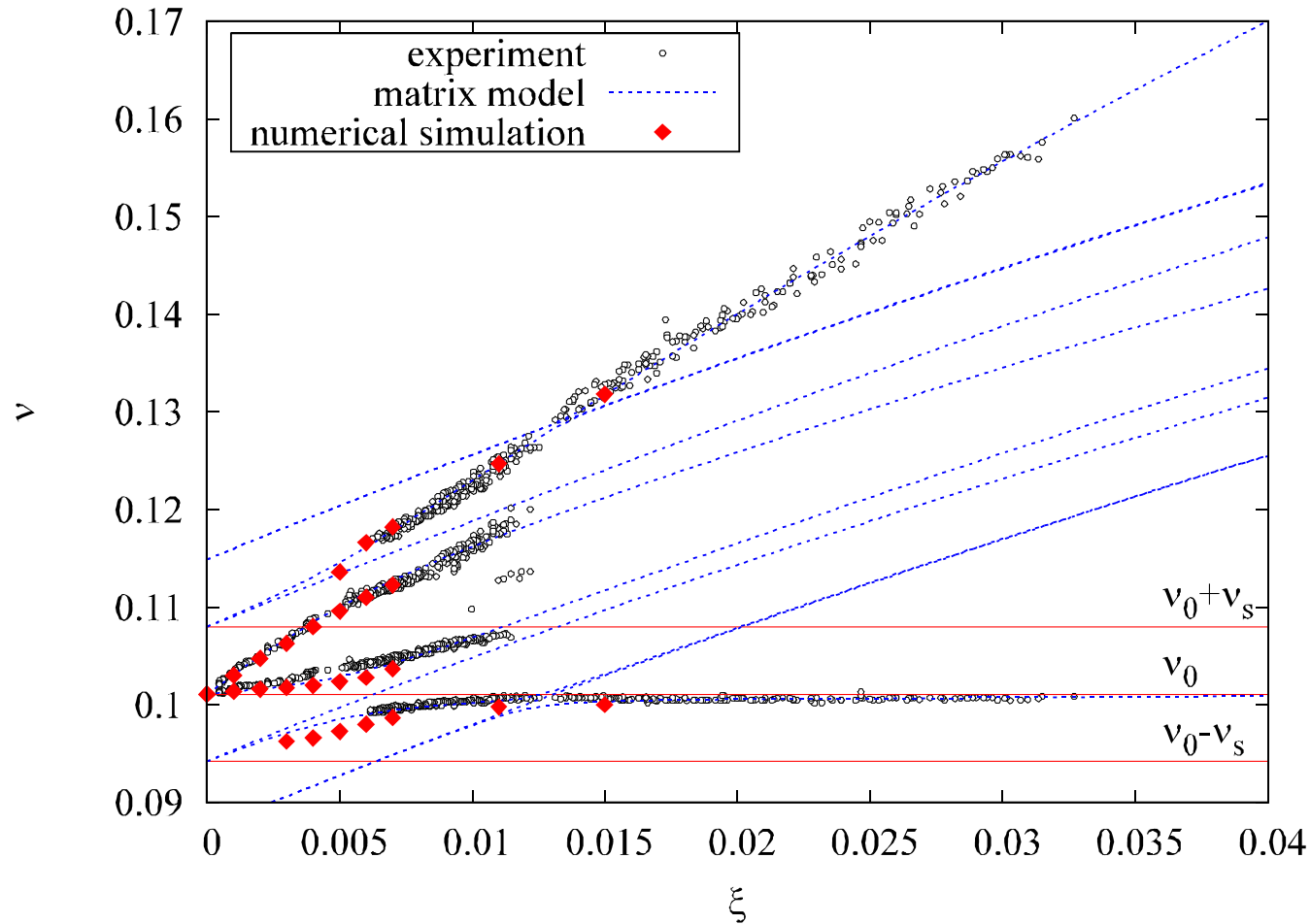
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Synchro-betatron modes

Nesterenko, et al.

