Emittance Measurements and Modeling for the Fermilab Booster *

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* X. Huang, et al. Phys. Rev. ST Beams 9, 014202 (2006)

Outline

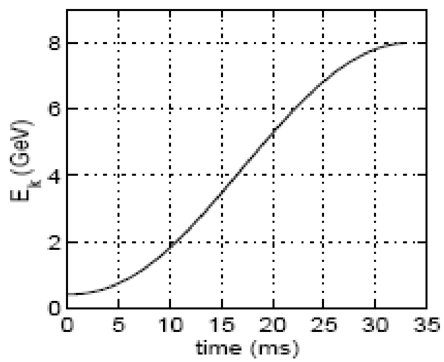
- Motivation
  - Characterize the emittance growth for the Booster synchrotron.
  - Understand the emittance growth mechanisms under space charge effects.
  - Identify main sources of emittance growths for possible cures.
- Measurements
  - Use ionization profile monitor for turn-by-turn beam size measurements.
  - Record machine conditions (rf gap voltage, tunes, etc) for full cycles.
- Modeling
  - Simplified Booster model for particle transport.
  - Random errors, sextupoles and space charge force as localized kicks.
  - Assume gaussian distribution to compute space charge force.

The Fermilab Booster

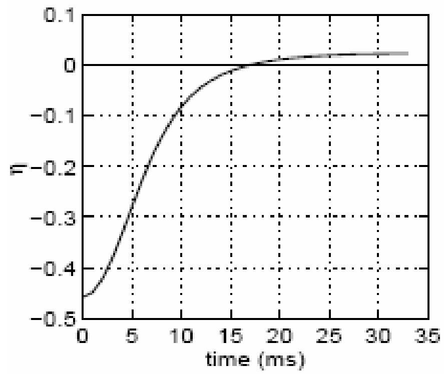
- A fast ramping (15 Hz) proton synchrotron (400 MeV – 8 GeV).
- Connect Linac and Main Injector.
- Multi-turn injection (10 or 11-turn injection in normal operations).
- High intensity (> 4×10^{12}/pulse) and high average proton flux.
- Circumference 474 m, 24 cells, rf 37-53 MHz, betatron tunes 6.7/6.8.
- Transition crossing at 17 ms (γ=5.5).
- Significant space charge effect in first few ms.

The space charge perveance paramater

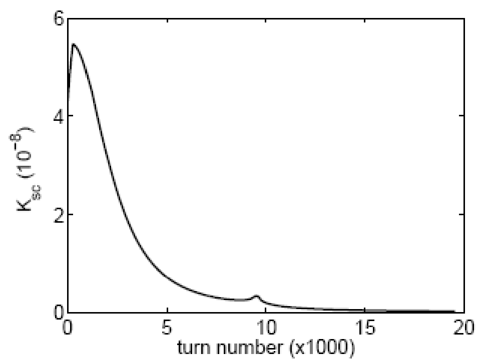
$$K_{sc} = \frac{2Nr_0}{\beta^2\gamma^3}$$



