Transverse-Transverse and Transverse-Longitudinal Phase Space Converters for Tailoring Beam Phase Spaces

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Contents

- Phase space converters:
  - Emittance Exchange (EEX)
  - Flat Beam Transform (FMT)
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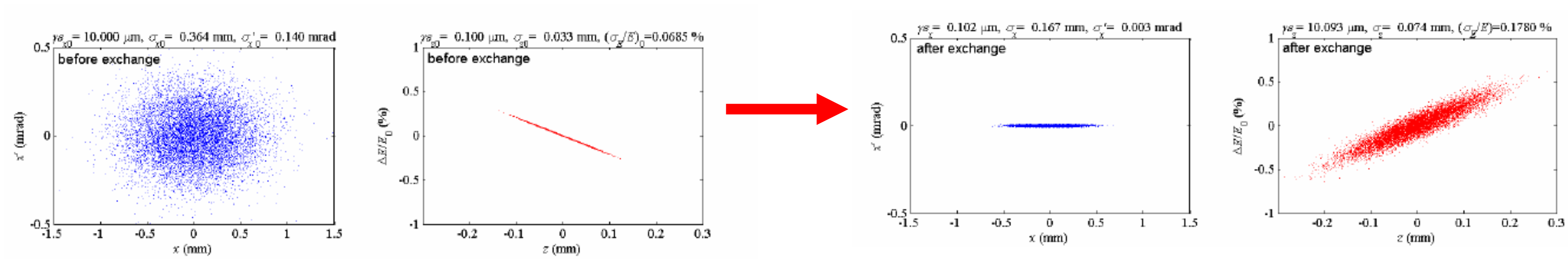
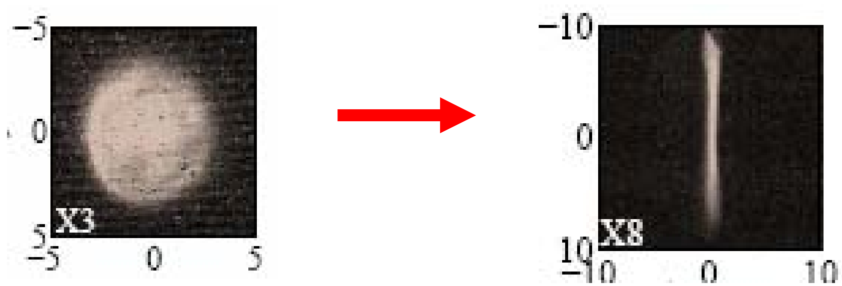
Emittance Exchange and Flat Beam Transform

- Emittance Exchange (EEX): Complete exchange of x- and z-phase spaces: $(\varepsilon_x,\varepsilon_z) \rightarrow (\varepsilon_z,\varepsilon_x)$

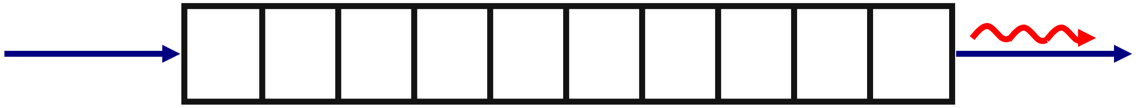


- Flat Beam Transform (FBT): Transform a round photo-cathode beam to a flat beam with a desired *emittance* ratio in (x,y) phase space



- *Applications often require a combination of these manipulations*

Producing Matched e-Beams for X-Ray HG FEL



- Electron beam emittance should be matched to the radiation emittance: $\varepsilon_x^n \sim \gamma\lambda/4\pi$

- For 1-Å with E=5 GeV, the matched emittance is $\varepsilon_x^n \sim 0.1$ μm, *which is smaller by an order of magnitude than the current state-of-the-art*

- In current HGFEL projects, the mismatch is dealt with by a high E (>15 GeV), high K (3.7), and high current (a few kA)

- However, noting that $\Delta E/E_{slice} < 10^{-6}$, two orders of magnitudes smaller than required, the FBT and EEX can be employed to produce a matched beam

A FBT-EEX for Improved X-Ray FEL Performance

(P. Emma, Z. Huang, P. Piot, and KJK, PRSTAB, 9, 100702, (2006))

- Measured slice energy spread from a photocathode gun for 4 nC beam is about 4 keV. Assume $\sigma_{\Delta E}$~1.5 keV (for 30pC) →$\sigma_{\Delta\gamma}$~0.003
- Use short electron beam σ_z = 34 μ (Q = 28 pC, I = 100 A)

$$\therefore \gamma\varepsilon_z = \sigma_z \sigma_{\Delta\gamma} = 33\mu \otimes 3\times 10^{-3} = 10^{-7}\, m$$

