



# Tomography for beams with intense space charge

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# Motivation



## The requirement:

Many applications need well regulated beams with **high** intensity and **low** emittance

## The challenge:

Beams are born at low-energies where space charge forces are an issue

- Can cause **emittance growth**
- **Halo** formation possible
- Propagating **waves**



# Approach

- To understand space charge we need an accurate **phase space diagnostic**
- **Tomography** is a good candidate, but to date, has been used only for beams with **little** space charge
- This study further **develops** and **uses** tomography for beams with intense space charge



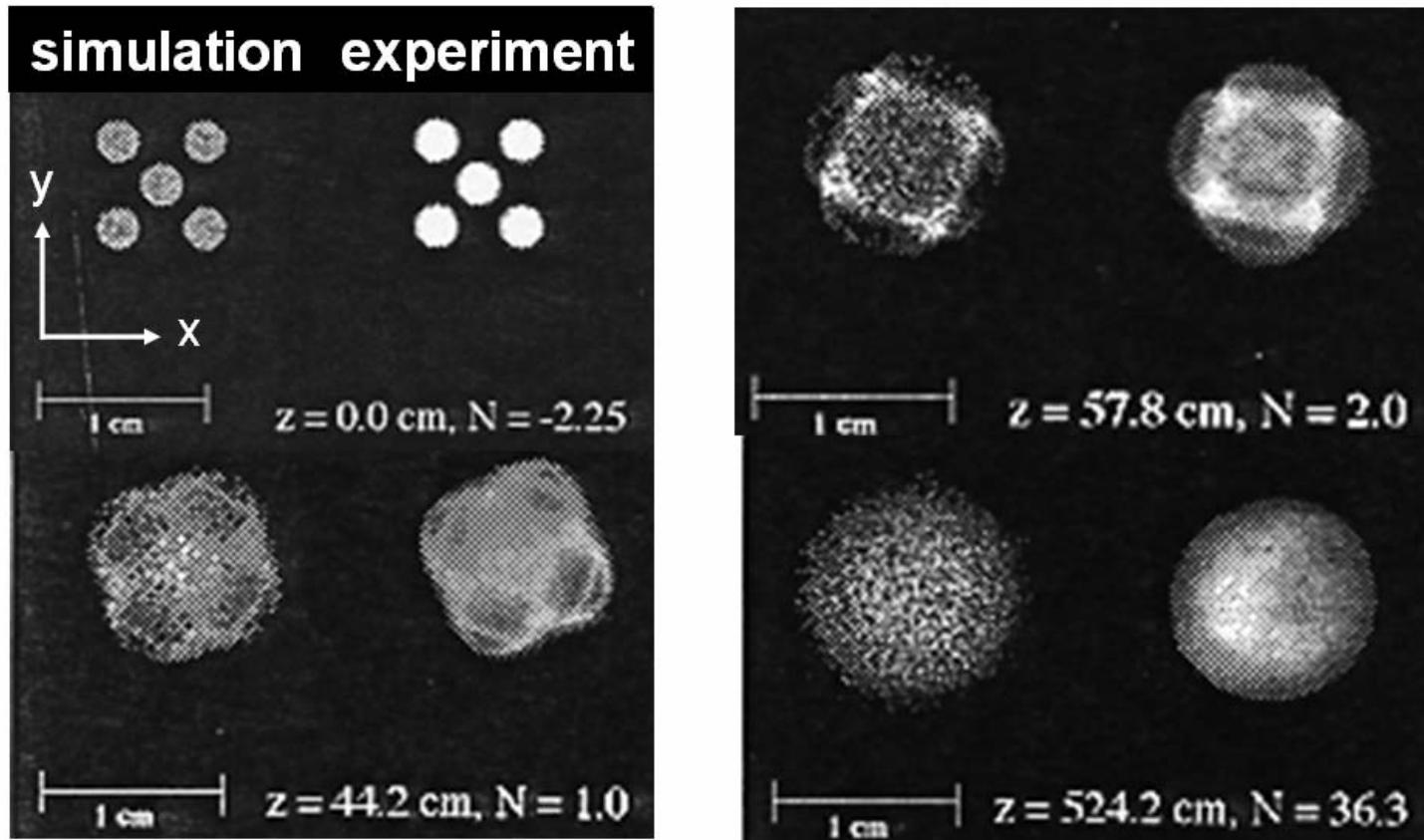
# Outline

1. Example
2. History/Overview
3. Extension to Beams with Space Charge
4. Simulation/ Validation of Tomography
5. Experimental Results



