INSTABILITIES OF COOLED ANTIPROTON BEAM IN RECYCLER

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PAC'07

Contents

- In the 3.3 km Recycler Ring, stacked 8.9 GeV/c $\bar{p}$ are cooled both with stochastic (transversely) and electron (3D) cooling.
- For ~ 20 hr about 4E12 pbars are stacked.
- The more beam is cooled, the less stable it is.
- Analysis of transverse coherent instabilities in the Recycler forced us to solve three theoretical problems:
  - Coherent instabilities near the coupling resonance $\nu_x \approx \nu_y$
  - Stability analysis with digital dampers
  - Coherent antiproton-electron instability

Head-Tail near Coupling Resonance

Head-Tail at Coupling Resonance

- As many machines, Recycler stays near $\nu_x = \nu_y$. Single-particle motion can be coupled, and so a conventional optical formalism be invalid.

- Optical modes are not plain x / y eigenvectors any more. Instead, general 4D eigenvectors have to be used.

- There are canonical coordinates and momentums – *normal variables* - associated with the eigenvectors.

- An elementary kick from a leading to a trailing particle has to be calculated in terms of their normal variables.

- After that, Vlasov equation is written in terms of the normal variables similar to conventional uncoupled case.

Coupled Eigenvectors

In Lebedev-Bogacz presentation (further development of Ripken-Mais), the general 4D eigenvectors are:

$$\mathbf{V}_1 = \left(\sqrt{\beta_{1x}},\ \frac{i(u-1)-\alpha_{1x}}{\sqrt{\beta_{1x}}},\ \sqrt{\beta_{1y}}e^{i\nu_1},\ \frac{-iu-\alpha_{1y}}{\sqrt{\beta_{1y}}}e^{i\nu_1} \right)^{\mathrm{T}}$$

$$\mathbf{V}_2 = \left(\sqrt{\beta_{2x}}e^{i\nu_2},\ \frac{-iu-\alpha_{2x}}{\sqrt{\beta_{2x}}}e^{i\nu_2},\ \sqrt{\beta_{2y}},\ \frac{i(u-1)-\alpha_{2y}}{\sqrt{\beta_{2y}}} \right)^{\mathrm{T}}$$

$$\mathbf{V}_{-m} = \mathbf{V}_m^*$$

$$\mathbf{R}\cdot\mathbf{V}_m = \exp(-i\mu_m)\mathbf{V}_m \qquad m = 1,\ 2,\ -1,\ -2\ ;$$

With **R** as the revolution matrix

Mode Amplitudes

- These basis vectors are orthogonal through the symplectic unit matrix **U** :

$$\mathbf{V}_m^{+} \cdot \mathbf{U} \cdot \mathbf{V}_n = -2i\delta_{mn}\,\mathrm{sgn}(m)$$

$$\mathbf{U} \equiv \begin{pmatrix} \mathsf{U}_\mathrm{d} & 0 \\ 0 & \mathsf{U}_\mathrm{d} \end{pmatrix} \;\; ; \;\; \mathsf{U}_\mathrm{d} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Any vector **X** in the 4D phase space can be expanded over **V** ’s:

$$\mathbf{X} = \sum_n C_n \mathbf{V}_n \;; \quad C_n = \frac{i}{2} \mathbf{V}_n^{+} \cdot \mathbf{U} \cdot \mathbf{X}$$

Elementary Kick

- Conventionally, the elementary kick of the trailing particle is expressed as

$$\Delta\theta_x = \frac{e^2 W_x(s) x}{p_0 v_0}; \quad \Delta\theta_y = \frac{e^2 W_y(s) y}{p_0 v_0}$$

- In terms of the phase space vector $\mathbf{X}$, this can be expressed as a perturbation $\Delta\mathbf{X} = \mathbf{W}\cdot\mathbf{X}$, and for the amplitudes:

$$\Delta C_n = \frac{i}{2}\mathbf{V}_n^+ \cdot \mathbf{U}\cdot\Delta\mathbf{X} \equiv \frac{i}{2}\sum_m G_{nm} C_m$$

$G_{nm} \propto W_{x,y}$ is the wake matrix in a basis of eigenvectors.