- Flat beam transformation in transverse phase space (units in m-rad)

$$\gamma\varepsilon_x \otimes \gamma\varepsilon_y : \quad \left(10^{-6}\right)^2 \rightarrow 10^{-5} \otimes 10^{-7}$$

- Emittance exchange (x ↔ z)

$$\gamma\varepsilon_x \otimes \gamma\varepsilon_y \otimes \gamma\varepsilon_z : \left(10^{-6}, 10^{-6}, 10^{-7}\right) \rightarrow \left(10^{-5}, 10^{-7}, 10^{-7}\right) \rightarrow \left(10^{-7}, 10^{-7}, 10^{-5}\right)$$

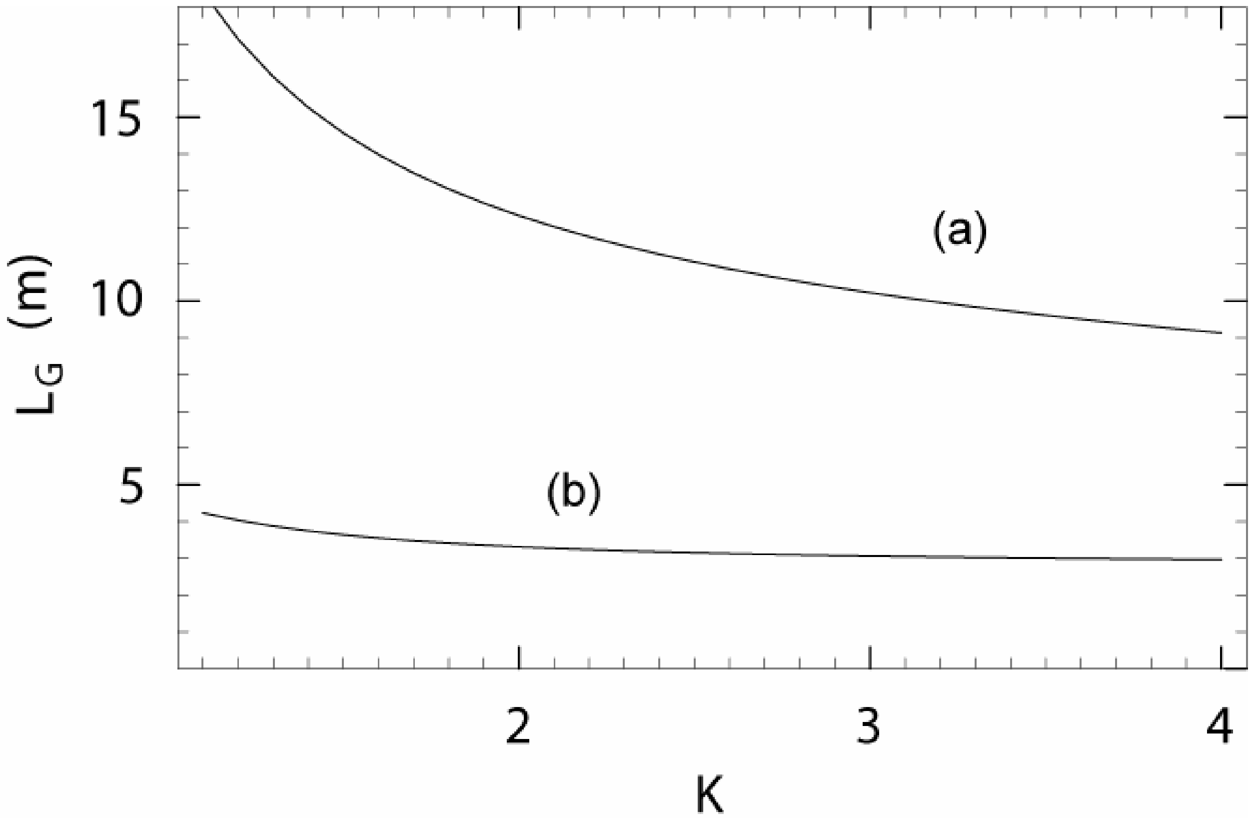
- Before the FEL, the energy spread is increased to $\sigma_{\Delta\gamma}$~3, corresponding to σ_δ~10^{-4} and σ_z~3.3 m (I=1000 A) at E=15 GeV.