# Motivation Example

- Multi-Beamlet Merger

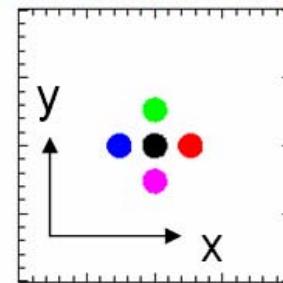


Haber, Kehne, Reiser and Rudd, Physical Review A (1991)

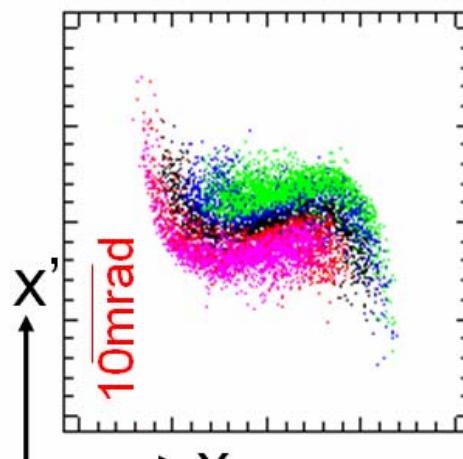
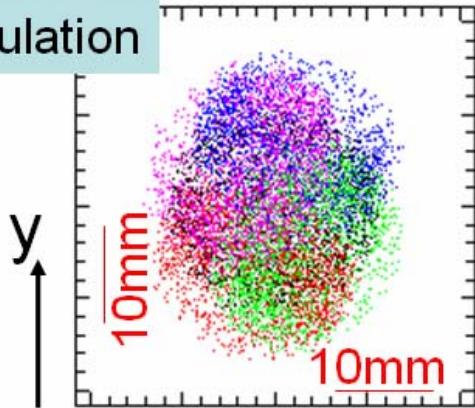
- What happens in phase space?

# Importance of Phase Space

- Initial distribution
- Downstream



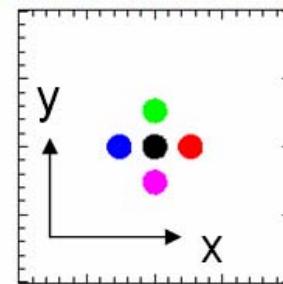
Simulation



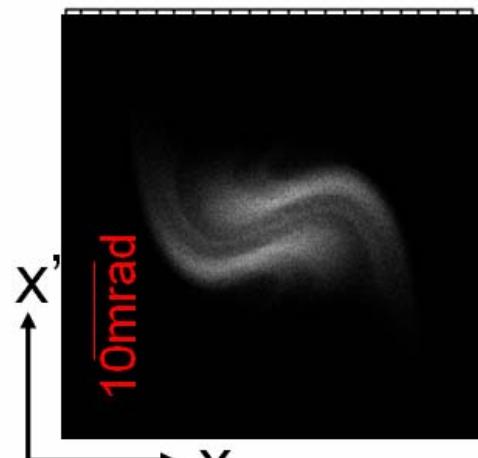
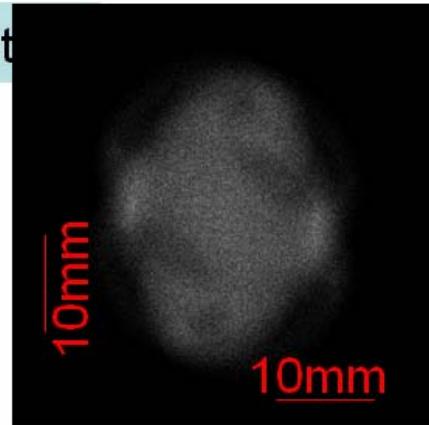
- Homogenization of a beam is different in configuration and phase space