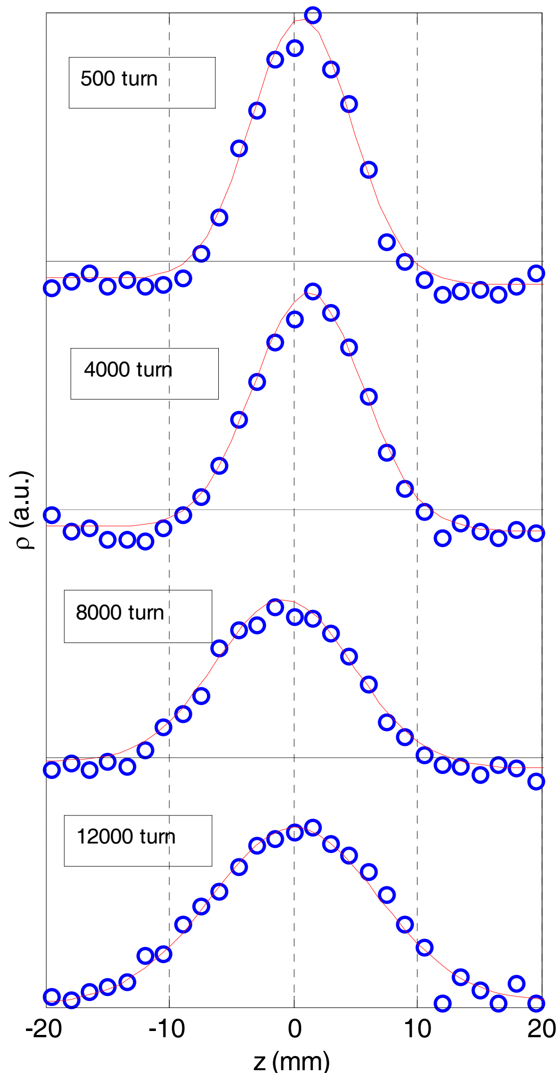
Energy



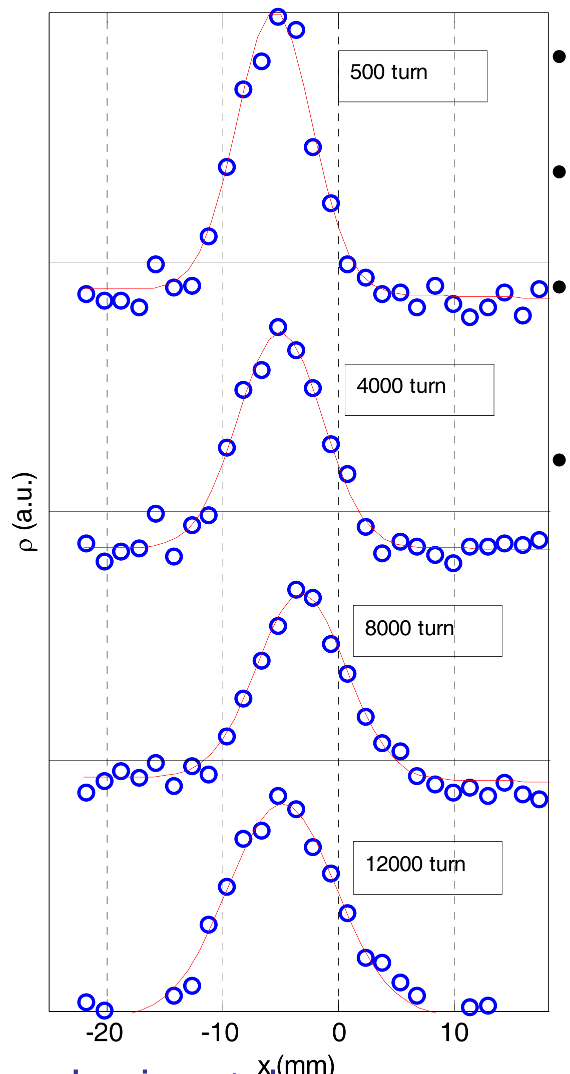
Phase slip factor



The IPM Measurements



vertical



horizontal

- Beam sizes were derived by fitting the profile to gaussian.
- Space charge effect (on ions) correction was applied*.
- IPM data were taken turn by turn for a full cycle (20,000 turns).
- Data under various intensity levels were taken (1-8 $\times 10^{12}$ protons/pulse).

$$\varepsilon_z = \frac{\sigma_z^2}{\beta_z},$$

$$\sigma_x^2 = \beta_x \varepsilon_x + D^2 \sigma_\delta^2$$

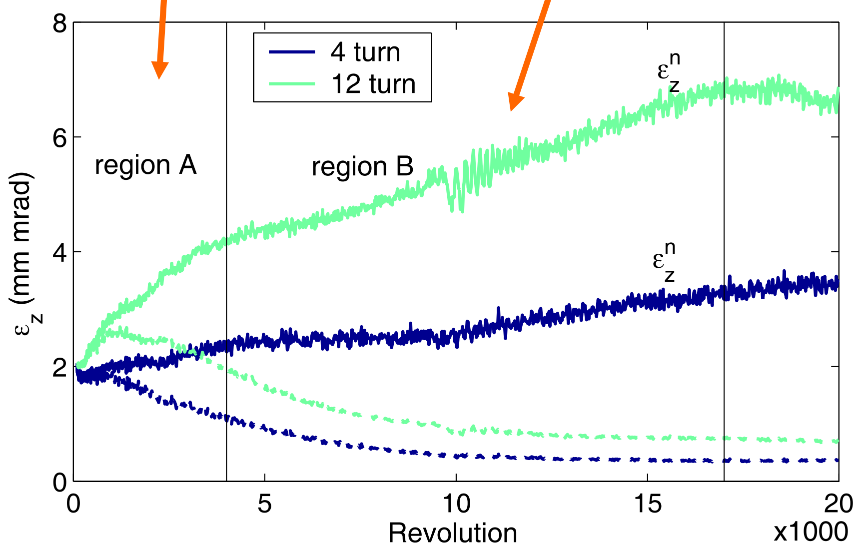
*J. Amundsen, et al. Phys. Rev. ST Beams 6, 102801 (2003)