$$\gamma\varepsilon_z = \gamma\sigma_z\sigma_\delta = \gamma\sigma_z \times 10^{-4} = 10^{-5}, \quad \gamma = 3\times 10^4$$

$$\sigma_z = 3.3\, 10^{-6} \Rightarrow \textit{Compression}\ \text{by}\ 10$$

$$I = 100 \rightarrow 1000\, A$$

Improved FEL Performance with matched beams

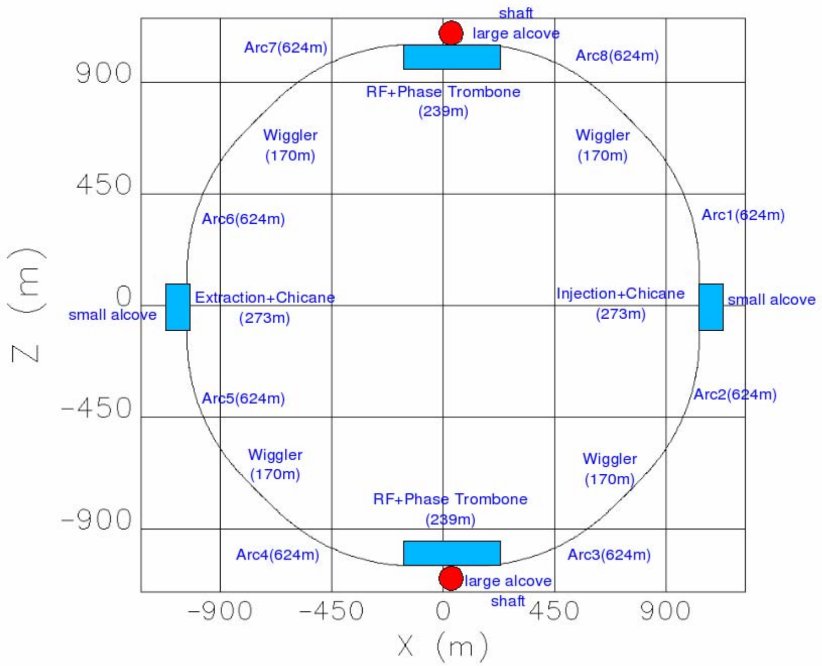


- **Higher gain**
- **The same gain with smaller K**
  - **Less magnets**
  - **Lower energy**

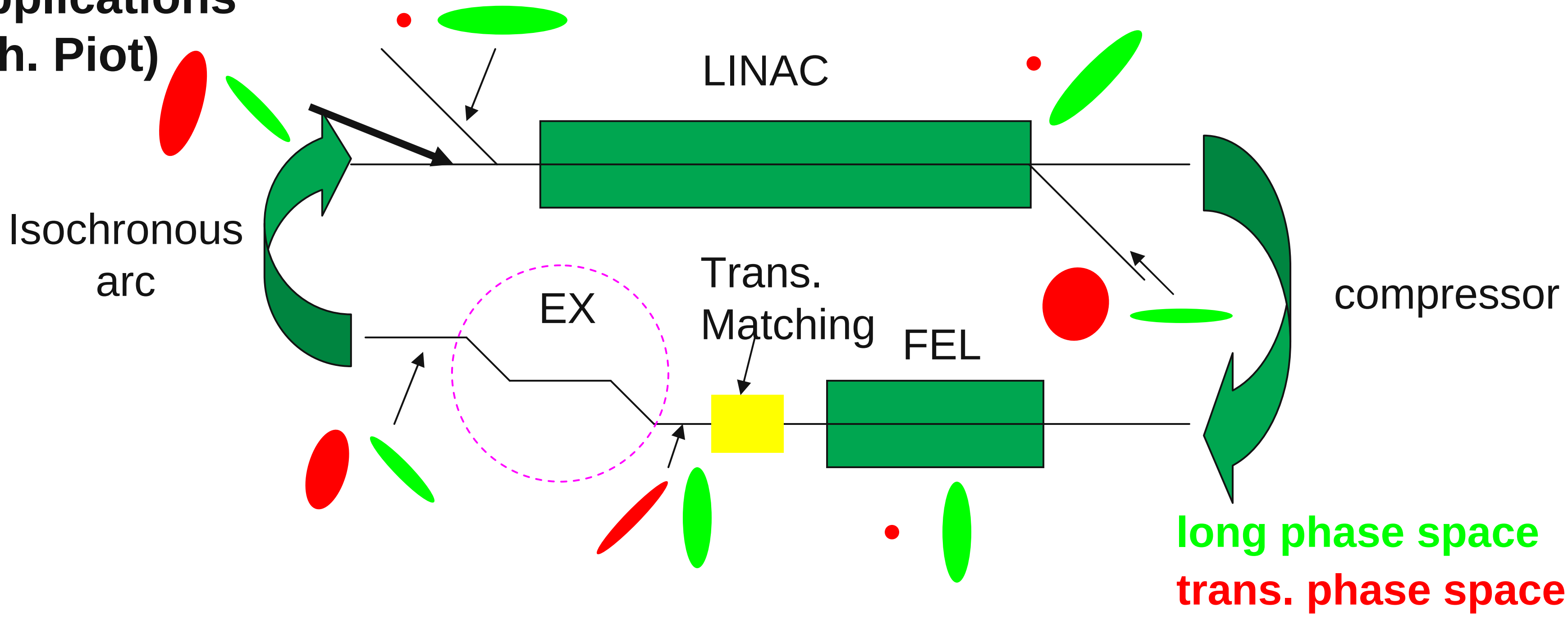
Power gain length *LG* of an x-ray FEL at 0.4 Å versus the undulator parameter *K* for (a) a beam with a normalized transverse emittance 1×10^{-6} m-r and a peak current 3.5 kA and (b) a beam with a normalized transverse emittance 1×10^{-7} m-r and a peak current 1 kA. The relative rms energy spread in both cases is 1×10^{-4} (courtesy of Z. Huang).