sample BeamBeam3d simulation

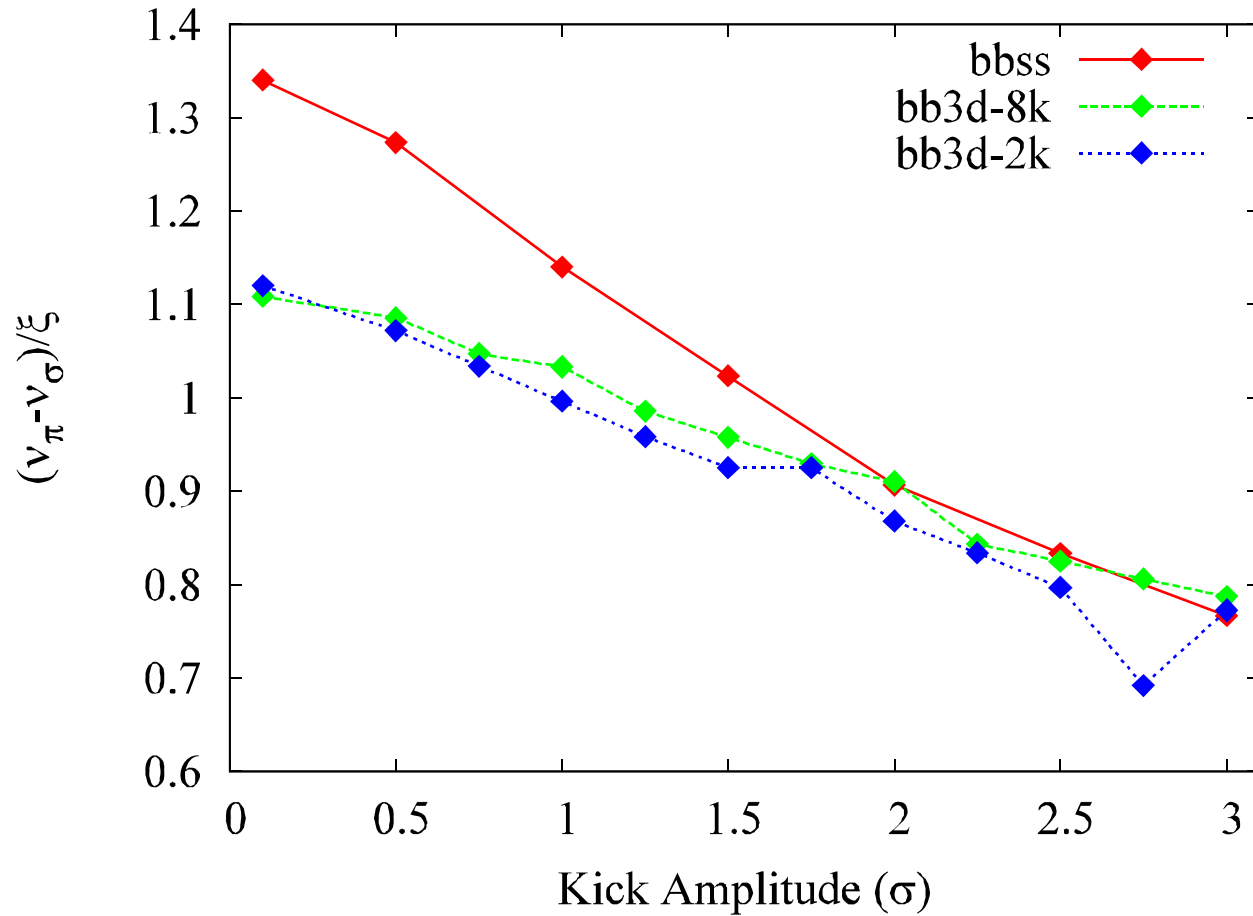FIG. 1. Notation for the synchrobetatron modes of colliding bunches.

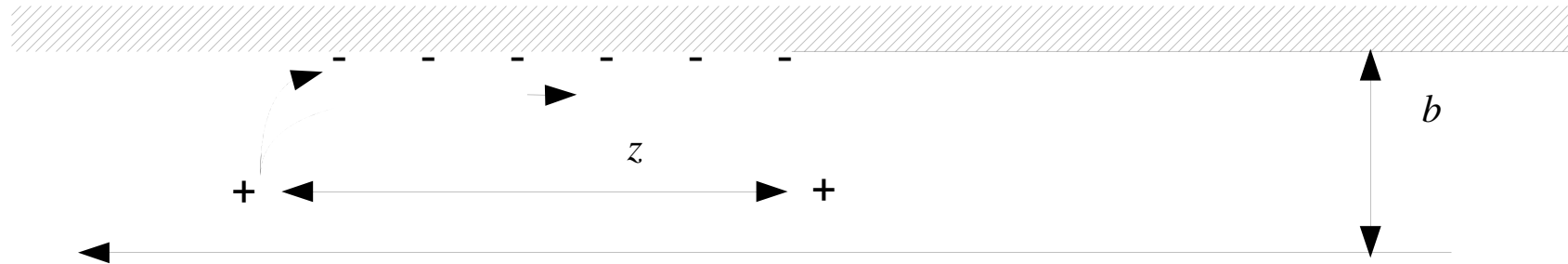# Simulation results vs. experiment, BeamBeam3d, Stern, Valishev