Emittance Evolution

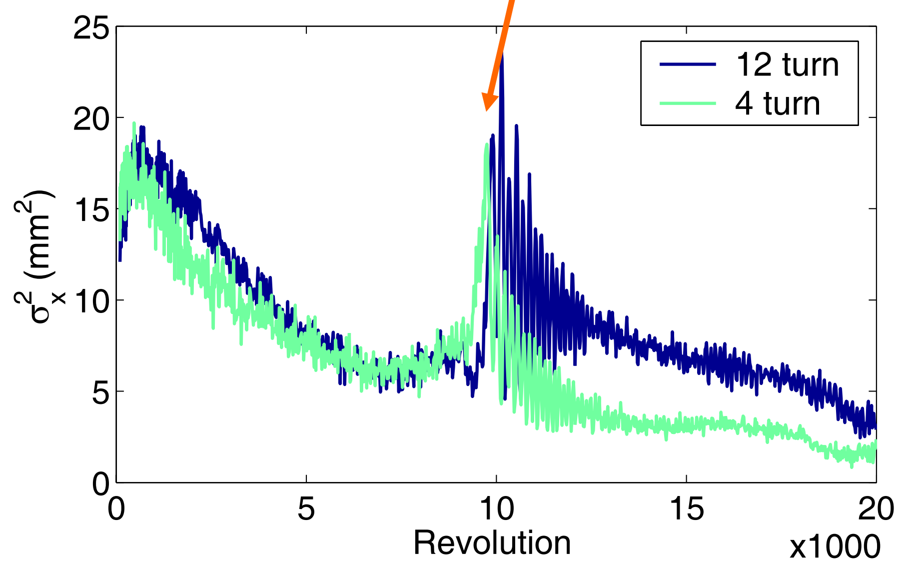
Space charge dominated region

Linear growth region

Transition crossing at $\gamma = 5.5$



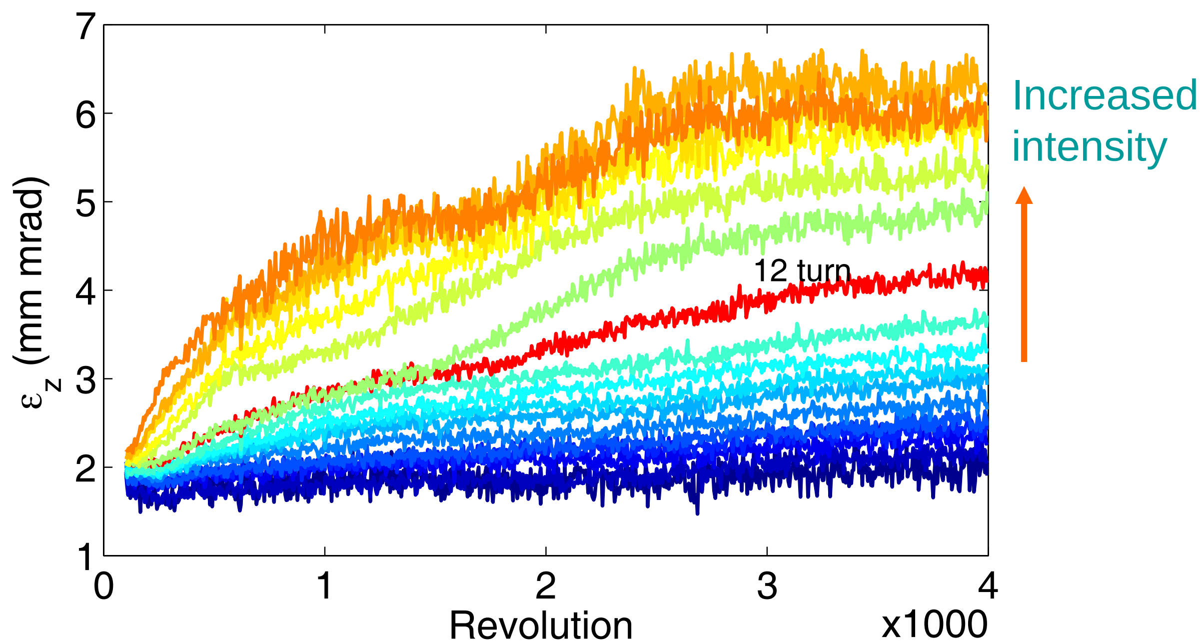
Vertical rms emittance and normalized rms emittance.

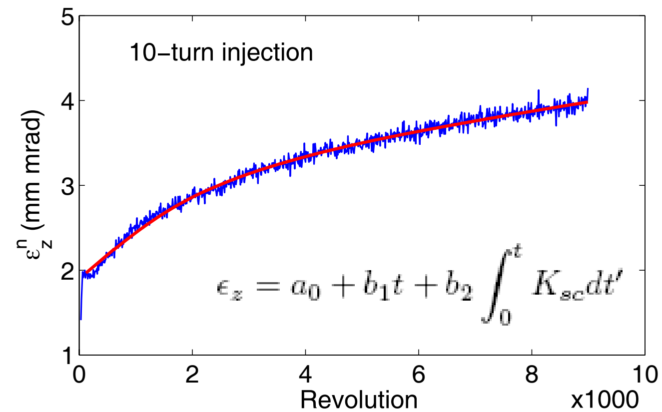


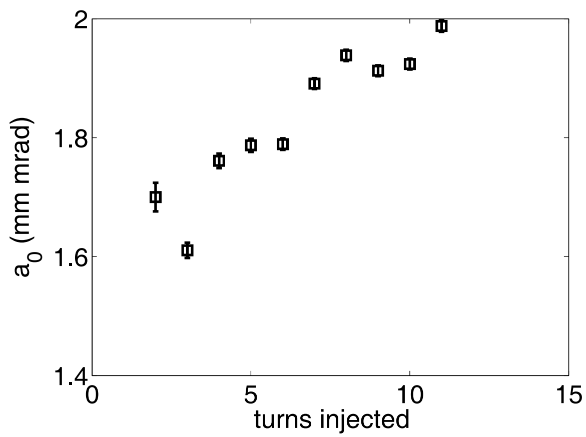
Horizontal beam size.

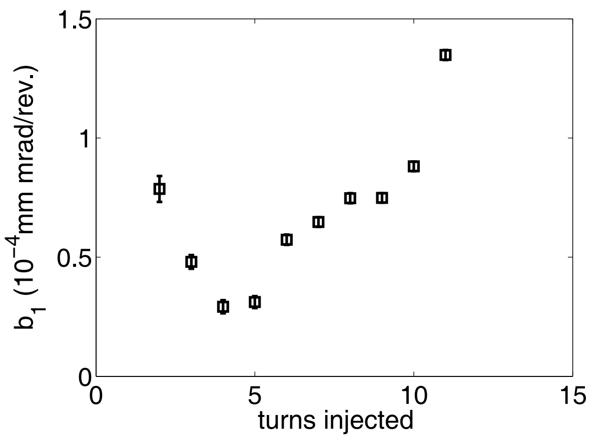
There is not as much emittance growth on the horizontal plane as on the vertical plane in the first few ms.