# Importance of Phase Space

- Initial distribution
- Downstream



Simula

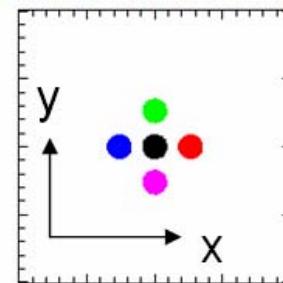


- Homogenization of a beam is different in configuration and phase space

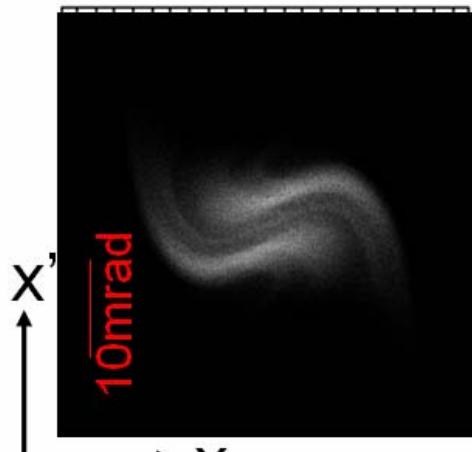
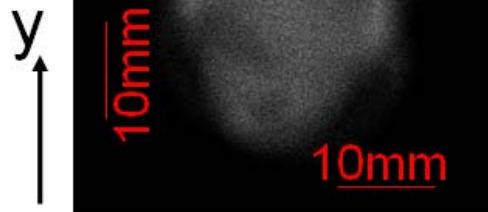
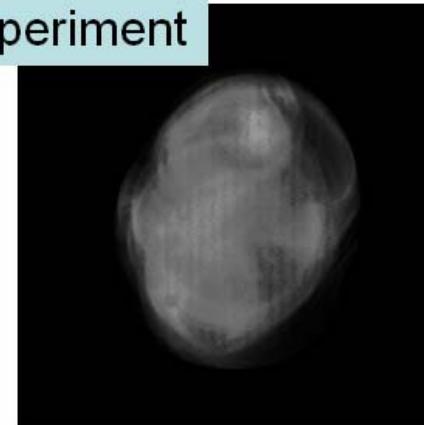
# Importance of Phase Space

- Initial distribution
- Downstream

Simulat



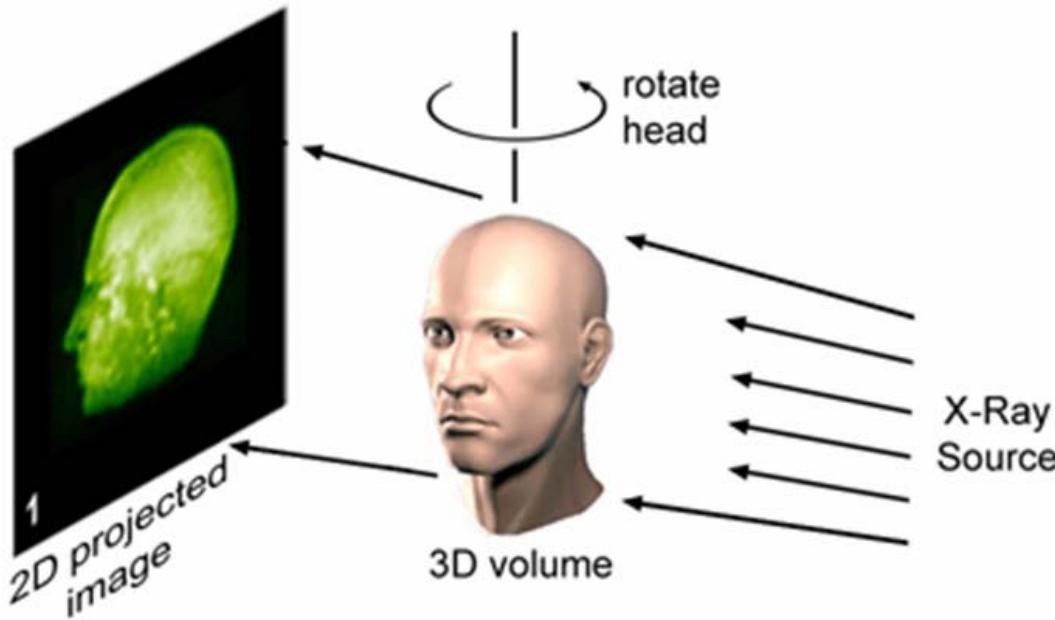
Experiment



- Homogenization of a beam is different in configuration and phase space

# Tomography (CAT Scan)

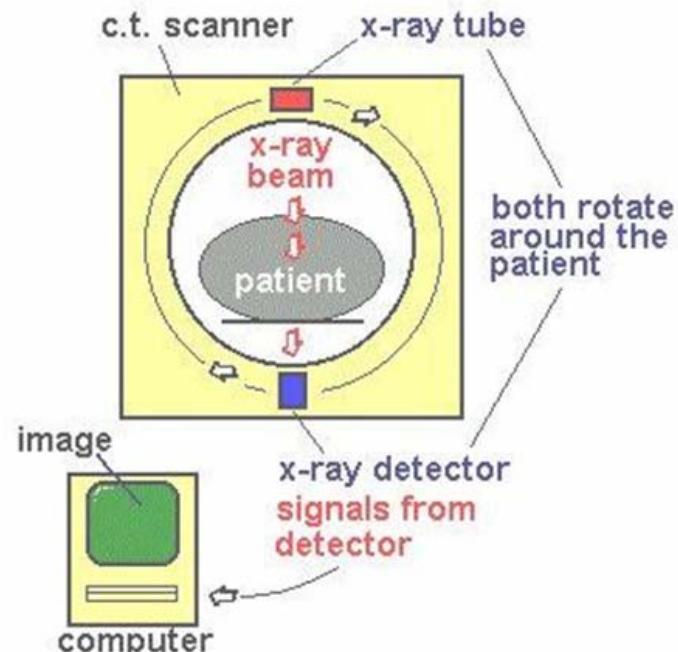
- Tomography is the technique of reconstructing an image from its projections