Initial offset in numerical simulation is 0.5σ

VEPP-2M experiment favors normalized tune split of 1.1 at small kicks

Impedance

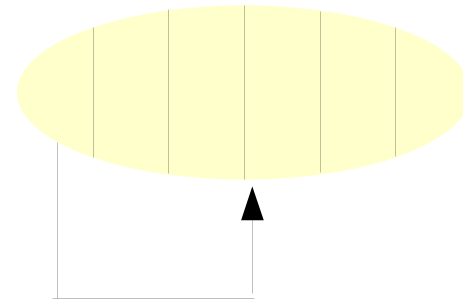A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators

A charged particle traveling through a pipe with finite conductivity walls induces opposite signed charges on the walls, leaving behind a wake field.

dipole resistive wall

kick

$$W = \left(\frac{2}{\pi b^3} \right) \sqrt{\frac{c}{\sigma}} \frac{L}{z^{1/2}}$$

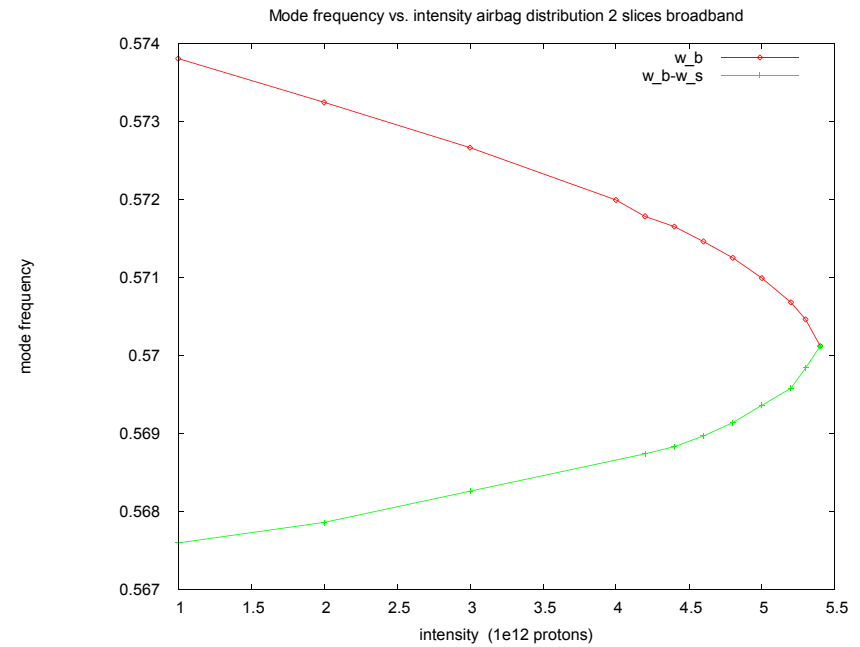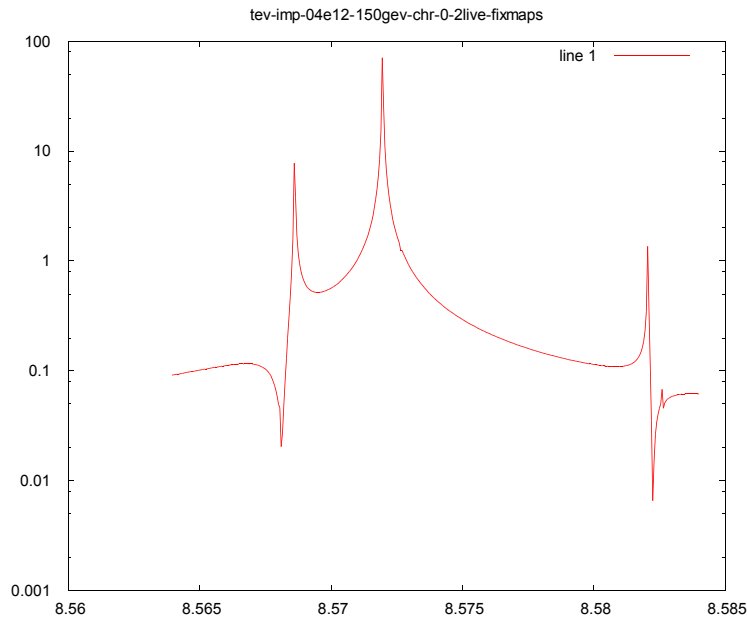$$\Delta y_2' = N y_1 r_p W$$