Diagonal Elements

- Perturbation theory over wake is built similar to Quantum Mechanics. By the same reason, when $|\nu_1 - \nu_2| >> \Delta\nu_{\mathrm{coh}}$,

Only diagonal matrix elements of $\mathbf{G}$ count:

$$G_{nn} = -\frac{e^2}{p_0 v_0}\left(\beta_{nx} W_x(s) + \beta_{ny} W_y(s)\right)$$

Compared with uncoupled case

$$G_x = -\frac{e^2}{p_0 v_0}\beta_x W_x(s)$$

This shows how the coupled problem is reduced to an uncoupled one.

Normal Variables

- The complex amplitudes C_n can be presented as

$$C_n = \frac{q_n}{2} + i\frac{p_n}{2}$$

A linear phase space transformation

$$(x, \theta_x, y, \theta_y) \rightarrow (q_1, p_1, q_2, p_2)$$

is canonical

since it is provided by a symplectic matrix,
composed from real and imaginary parts of the eigenvectors $\mathbf{V}$

Thus, $q_{1,2}$ and $p_{1,2}$ are normal coordinates and momenta.

Kick for Normal Momenta

- The elementary kick results in

$$\Delta q_n = 0 \qquad\qquad n = 1,\, 2$$

$$\Delta p_n = G_{nn} q_n = -\frac{e^2}{p_0 v_0}\left(\beta_{nx} W_x(s) + \beta_{ny} W_y(s)\right) q_n$$

- For uncoupled case, in particular:

$$\Delta q_x = 0$$

$$\Delta p_x = -\frac{e^2}{p_0 v_0}\beta_x W_x(s)\, q_x$$

- After that, the Vlasov equation in the phase space $(q_n\,,\,p_n)$ is exactly identical to the uncoupled case $(q_x\,,\,p_x)$.

Substitution Rules

- Thus, solution of any coupled head-tail stability problem follows from the corresponding uncoupled case applying the substitution rules for tunes, wakes and impedances:

$$
\begin{aligned}
&\nu_x \to \nu_n \\
&\beta_x W_x(s) \to \beta_{nx} W_x(s) + \beta_{ny} W_y(s) \\
&\beta_x Z_x(s) \to \beta_{nx} Z_x(s) + \beta_{ny} Z_y(s)
\end{aligned}
\qquad n = 1, 2
$$

- This is valid when $|\nu_1 - \nu_2| >> \Delta\nu_{\mathrm{coh}}$ (in practice, it is normally so).

- In an opposite case, uncoupled Twiss parameters have to be used.