Abel (1826)



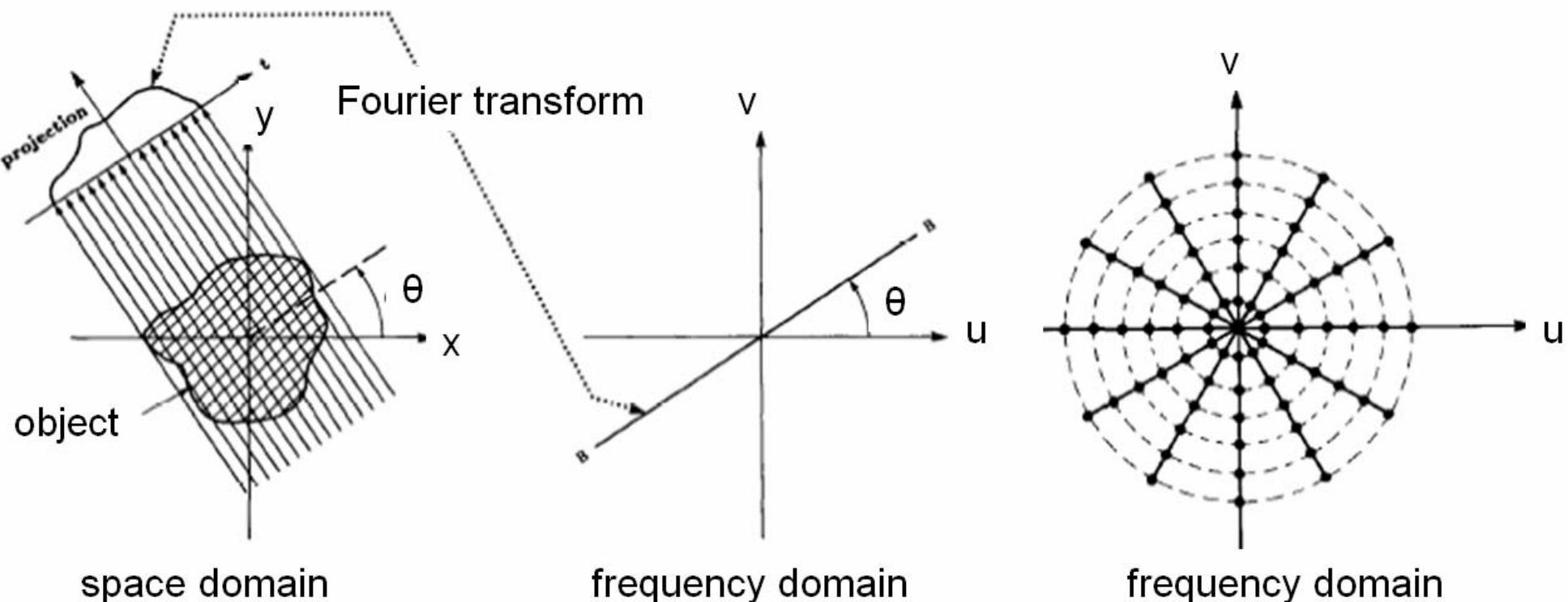
Radon (1917)



# Tomography Algorithm

## Fourier Slice Theorem

Fourier transform of a parallel projection is equal to a slice of the two-dimensional Fourier transform of the original object.

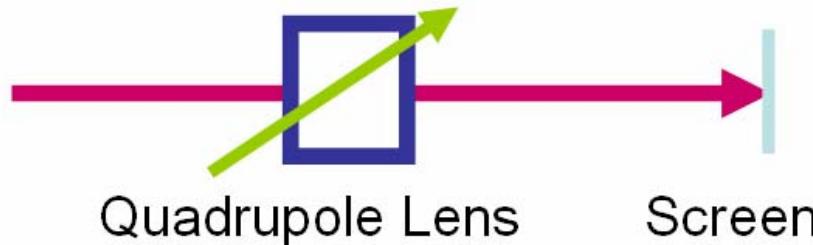




# Tomographic Examples

- Multi-screen method
  - Sander et al. 1979
  - Minerbo et al. 1981
  - Honkavaara et. al. 2005
  - Holder et. al. 2006
- Multi-turn
  - Hancock et al. 1999
  - Connolly et al. 2000
- Multi-slit
  - Raparia et. al. 1997
  - Adachi et. al. 1998
  - Anderson et. al. 2002
  - Friedman et. al. 2004
- Cherenkov Radiation
  - Chen et al. 2003
- Quad-scan Method
  - Fraser et al. 1979
  - McKee et al. 1995
  - Sawamura et al. 1998
  - Geitz et al. 1999
  - Brunken et al. 2000
  - Yakimenko et. al. 2003
  - Montag et. al. 2004
  - Ohgaki et. al. 2004
  - Li, PhD Dis. et al. 2004
  - Zhou et al. 2006

# Quad-Scan Tomography



- Quadrupoles **rotate** the phase space distribution

- Single particle:

$$\ddot{x} = -\kappa x + F_{SC}$$

$$\kappa = \frac{qB}{\gamma m a v}$$

- No SC:  $\ddot{x} = -\kappa x$

Is this equation  
familiar?