Impedance generated instabilities

strong head-tail

airbag distribution
two-particle model, fixed 20 cm separation
150 GeV
3 cm pipe

Stable motion when

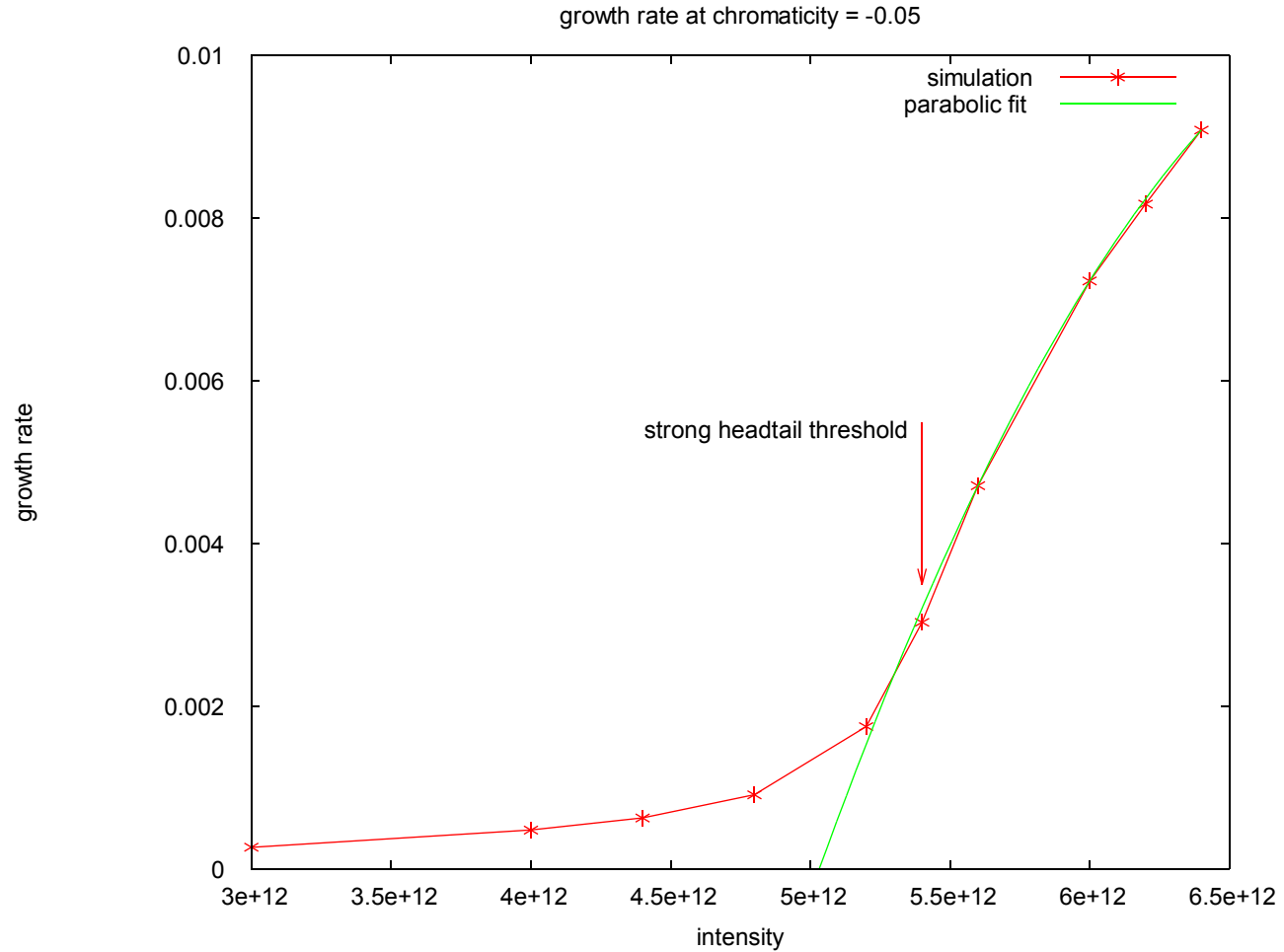$$\frac{\pi N r_0 W_0 L}{4(2\pi)^2 \gamma \nu_\beta \nu_s} < 2$$