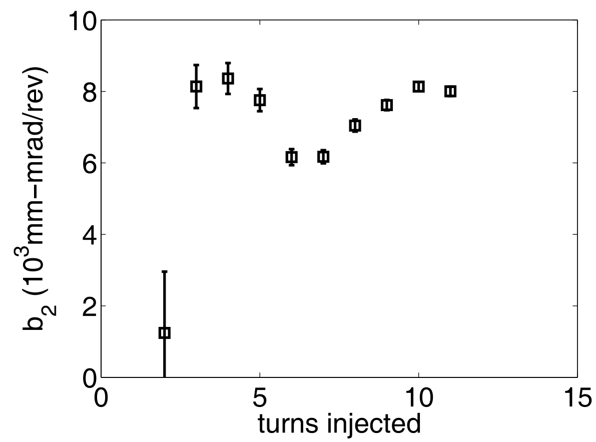
A Closer Look at the Vertical Plane











Emittance Growth Modeling

Goal: what are the mechanisms that lead to emittance growth with space charge?
Approach: multi-particle tracking with a simplified model.
Candidates for the mechanisms: half-integer resonance stopband, Montague resonance ($2Q_x - 2Q_y=0$), linear coupling resonances (sum and difference), decoherence of dipole kicks under nonlinearities.

Modeling algorithm – transverse:

- 24 FODO cells, using transfer matrices for particle transport. Measured betatron functions and tunes are used to build transfer matrix*.
- Systematic sextupoles and random dipole, quadrupole and skew quadrupole errors as localized kicks.

$$x'' + K_x x = b_0 + b_1 x + a_1 z - \frac{1}{2} b_2 (x^2 - z^2),$$
$$z'' + K_z z = -a_0 - b_1 z + a_1 x + b_2 x z,$$

Sextupoles (b_2) derived from field measurements; random errors (a_0, b_0, a_1, b_1) controlled by amplitudes; amplitudes damped down to model decoherence.

$$A_{b0}(n) = A_{b0}(0) \exp\{-\frac{n}{N_{dipole}}\}$$

*X. Huang, et al. Phys. Rev. ST Beams 8, 064001 (2005)

Emittance Modeling

Modeling algorithm – longitudinal:

- linear magnetic field ramping; rf voltage ramping curve as measured; energy gain distributed over 18 cavities.

Modeling algorithm – space charge force:

- assume gaussian bunch distribution.

$$\rho(x,z) = \frac{Ne}{2\pi\sigma_x\sigma_z}\exp\{-\frac{x^2}{2\sigma_x^2} - \frac{z^2}{2\sigma_z^2}\},$$

distribution

$$\begin{aligned}\Delta x' &= \frac{2Nr_0\ell}{\beta^2\gamma^3\sigma_x(\sigma_x+\sigma_z)} x\exp\{-\frac{x^2+z^2}{(\sigma_x+\sigma_z)^2}\},\\ \Delta z' &= \frac{2Nr_0\ell}{\beta^2\gamma^3\sigma_z(\sigma_x+\sigma_z)} z\exp\{-\frac{x^2+z^2}{(\sigma_x+\sigma_z)^2}\},\end{aligned}$$

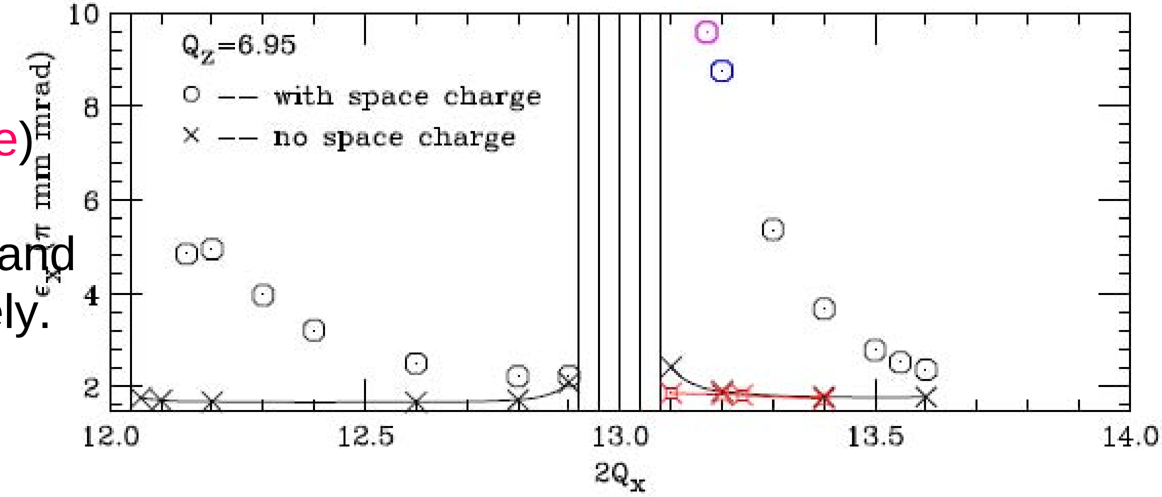
Horizontal and vertical kicks per half cell (length l) derived from the space-charge potential*.

- Inconsistent, but a valid approach to gain physics insights.