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos \sqrt{\kappa_x} z & \frac{1}{\sqrt{\kappa_x}} \sin \sqrt{\kappa_x} z \\ -\sqrt{\kappa_x} \sin \sqrt{\kappa_x} z & \cos \sqrt{\kappa_x} z \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

- With SC: Very complicated! Need **approximations**



# Beam Tomography with space charge



- Single particle equation:

$$\ddot{x} = -\kappa x + F_{SC}$$

- Assume linear forces:

$$\ddot{x} = -(\kappa_{x,0} - \frac{2K}{X(X+Y)})x$$

- Find X, Y by solving envelope equations:

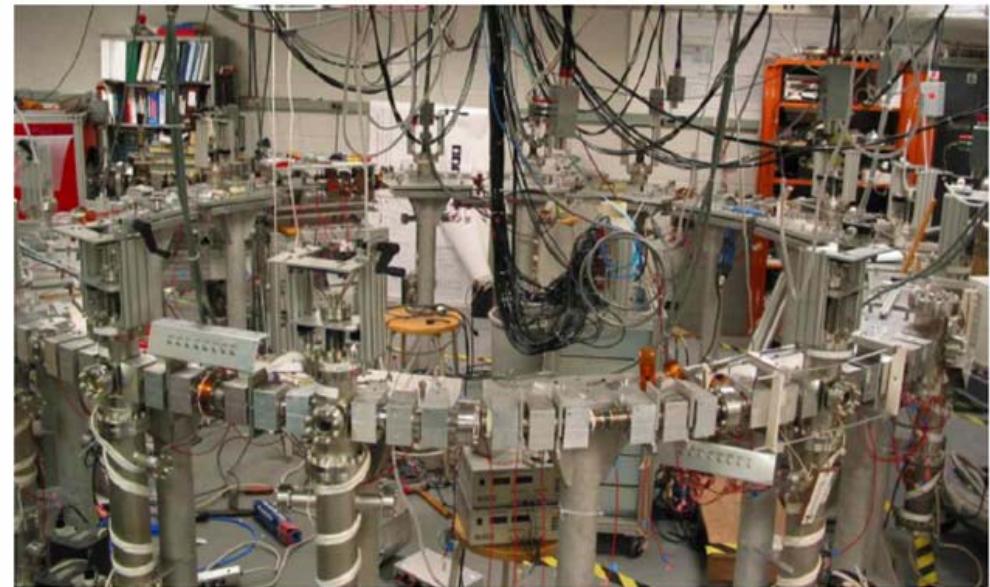
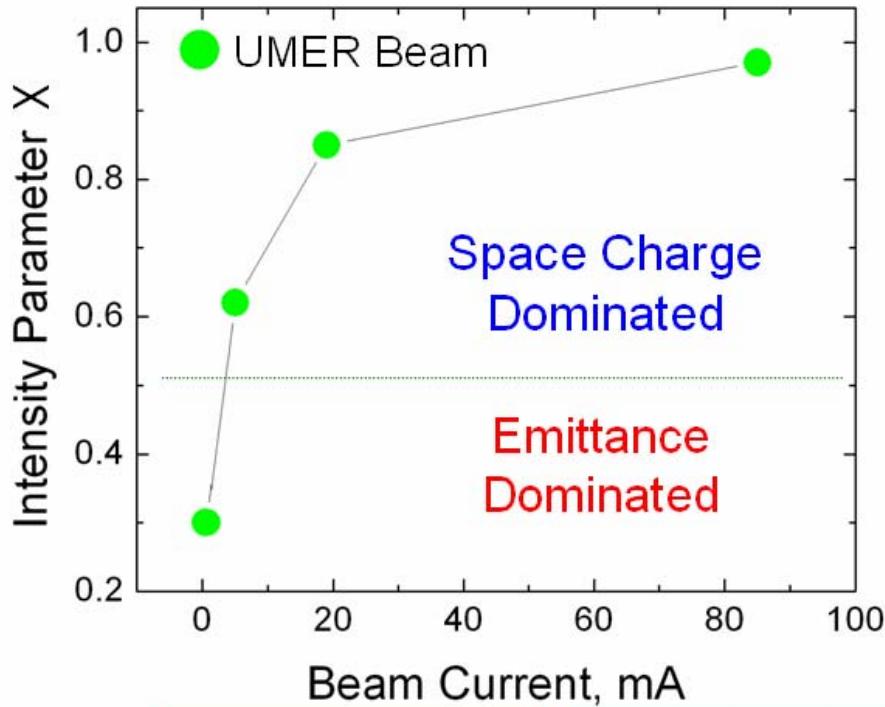
$$X'' + \kappa_x X - \frac{2K}{X+Y} - \frac{\epsilon_x^2}{X^3} = 0$$