FBT-EEX to Produce ILC e-Beams without Damping Ring

- **Emittances of ILC electron bunches are (ε_x, ε_y, ε_z)=(8, 0.02, 3000) μm at 3 nC**
- **These bunches are produced by a 5 GeV, 6 km damping ring**
- The emittance $\varepsilon_T = (\varepsilon_x \varepsilon_y)^{1/2}$ **is too small for photocathode**
- **A possibility: (1, 1, 8)→(50,0.02, 8)→(8, 0.02,50) (Ph. Piot)**

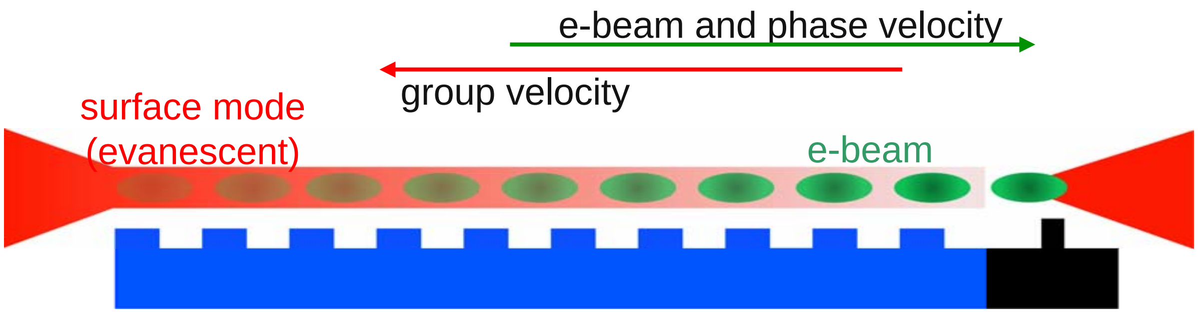


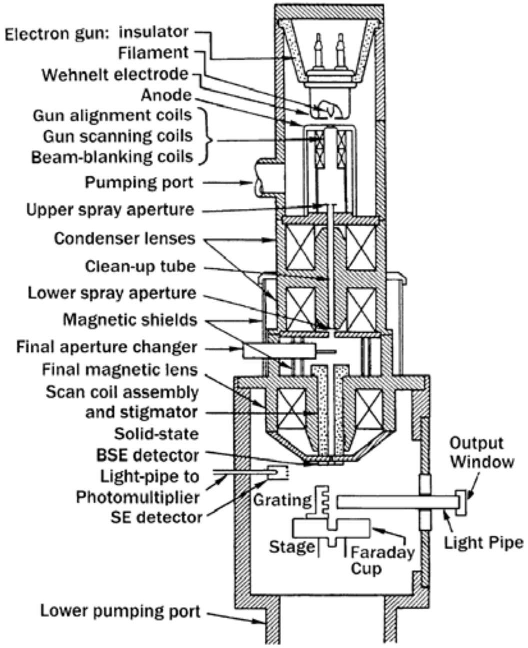
A possible use of EEX for MW-Class IR FEL for Industrial/Defense Applications (Ph. Piot)



- A major problem for MW-Class FEL is disposing the beam to dump with a minimal loss
- EEX technique may provide a better solution than that based on longitudinal manipulation