* S. Kheifeit, PETRA Note No. 119, 1976

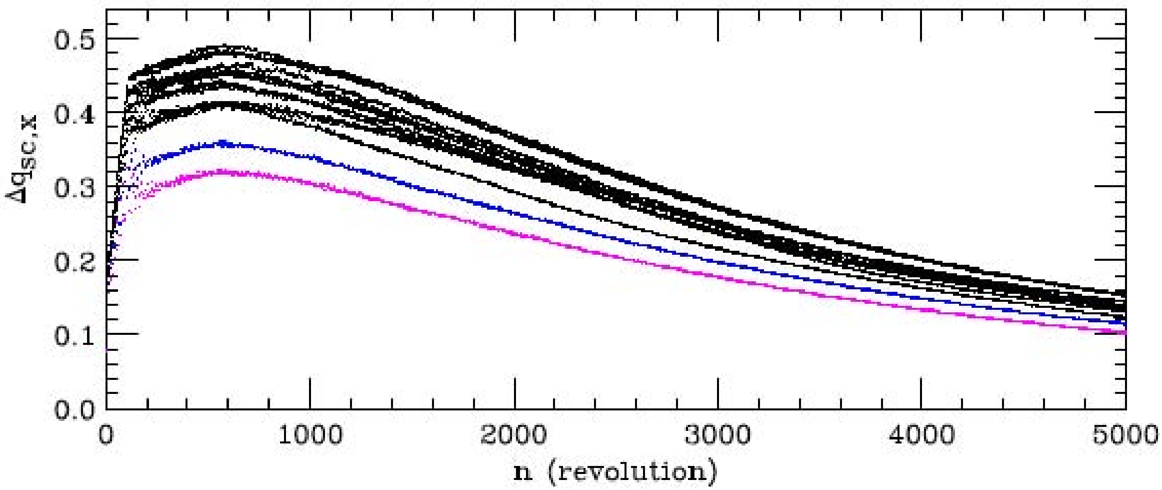
Effects of the Half-integer Stopband

- Turn on random quadrupole errors.
- Include nonlinearities.
- Set amplitude 4×10^{-3} /m (black circles) and 4×10^{-4} /m (red square)
- Choose seeds such that harmonics 12 and 13 have stopband width about 0 and 0.1, respectively.
- Fix Q_z and vary Q_x.



Observations:

- No vertical emittance growth.
- Horizontal emittance growth at large random quadrupole errors.

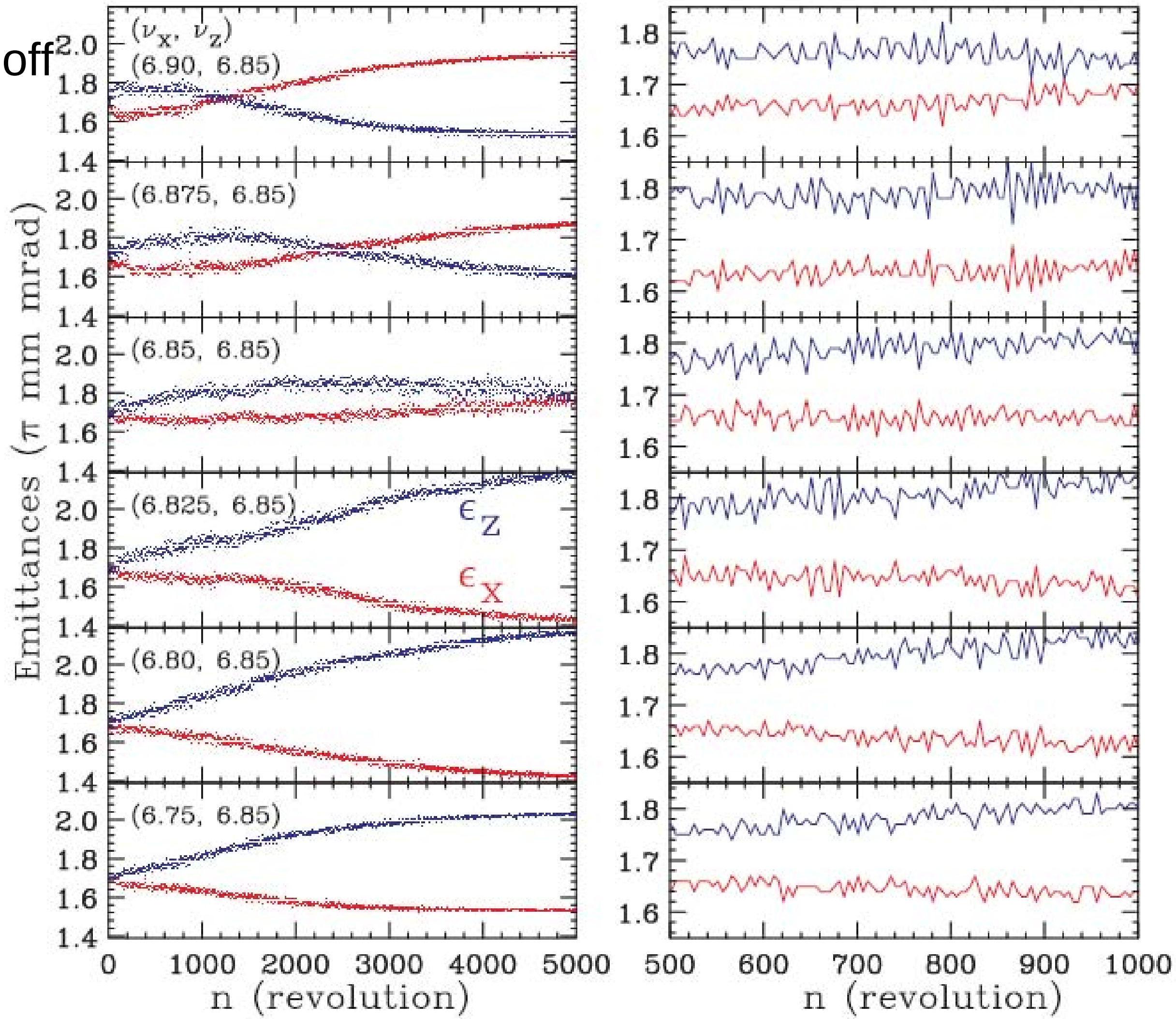


At the estimated random quadrupole error level (4×10^{-4} /m), half-integer stopband causes little emittance growth.