$$Y'' + \kappa_y Y - \frac{2K}{X+Y} - \frac{\epsilon_y^2}{Y^3} = 0$$

- Get transport matrix

Stratakis et al. Physical Review ST - AB 9, 112801 (2006)  
Geitz et al. PAC 1999

- UMER is serving as a low-cost model of high intensity accelerators



$$K = \frac{qI}{2\pi\epsilon_0 m(c\beta\gamma)^3}$$

$$\chi \equiv \frac{1}{1 + \frac{\beta\gamma l_0}{2I} \left( \frac{\epsilon_n}{a^2} \right)}$$

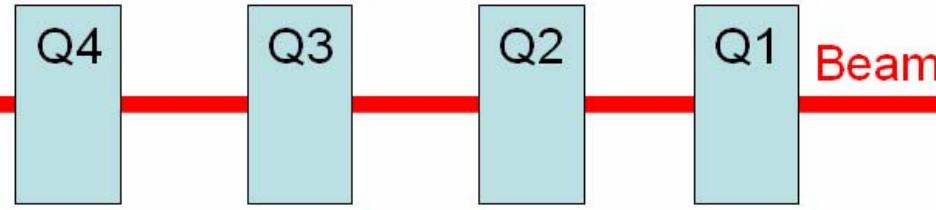
Kishek (TUZBAB03) – talk  
 Walter(TUPAS047), Bernal(THPAS030) –  
 posters

Energy	10 keV
Current range	0.6-100 mA
rms Emittance, norm	0.2-3 $\mu$ m
Zero-Current Tune	7.6
Depressed Tune	1.5 – 6.5