SEM-Based Smith-Purcell BWO for Intense Terahertz Radiation





- For tight beam-grating coupling, electrons must travel close to the grating surface (<10 mm)
- →Flat beam with emittance ratio of 1000
- (KJK and V. Kumar, submitted to PRSTAB)

Hamiltonian System

- Canonical variables (4-D phase space) (x, x', z, δ) (x, x', y, y')
- X $= \begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix}$
- Beam Matrix: $\sum = \langle x\tilde{x} \rangle = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xz \rangle & \langle x\delta \rangle \\ \langle xx' \rangle & . & . & . \\ \langle xz \rangle & . & . & . \\ \langle x\delta \rangle & . & . & \langle \delta^2 \rangle \end{bmatrix}$
- $\sum = \begin{bmatrix} \Sigma_x & \Sigma_c \\ \tilde{\Sigma}_c & \Sigma_z \end{bmatrix}$, $\Sigma_x, \ldots\ 2 \times 2$ submatrices
- Emittances (“projected” for non-vanishing Σ_C)

$$\varepsilon_x{}^2 = Det\,\Sigma_x, \quad \varepsilon_z{}^2 = Det(\Sigma_z)$$

Emittance Exchange Theorem

(E. Courant, in H. Bethe Symposium, 1966)

- Symplectic condition for linear transport

$$X_0 \to X = MX_0; \quad M^T JM = J \qquad J = \begin{bmatrix} J_{2D} & 0 \\ 0 & J_{2D} \end{bmatrix}, \quad J_{2D} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- Two conserved quantities

$$\varepsilon_{4D} = Det(\Sigma) \qquad I^{(2)} = -\frac{1}{2} T_r(\Sigma J \Sigma)$$

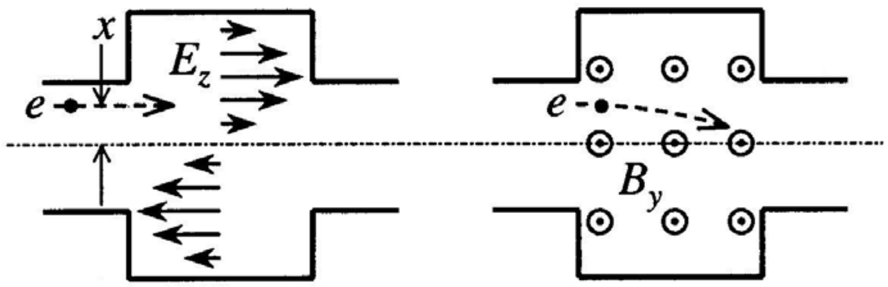
- *For an uncoupled case (Det(Σ_C)=0), emittances are either completely exchanged or conserved.*

$$(\varepsilon_{xo}, \varepsilon_{zo}) \to (\varepsilon_{xo}, \varepsilon_{zo}) : \text{conservation}$$

$$\to (\varepsilon_{zo}, \varepsilon_{xo}) : \text{exchange}$$

Building Blocks for EEX

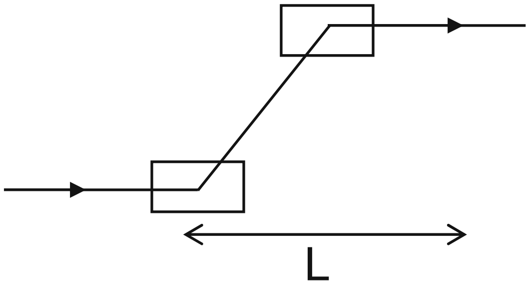
- Dipole mode cavity to produce x-dependent kick:



$$\Delta\delta = kx,\ \Delta x' = kz;\ k = eV_o/eE$$

$$M_C(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}$$

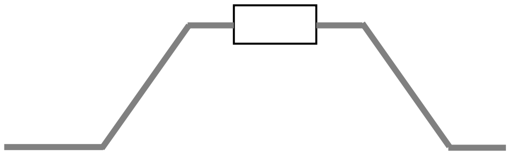
- Dog leg to produce dispersion



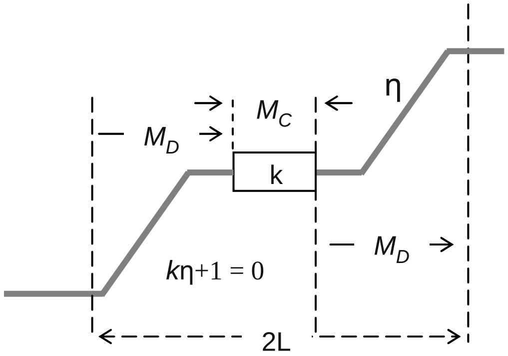
$$M_D(\eta,\xi,L) = \begin{bmatrix} 1, & L, & 0, & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Construction of Exact (x,z) Exchange