$$N \approx 4.72 \cdot 10^{12}$$

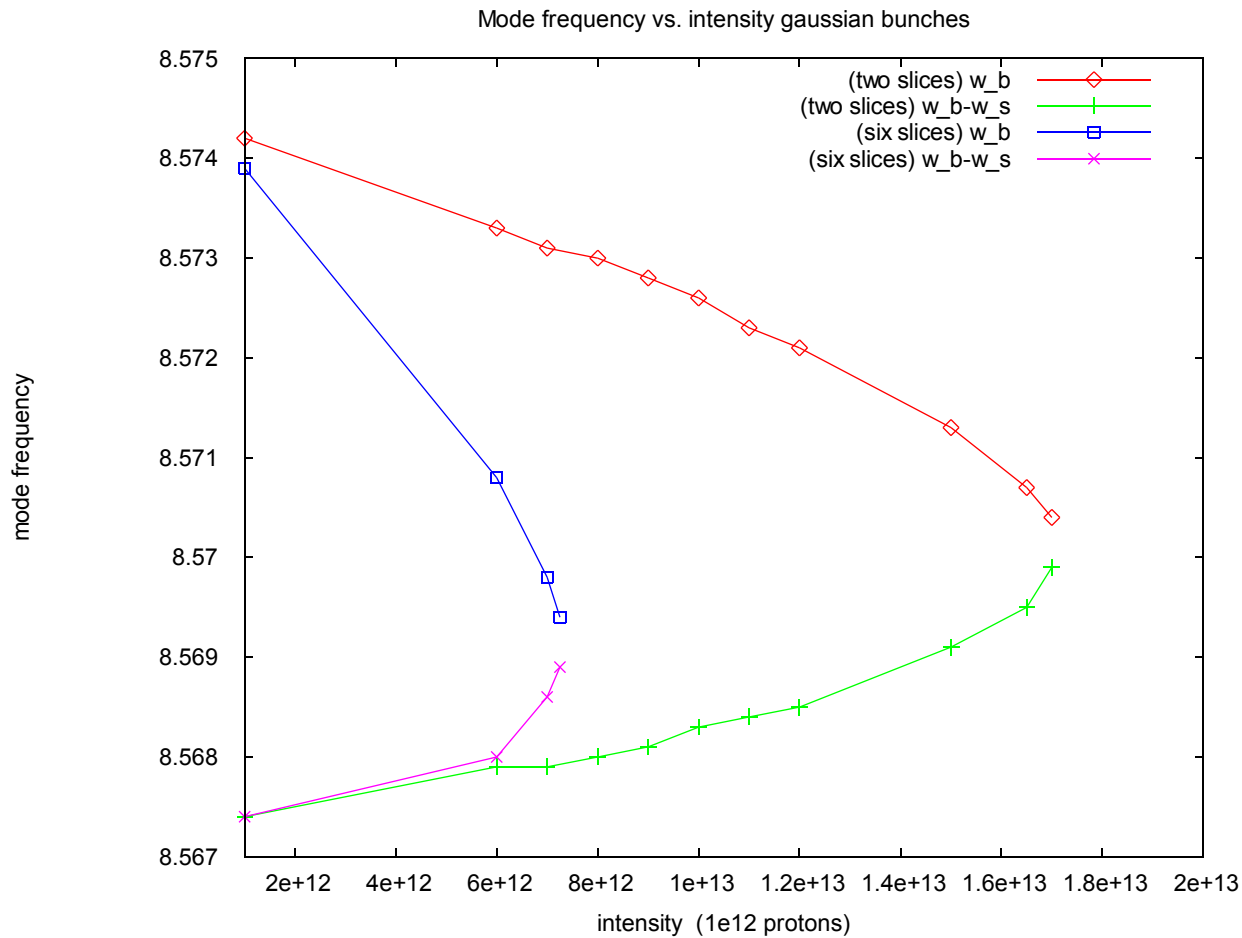tev-imp-04e12-150gev-chr-0-2live-fixmaps

Mode frequency vs. intensity airbag distribution 2 slices broadband

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Parabolic growth after instability threshold

growth rate at chromaticity = -0.05

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A little more realistic case