Effects of the Montague Resonance

• Keeping only systematic sextupoles (in dipoles). Turning off all linear errors.

• Fix vertical tune and vary horizontal tune around it.



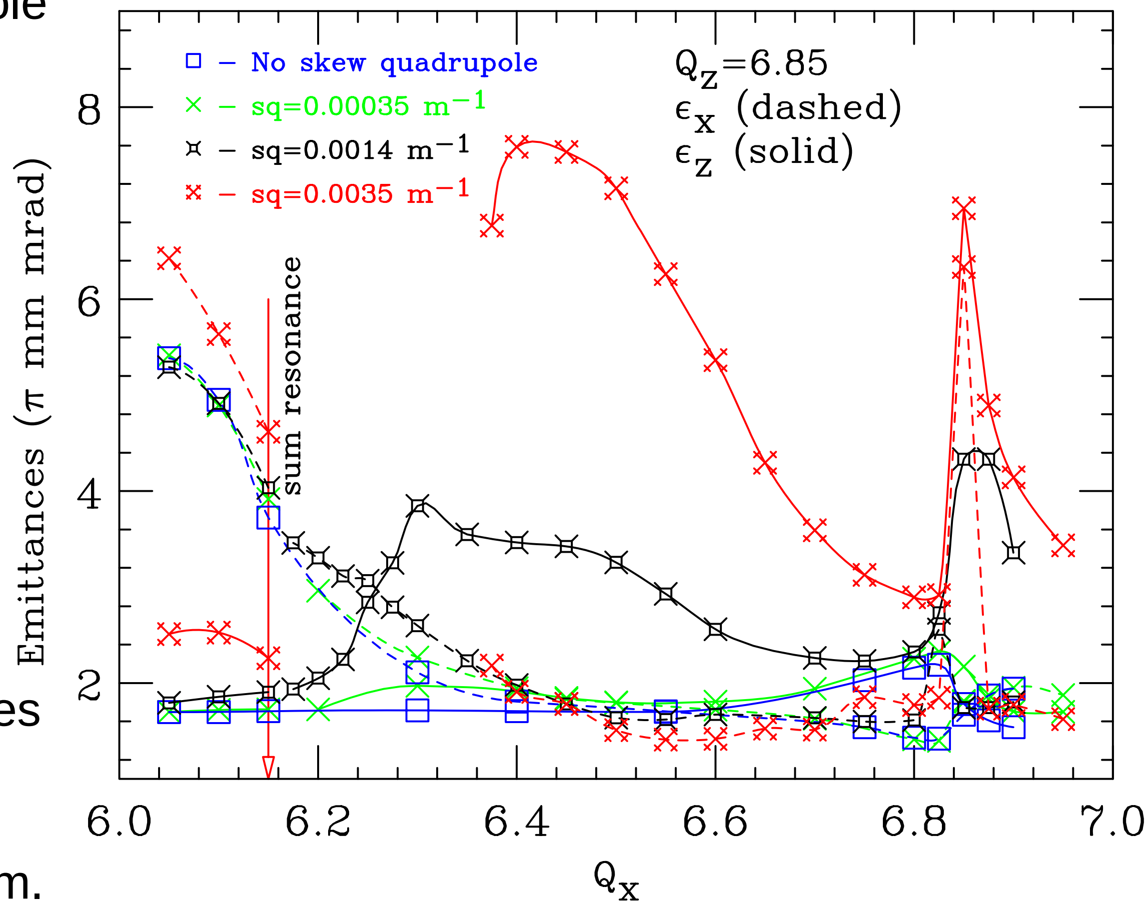
Observations:

• Emittance exchanges.

• Not much overall growth.

Effects of Linear Coupling Resonances – 1

- Turn on random skew quadrupole errors.
- Set amplitude to various levels.
- Assume 12-turn injection.
- Fix Q_z and vary Q_x.