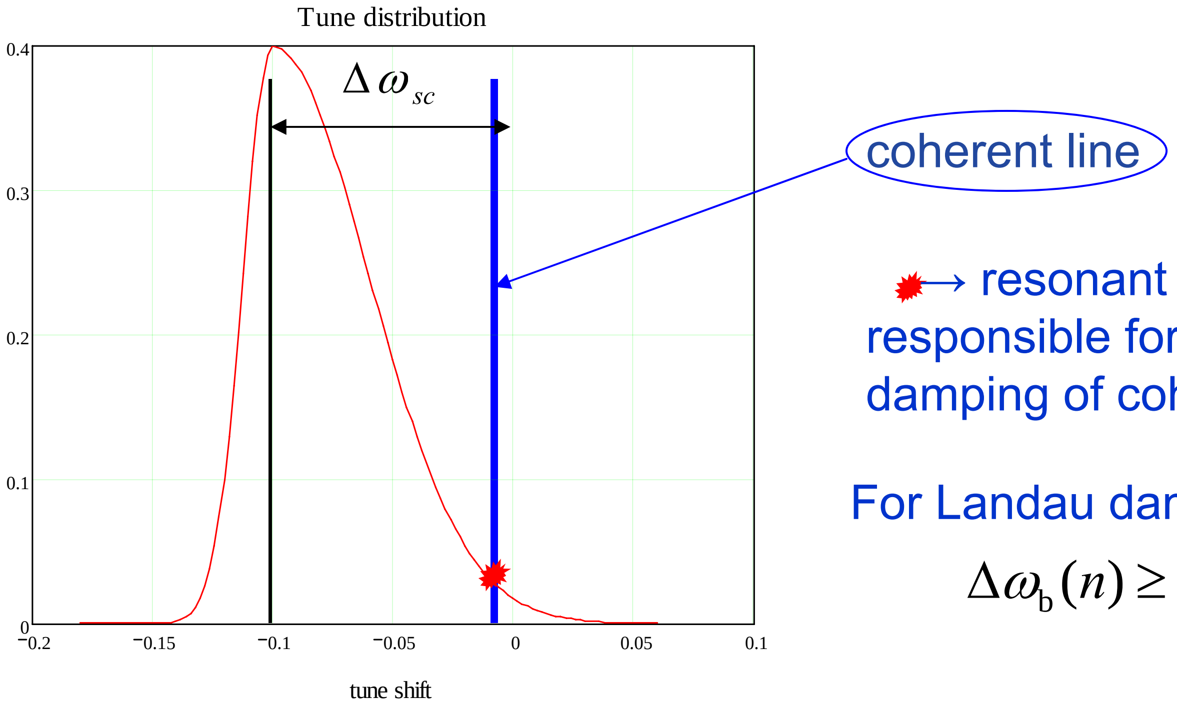
Beam Stability with a Digital Damper

Damper: Space Charge

- Beam space charge (SC) separates coherent and incoherent frequencies by (coasting beam, max):

$$\Delta\omega_{sc} = \frac{N r_0}{2\gamma^2 \varepsilon_\perp T_0}$$

- Chromatic tune spread: $\Delta\omega_b = \omega_0 \,|\, \eta n - \xi \,|\, \delta p / p$



→ resonant particles density, responsible for the Landau damping of coherent oscillations.

For Landau damping:

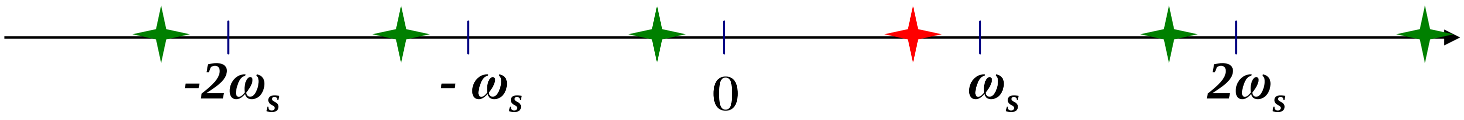
$$\Delta\omega_b(n) \geq (0.2-0.3)\Delta\omega_{sc}$$

For us, it means that frequencies < 100 – 200 MHz can be unstable due to the ring impedance.

Alias Frequencies

- To stabilize these broad band of beam frequencies, a digital damper was installed at Recycler.

- Digitizing goes with sampling frequency, so it adds to incoming frequency ω sequence of all alias frequencies $\omega + q\omega_s$.