Gaussian particle distribution
two slices and six slices

Mode frequency vs. intensity gaussian bunches

Chao's formula predicts
stability limit $\approx 9.42 \cdot 10^{12}$

Chromaticity induced head-tail instability

How to simulate chromaticity with the tevatron coupled maps

Chao's definition of chromaticity

$$y'' + \omega_\beta^2 y = 0 \qquad \omega_\beta = \omega_0 (1 + \delta \xi)$$

Tevatron map from point 1 to point 2

$$M_{12} = -V_2 P S V_1^T S$$

to apply chromaticity, multiply by map:

$$P = \begin{vmatrix} \cos\mu_1 & \sin\mu_1 & 0 & 0 \\ -\sin\mu_1 & \cos\mu_1 & 0 & 0 \\ 0 & 0 & \cos\mu_2 & \sin\mu_2 \\ 0 & 0 & -\sin\mu_2 & \cos\mu_2 \end{vmatrix}$$
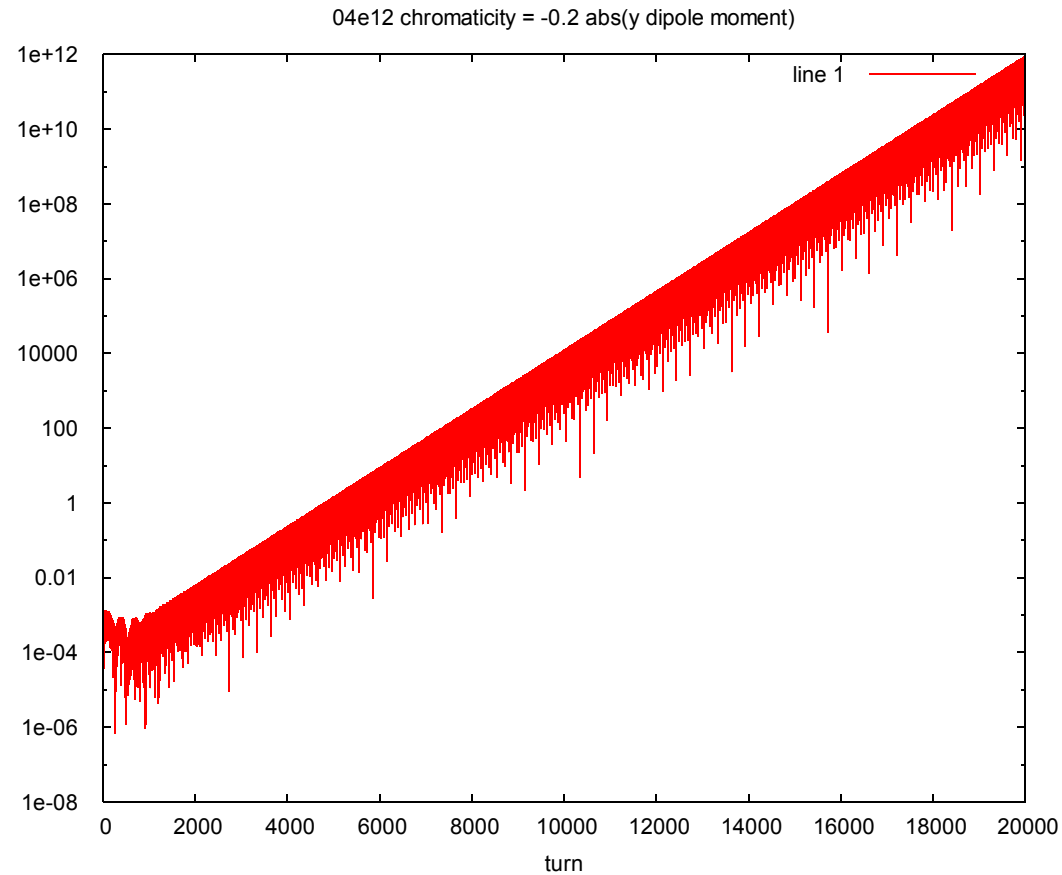
$$M_\xi = -V_2 P_\xi S V_2^T S$$