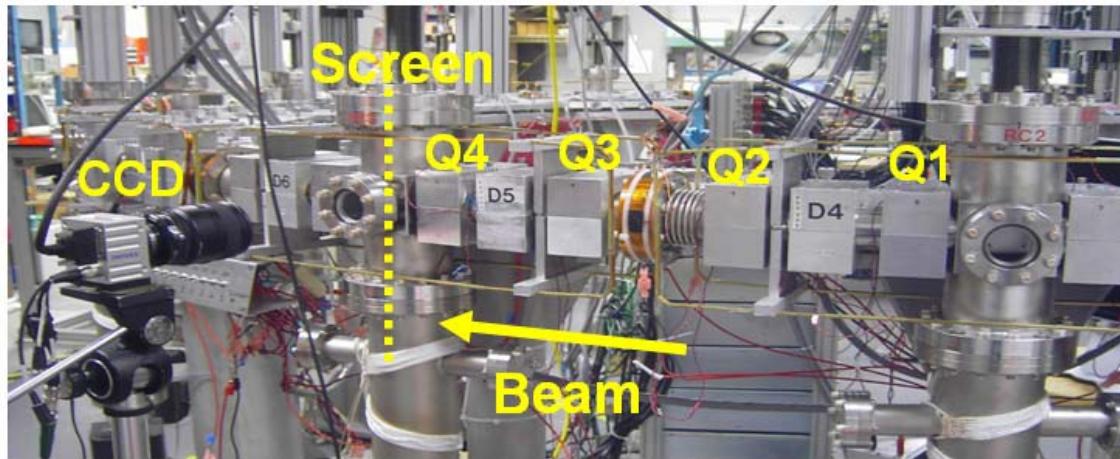
# Tomography Diagnostic Configuration

Photo Capture

Phase space  
recovery



Simplified  
configuration



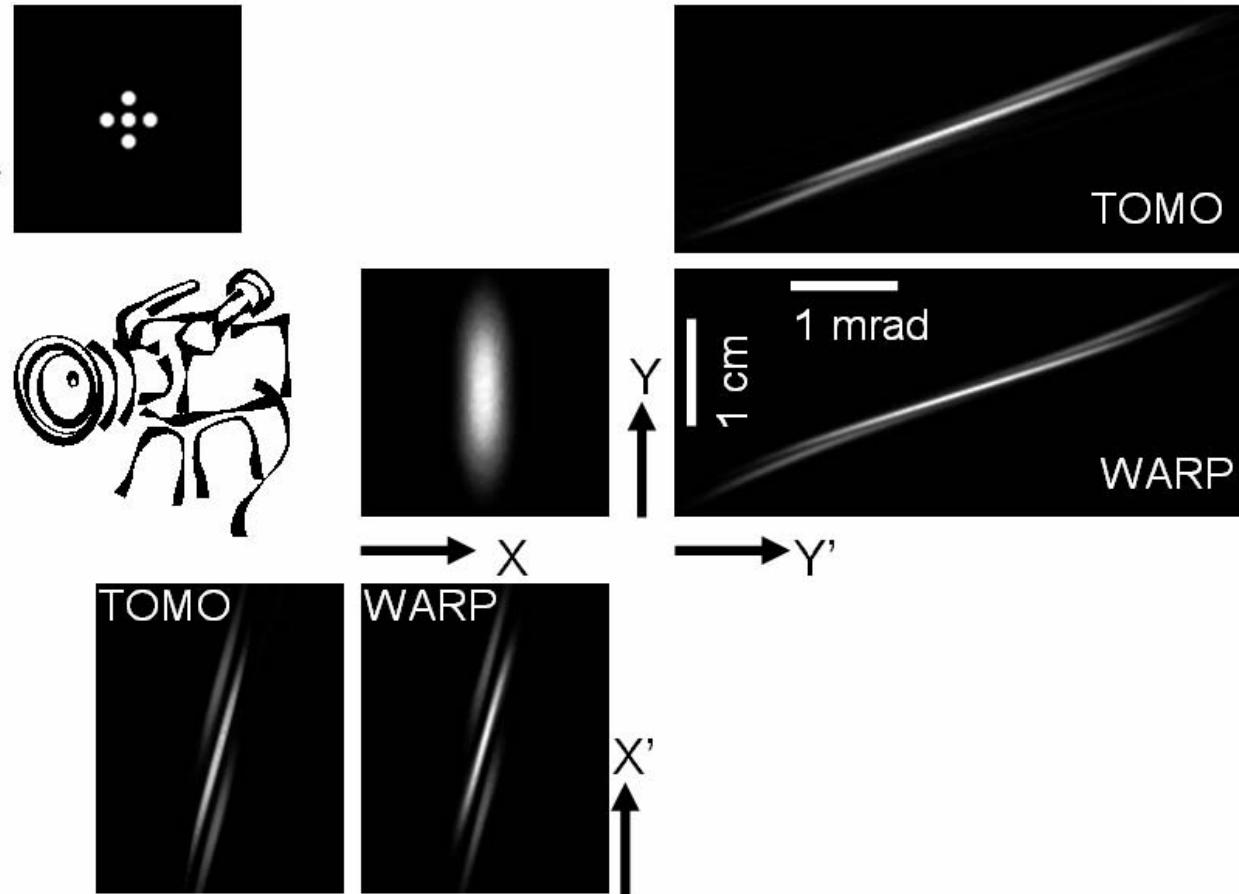
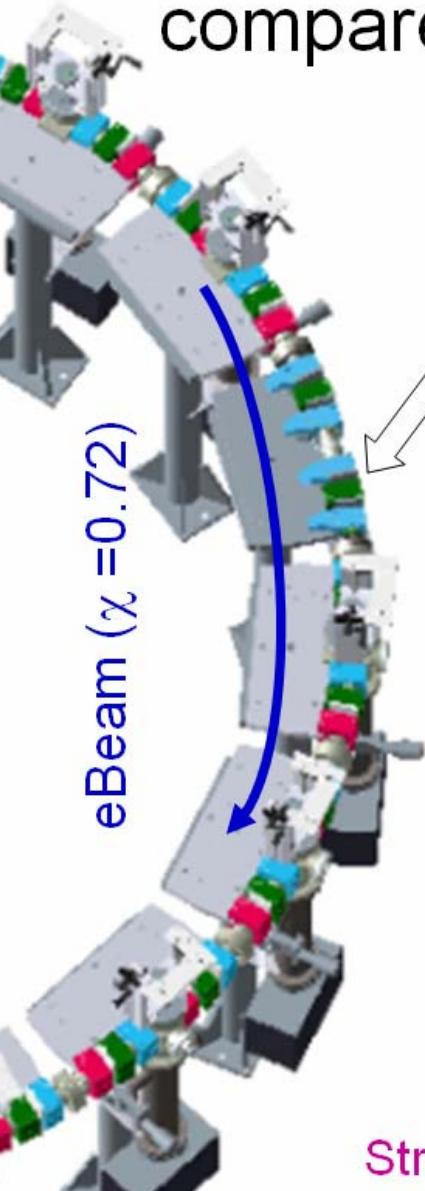
Experimental  
configuration