- The original scheme by M. Cornacchia and P. Emma (PRSTAB, 5, 084001 (2002)) produces an approximate exchange



- An exact scheme is possible with two identical dog-legs:



- A necessary condition for exact exchange: an arbitrary initial energy off-set δ must be cancelled, (0,0,0,δ)→(0,0,0,0)

→Require a dispersion η and x-dependent kick so that

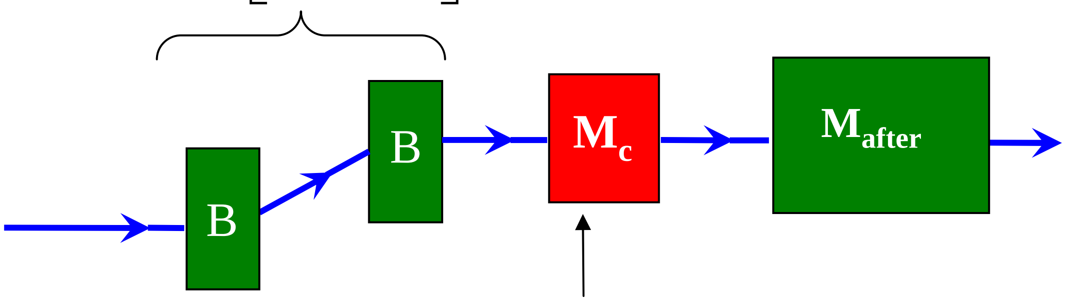
$$\delta_{final} = \delta + k \times \eta\delta \Rightarrow 0$$

$$\therefore 1 + k\eta = 0$$

Construction of An Exact Exchange (cont'd)

Dog-Leg for generating dispersion

$$M_D = \begin{bmatrix} 0 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Thin Trasverse Cavity

$$M_C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 0 \end{bmatrix}, \quad k = -1/\eta$$

A general static matrix (R. Fliller):

$$M_{after} = \begin{bmatrix} e & f & 0 & D \\ g & h & 0 & D' \\ gD - eD' & hD - fD' & 1 & X \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Condition for exact exchange

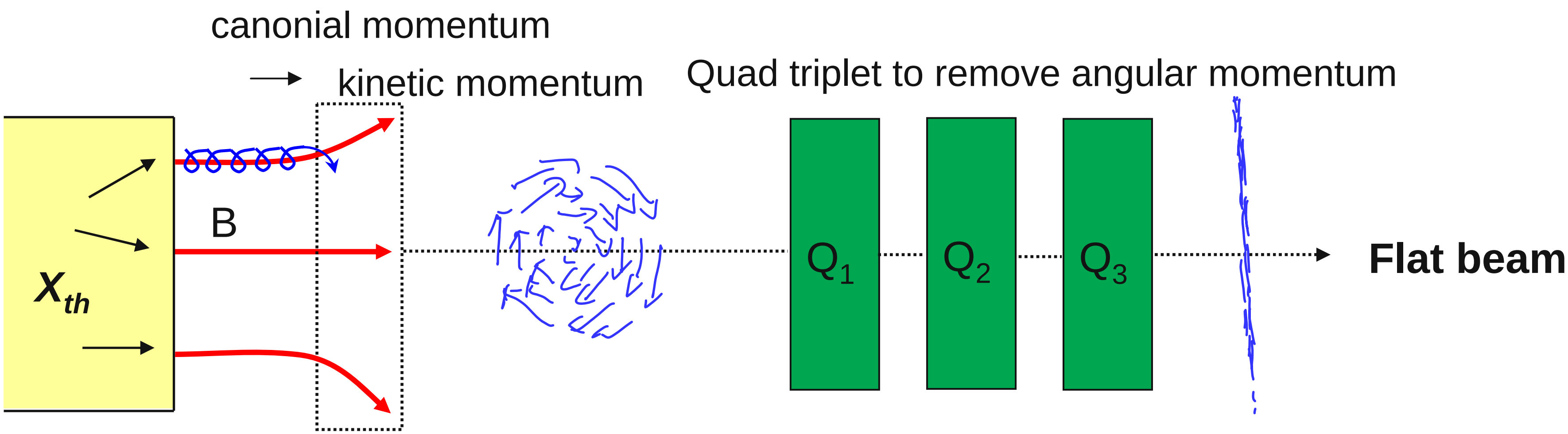
$$\begin{pmatrix} e & f \\ g & h \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} D \\ D' \end{pmatrix}$$