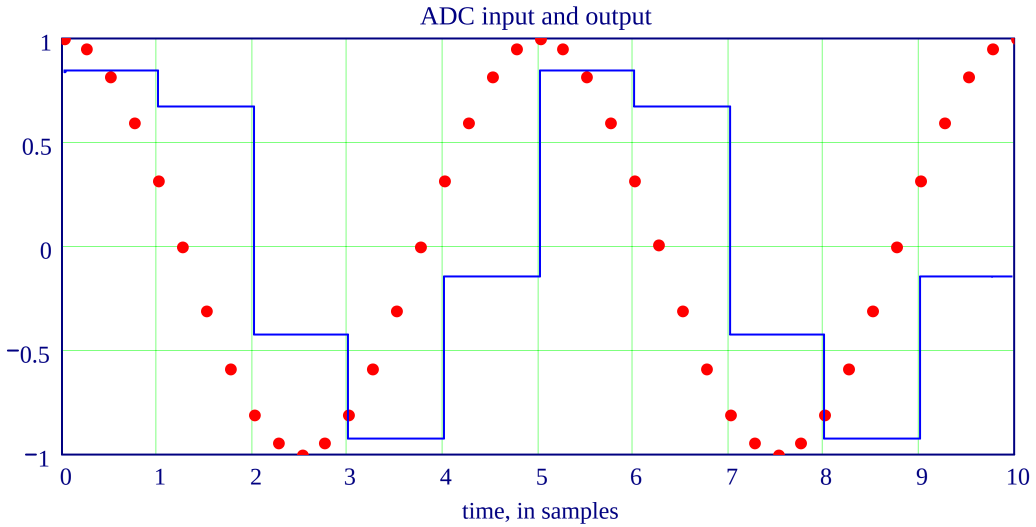


- Thus, longitudinal mode structure is changed by the damper. For coasting beam, space harmonics $\propto \exp(in\theta)$ are not the case any more –except low frequencies $n << \omega_s / \omega_0$.

Analog-Digital Converter (ADC)

- An output of ADC was originally at a sample frequency 53 MHz, being exactly 588 harmonic of the revolution (to filter out all the revolution harmonics).

- The input signal was detected at 4 times higher frequency, and then an average of these 4 numbers went as an output.



ADC input (red dots) and output (blue steps) for 53/5=10.6 MHz input signal

ADC Matrix

- In other words the ADC transformation $\hat{T}$ works as:

$$\hat{T}\exp(-i\omega_p t) = \sum_q T_{pq}\exp(-i\omega_q t)$$

- With the matrix elements

$$T_{pq} = \frac{2}{N}\frac{\sin^2\left(\omega_p\tau_s/2\right)}{\omega_q\tau_s\sin\left(\omega_q\tau_s/(2N)\right)}$$

where $N = 4$ is the averaging number, and $\tau_s = 1/f_s = 2\pi/\omega_s \approx 20$ ns is the output sampling time.

Mode Evolution

- Interplay of the damper, Landau damping and impedance determines stability of the beam modes:

$$\frac{dA_p}{dt} = -\Gamma_0 \sum_q T_{qp} A_q - \Lambda_p A_p - i(\Delta\omega_{\text{coh}})_p A_p$$

- The ADC matrix T is strongly degenerated: all its eigenvalues but one are exact zeroes.

- With impedance, half of these zeroes are getting unstable; they can be stabilized by the Landau damping.

- Landau damping (Gaussian distribution) and coherent shift:

$$\Lambda_n = \sqrt{\frac{\pi}{2}} \Delta\omega_{\text{sc}} x_n \exp(-x_n^2/2); \quad x_n \equiv \frac{\Delta\omega_{\text{sc}}}{\Delta\omega_{\text{b}}(n)}$$

$$\Delta\omega_{\text{coh}} = -i \frac{N r_0 \beta_x}{2\gamma T_0^2} Z(\omega_b + n\omega_0)$$

Two-Beam Instability in Electron Cooling

Ion-electron interactions

- Electron cooling is a method to increase a phase space density of a hot (ion / pbar) beam by merging it with a co-moving cold electron beam at a small portion of the pbar trajectory (20 m from 3.3 km at Recycler – details at Lionel Prost poster).