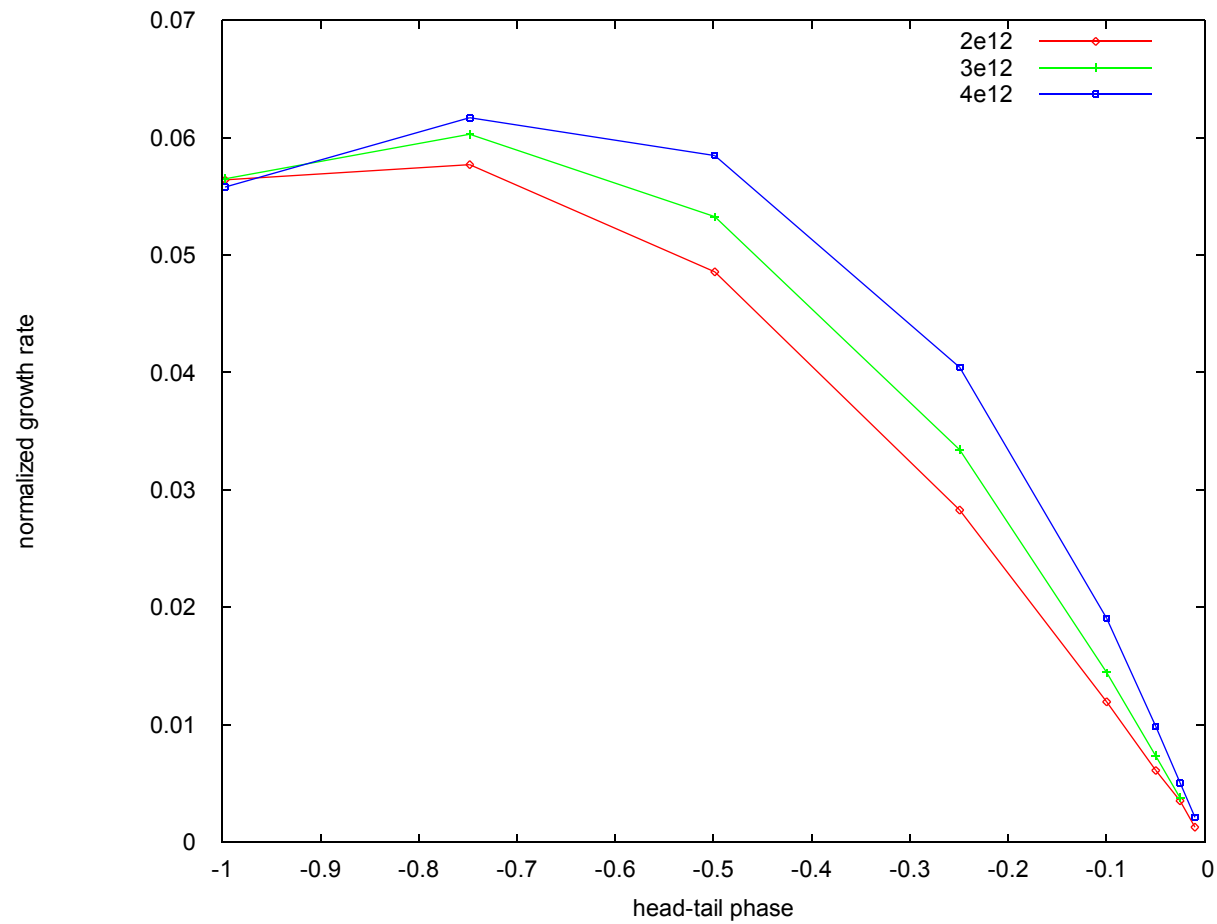P_ξ calculated with phase advance $\mu_1 = \mu_x \delta \xi$

V_x transforms uncoupled representation of particle coordinates into physical coupled coordinates at point x using Twiss parameters

Turn on chromaticity and simulate...

04e12 chromaticity = -0.2 abs(y dipole moment)

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head-tail phase $\overset{\text{def}}{=} \chi = \dfrac{\xi \, \omega_\beta \, \hat{z}}{c \, \eta} = \dfrac{2 \, \pi \, \xi \, Q_\beta \, \hat{z}}{L \, \eta}$

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Conclusions:

Pieces are in place to run combined strong-strong beam-beam simulations including effects including instability causing effects such as chromaticity and wakefields. Stay tuned for more results soon!