Solution I: $M_{after} = M_D$ (KJK)

Solution II: $M_{after} = B \bullet D2 \bullet Q \bullet D1$ (H. Edwards)

Flat Beam Generation

(Y. Derbenev, 1998), (R. Brinkmann, Y. Derbenev, K. Flöttmann, 2001)



Axial magnetic field giving use to angular momentum dominated beam

$$\begin{pmatrix} x \\ y \end{pmatrix}'' + \kappa^2 \begin{pmatrix} x \\ y \end{pmatrix} = 0 \ , \quad \kappa = \frac{qB}{mc}$$

$$X_{\mathrm{th}} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}, X_0 = \begin{pmatrix} x \\ x' - \kappa y \\ y \\ y' + \kappa x \end{pmatrix}$$

Does Flat Beam Technique Violate the Emittance Exchange Theorem?

- Thermal distribution before emission

$$\Sigma_{th} = \left\langle X_{th} \tilde{X}_{th} \right\rangle = \begin{bmatrix} \varepsilon_{th} T_{th,} & 0 \\ 0, & \varepsilon_{th} T_{th,} \end{bmatrix}, \varepsilon_{th} = \sigma_x \sigma_{x'}, T_{th} = \begin{bmatrix} \beta_{th} & 0 \\ 0 & 1/\beta_{th} \end{bmatrix}$$

- Distribution after emission

$$\Sigma_o = \left\langle X_o \tilde{X}_o \right\rangle = \begin{bmatrix} \varepsilon_{eff} T_o & \mathcal{L} J \\ -\mathcal{L} J, & \varepsilon_{eff} T_o \end{bmatrix}$$

$$\varepsilon_{eff} = \sqrt{\varepsilon_{th}{}^2 + \mathcal{L}^2}, T_o = \begin{bmatrix} \beta_o & 0 \\ 0 & 1/\beta_o \end{bmatrix}, \mathcal{L} = \kappa \sigma_x{}^2$$

- The theorem is not violated since
  - *$X_{th} \rightarrow X_o$ non-symplectic or Σ_o is coupled*

Removing Angular Momentum

- General form of cylindrically symmetric beam matrix

$$\Sigma = \begin{bmatrix} \varepsilon_{eff} T_0 & \mathcal{L} J \\ -\mathcal{L} J & \varepsilon_{eff} T_0 \end{bmatrix}, Det(\Sigma) = \varepsilon_{eff}{}^2 - \mathcal{L}^2$$

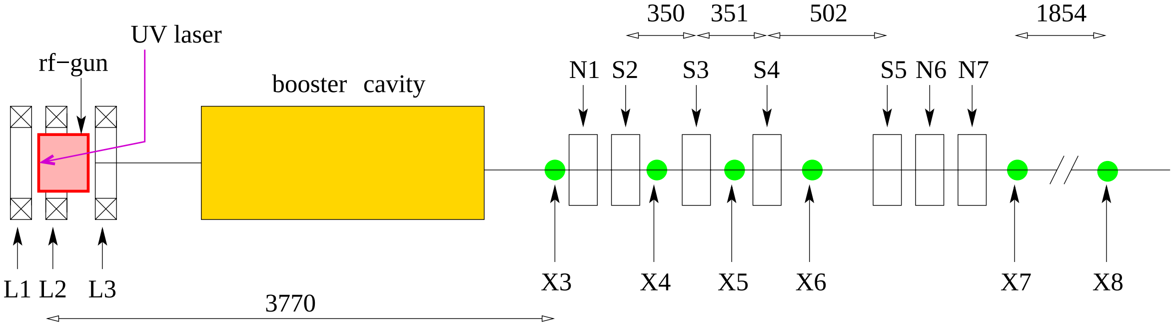
- Use symplectic matrix M to remove angular momentum

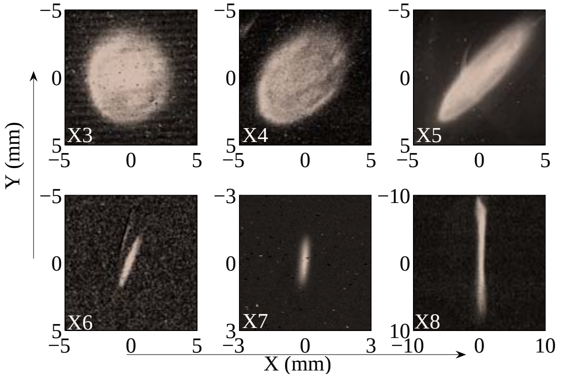
$$M\Sigma\tilde{M} = \begin{bmatrix} \varepsilon_+ T_+ & 0 \\ 0 & \varepsilon_- T_- \end{bmatrix}, T_\pm = \begin{bmatrix} 1/\beta_+ & 0 \\ 0 & 1/\beta_- \end{bmatrix}$$