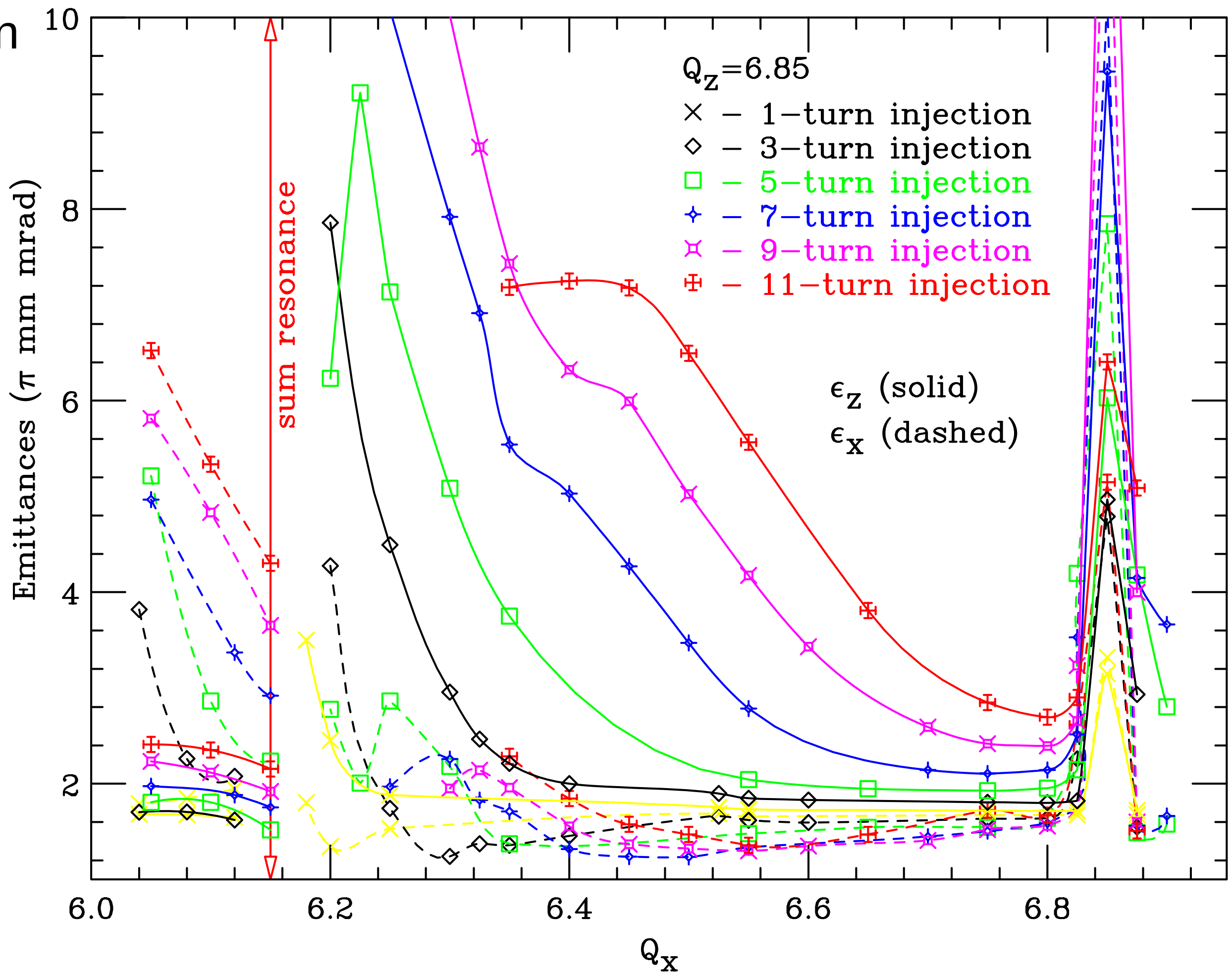


Observations:

- Sum resonance causes vertical emittance growth even when tunes are far away.
- Emittance growth when skew quadrupole amplitude >=0.0014/m.

Effects of Linear Coupling Resonances – 2

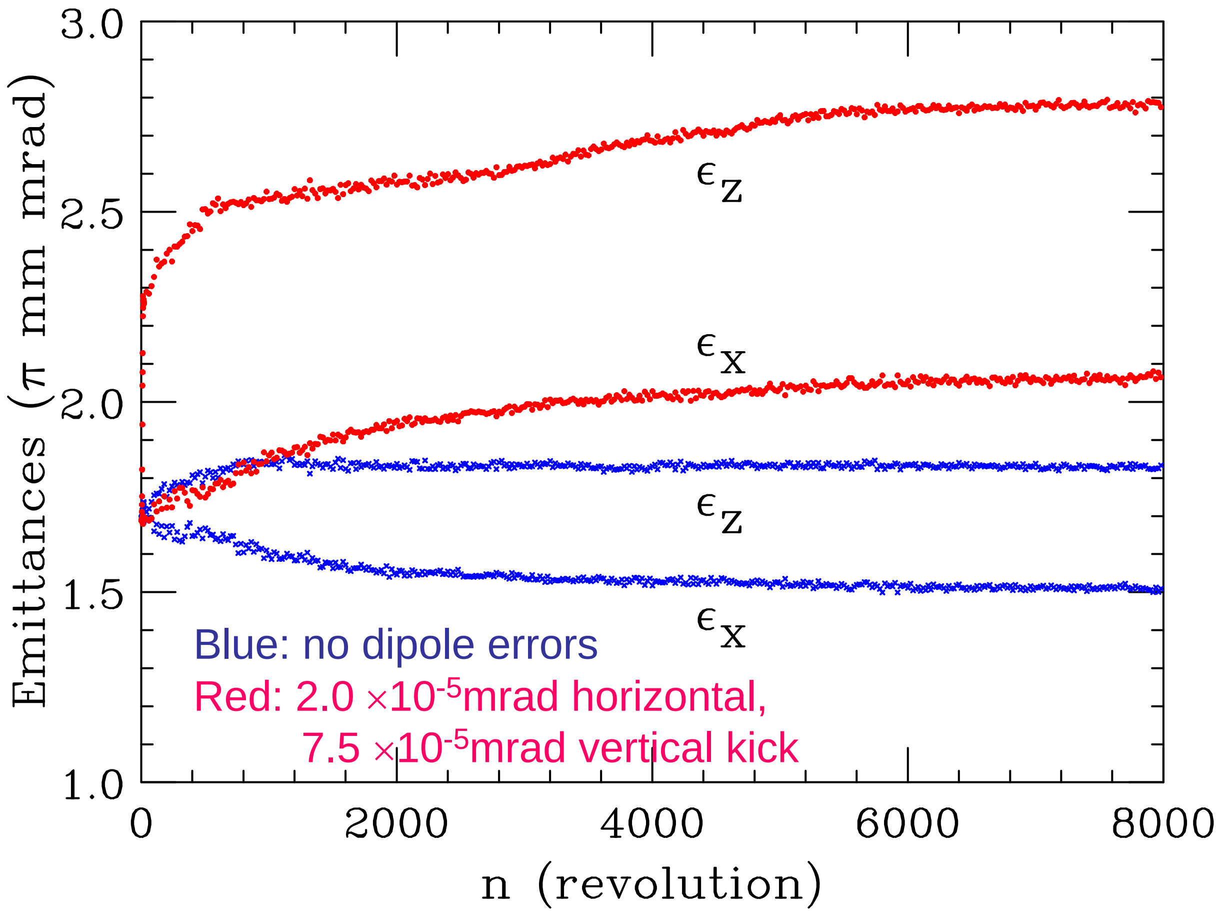
• We vary the beam intensity from 1-turn injection to 11-turn injection.
• Assume skew quadrupole amplitude 0.0035 /m.