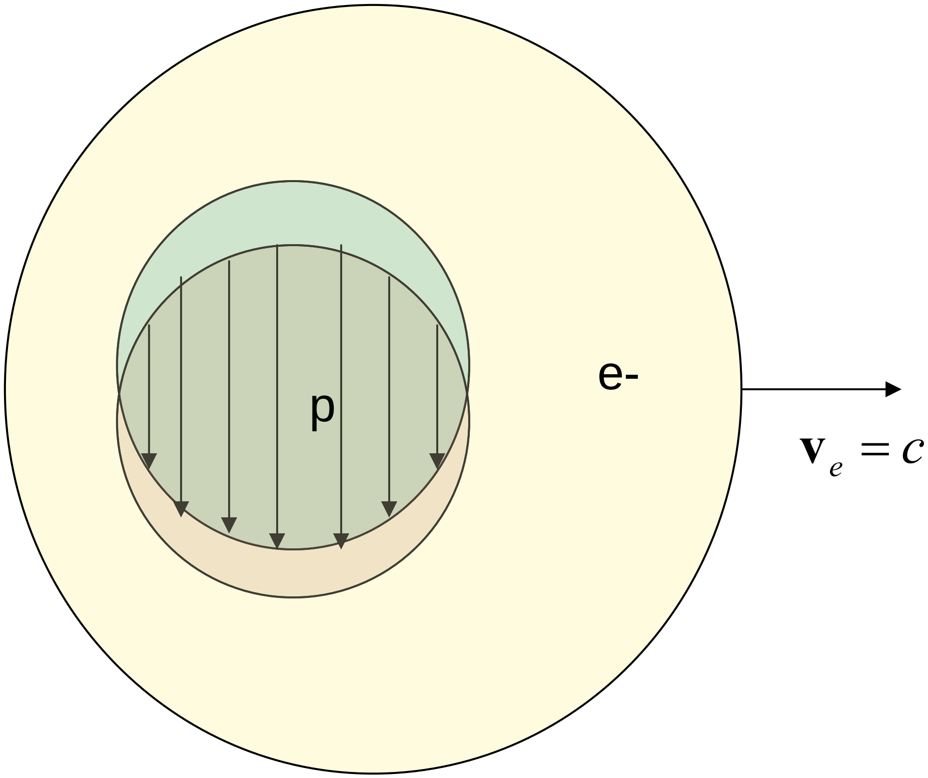
- Cooling may cause several detrimental phenomena:
  - Coherent instability due to lack of Landau damping;
  - Excitation of single-particle resonances by the cooled $\bar{p}$ beam or cooling e-beam $\Longrightarrow$ lifetime degradation;
  - Coherent $\bar{p}-e^{-}$ instability

Main Steps

- Electron beam responds to an initial pbar beam offset.
- The beams are comoving, so the response is local.
- Being local and linear, this response can be presented as a perturbation of the pbar revolution matrix.
- This perturbation is a non-symplectic matrix, proportional to a product of antiproton and electron currents.
- Perturbation theory allows to find eigenvalues of the coherent revolution matrix.

Electron Drift Response

Due to a solenoidal field in the cooler, electron response is essentially a drift in a direction orthogonal to the pbar offset.



$$\mathbf{v}_e = c\,\mathbf{E} \times \mathbf{B} / B^2$$

Dipole motion in the cooler

Rotation symmetry in the cooler allows to use $\xi_{i,e} \equiv x_{i,e} + iy_{i,e}$:

$$\xi_i'' = -k_{ie}^2(\xi_e - \xi_i) + ik_{iL}\xi_i' \qquad \xi_i(0) = \xi_{i0};\ \ \xi_i'(0) = \xi_{i0}'$$

$$\xi_e' = -ik_{ed}(\xi_i - \xi_e) \qquad \xi_e(0) = 0$$

with $k_{ie}^2 = 2\pi n_e Z_i r_p / \left(\gamma^3 \beta^2 A_i\right)$; $k_{iL} = Z_i eB/(p_i c)$

$$k_{ed} = k_{ei}^2 / k_{eL} \propto Z_i n_i / B$$

The interaction parameter: $\alpha = (k_{ie}^2 l^2)(k_{ed} l) = \psi_{ie}^2 \psi_{ed} \propto I_e I_i$

l – cooler length

From here, the cooler matrix can be found.

Coupling is Important

- In practice, all the 3 phases (*ie, iL, ed*) are small, $\psi \equiv kl << 1$. In a leading order:

$$\xi_i'' = -k_{ie}^2 \xi_e$$

$$\xi_e' = -ik_{ed} \xi_i$$

- Electron response is orthogonal to pbar offset. Thus, for conventional planar (uncoupled) pbar modes, a work of the electron response is zero:

$$\vec{F}_{ie} \vec{v}_i = 0$$

- Thus, the instability, if reveals itself at all, has to be strongly sensitive to x-y coupling of pbar optics.

Perturbation Theory

The entire revolution matrix: $$\mathbf{R} = \left(\mathbf{I} + \mathbf{P}\right)\cdot \mathbf{R}^{(0)}$$

$\mathbf{I}$ – identity matrix, $\mathbf{P}$ – perturbation.