- Invariance of I_2 and *Det(Σ):*

$$\varepsilon_\pm = \varepsilon_{eff} \pm \mathcal{L}$$

Flat Beam Production Has Been Demonstrated Experimentally

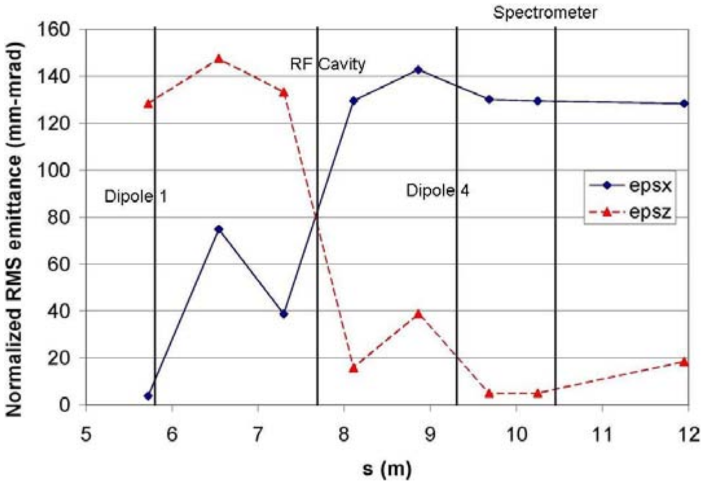


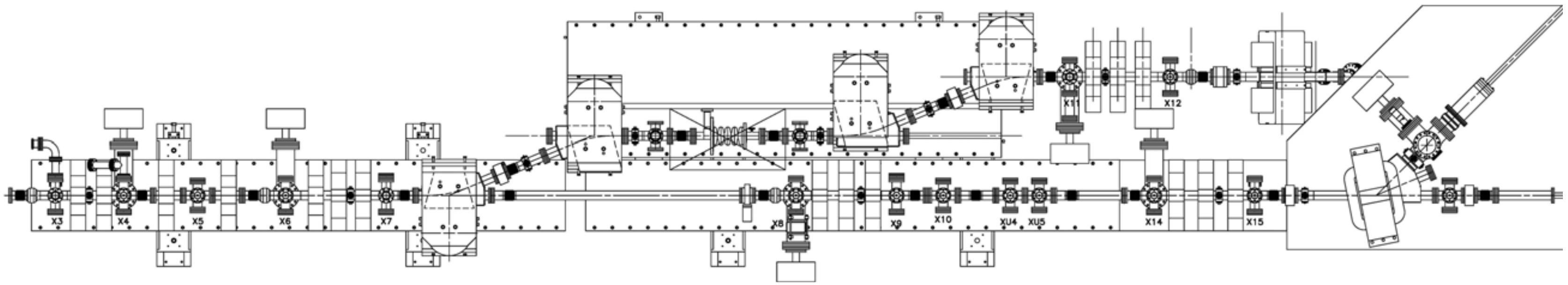


- D. Edwards, et. al., emittance ratio of 40 at Fermilab A0 (Linac2000, PAC2001)
- Yin-e Sun, U of C thesis (2005)
- Ph. Piot, Y.-e. Sun, and KJK, emittance ratio>100 (PRSTAB 9, 031001, 2006)

EEX demonstration experiment at FNAL-A0

- $(\varepsilon_x, \varepsilon_y, \varepsilon_z)=(4, 4,130)\ \mu m \rightarrow (130, 3,3)\ \mu m$
- 3.9 GHz, 5-cell, LiN-cooled copper cavity (adopted from CKM)
- Please visit:
  - " Transverse to longitudinal emittance exchange beamline at the A0 photoinjector", R.P. Fliller, et. al, THPAS094
  - "A TM110 cavity for longitudinal to transverse emittance exchange", T. Koeth, et. al.,THPAS079





Demonstration Experiment for EEX at Argonne AWA

- **AWA- and APS-ANL, NIU, and Tsinghua U collaboration**
- **L-band, TM110, (1/2*,1,1/2*)-cell cavity from Tsinghua U**

“Design study of a transverse-to-londitudinal EEX POP experiment”, Y. Sun, et. al.,THPAN094