Observations:
The stopband width of the sum resonance increases as intensity goes up.

Effects of Dipole Field Errors

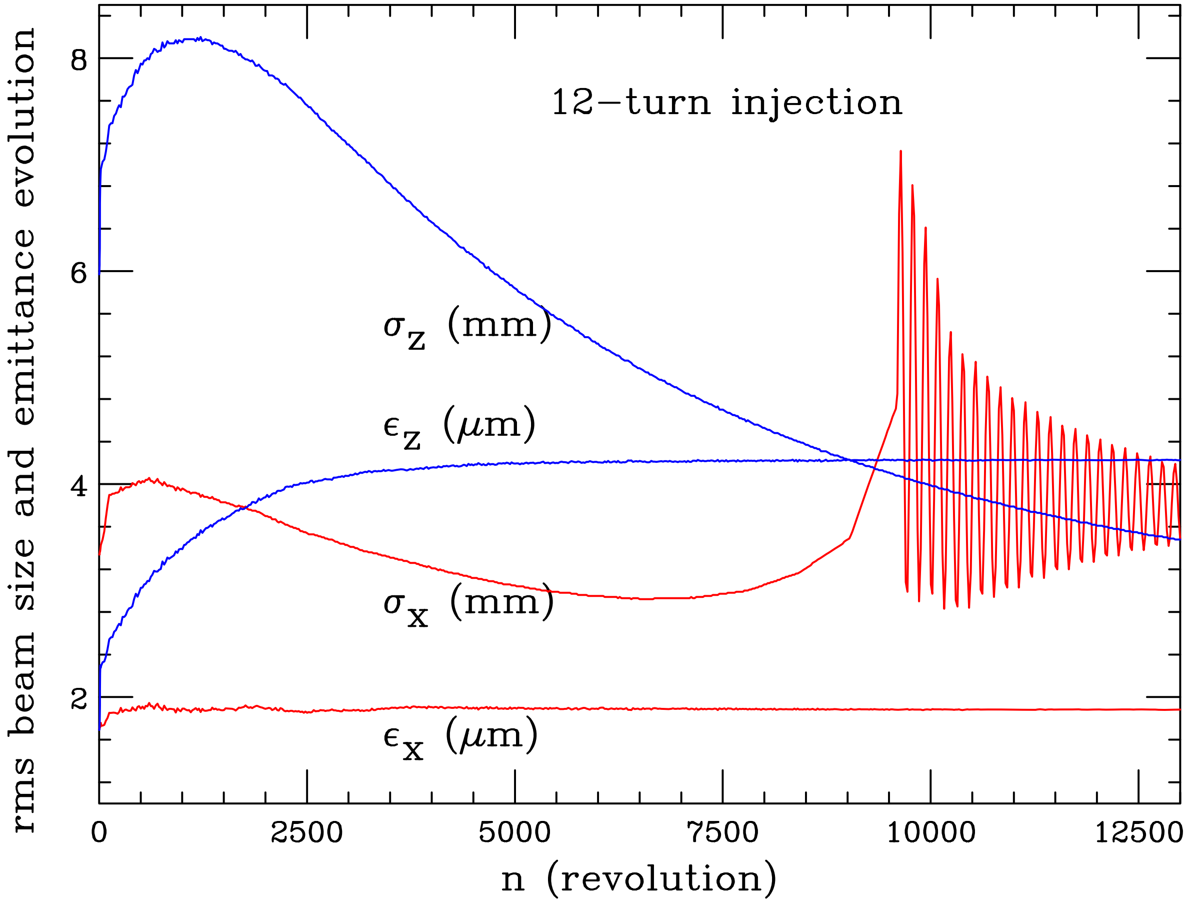
- At 12-turn injection, with space charge, sextupole components.
- With (red) and without (blue) random dipole errors.



There are larger horizontal field errors (i.e. vertical kicks) than the vertical plane (horizontal kicks) because the former can easily come from dipole rolls.

Putting Together

Our estimation:
Quadrupole error amplitude 4×10^{-4} /m; skew-quadrupole error amplitude 35×10^{-4} /m; Dipole field errors 2×10^{-5} mrad horizontal kicks, 7.5×10^{-5} mrad vertical kicks.



This closely reproduces the measurements.

Conclusions

1. We measured the emittance/beam size evolution for a fast ramping proton synchrotron for both transverse planes under various intensity levels on a turn-by-turn basis with ionization profile monitors (IPMs).
2. We analyzed the data to extract emittance growth and its dependence on space charge.
3. We conducted multi-particle simulation to model the emittance growth under space charge. The main sources of emittance growth were determined to be the linear coupling resonances due to random skew quadrupole errors and decoherence of kicks received from random dipole errors. The linear sum resonance plays a very important role in emittance growth in beams with large space charge tune shift
4. Further simulation with consistent space charge treatment may be needed.

Acknowledgments

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