The perturbation theory is constructed very similar to the Quantum Mechanics.

The tune shift is given by the diagonal matrix element:

The complex phase shifts: $$\delta\mu_n = -\mathbf{V}_n^+ \cdot \mathbf{U} \cdot \mathbf{P} \cdot \mathbf{V}_n \,/\, 2$$

Where $\mathbf{V}$ are the 4D eigenvectors, and $\mathbf{U}$ – the symplectic unit matrix

The growth rates: $$\Lambda_n = \operatorname{Im} \delta\mu_n / T_0 = -\operatorname{Im}\left(\mathbf{V}_n^+ \cdot \mathbf{U} \cdot \mathbf{P} \cdot \mathbf{V}_n\right)/(2T_0)$$

Useful relation: $$2T_0(\Lambda_1 + \Lambda_2) = \det(\mathbf{R}) - 1 = \operatorname{tr}(\mathbf{P})$$

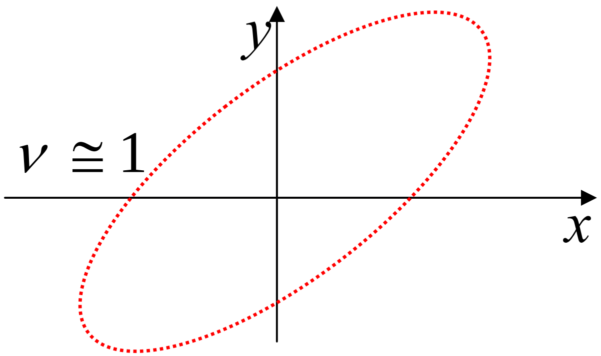
Ion-Electron Growth Rates

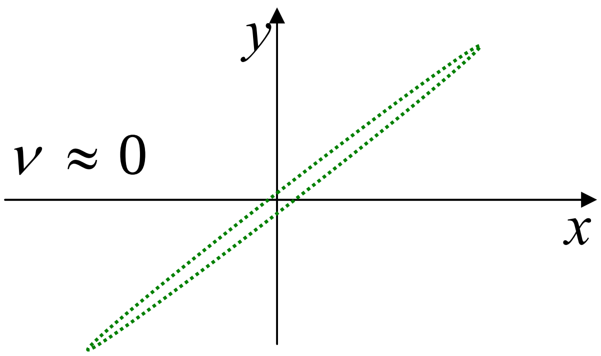
The growth rate follows:

$$\Lambda^{c}_{1,2} = \pm \frac{\alpha \kappa_{xy}}{2T_0}$$

coupling parameter:

$$\kappa_{xy} = \sqrt{\beta_{1x}\beta_{1y} / l^2} \sin(\nu_1) = \sqrt{\beta_{2x}\beta_{2y} / l^2} \sin(\nu_2)$$





In case coupling results from the solenoid only:

$$\Lambda = \frac{\alpha \beta_0}{4T_0 l} \frac{1}{\sqrt{1 + (\mu_x - \mu_y)^2 / \psi_{iL}^2}}$$

$$\psi_{iL} = Bl / (B\rho)$$

Recycler Experience

- Originally, Recycler stayed at coupling resonance (0.42, 0.42) . Lifetime degradation and transverse emittance growth of the cooled pbar beam was observed. The phenomenon was seen to be sensitive on the pbar linear density and on the beams offset.
- The described theory pushed me to insist on more separation of the tunes.
- To have more tune space for stepping out the coupling resonance, the tunes were moved to (0.46, 0.45) . At these tunes, no emittance growth was seen (always cooling), and the lifetime behavior was much better.
- However - the phenomenon did not show any visible dependence on the distance from the coupling resonance – at (0.46, 0.46) it was as good!
- So, the two-beam instability is excluded at Recycler. Our current conjecture for the lifetime degradation is excitation of single-particle resonances by an overcooled core of pbar beam.

Summary

- Three general theoretical problems are solved:
  - Head-tail with x-y coupling;
  - Beam stability with a digital damper;
  - Two-Beam Instability in Electron Cooling.

Everybody is welcome to use that!