- Four magnets were employed for the tomographic recovery of the phase space

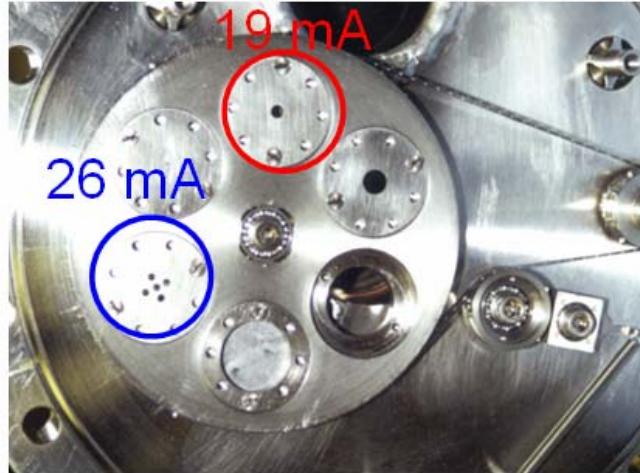
# Tomography Simulation/ Validation

- Reconstructed phase space by Tomography is compared to that generated directly by WARP.

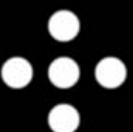
Non-uniform spatial distribution



# Experiments with Intense Beams

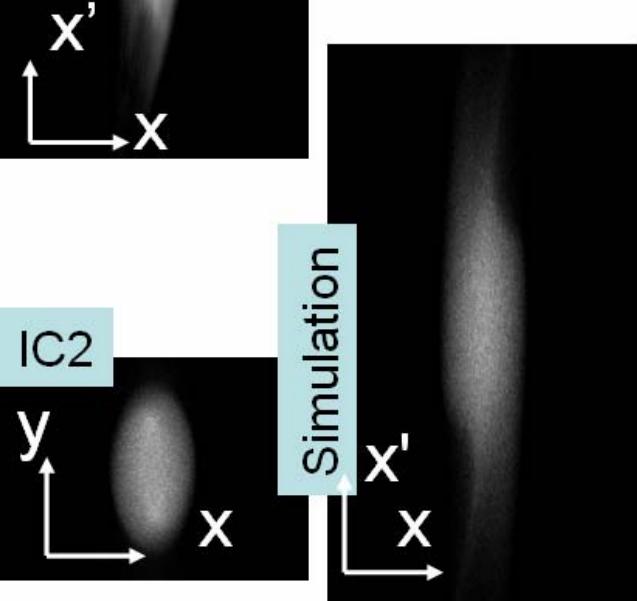
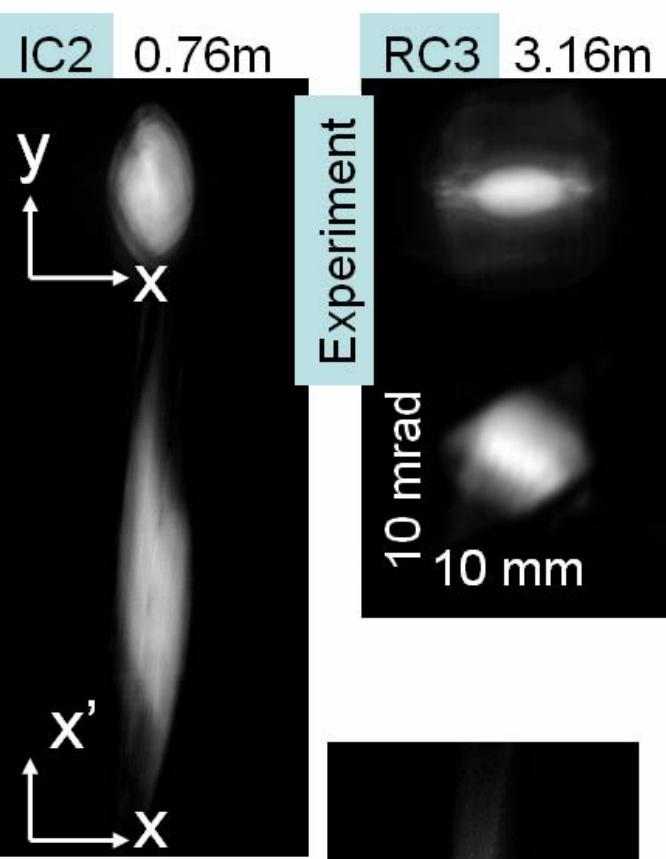


- Experiment 1:  
Uniform beam evolution (19mA,  $\chi=0.85$ ).
- Experiment 2:  
Nonuniform beam evolution (26mA,  $\chi=0.91$ ).

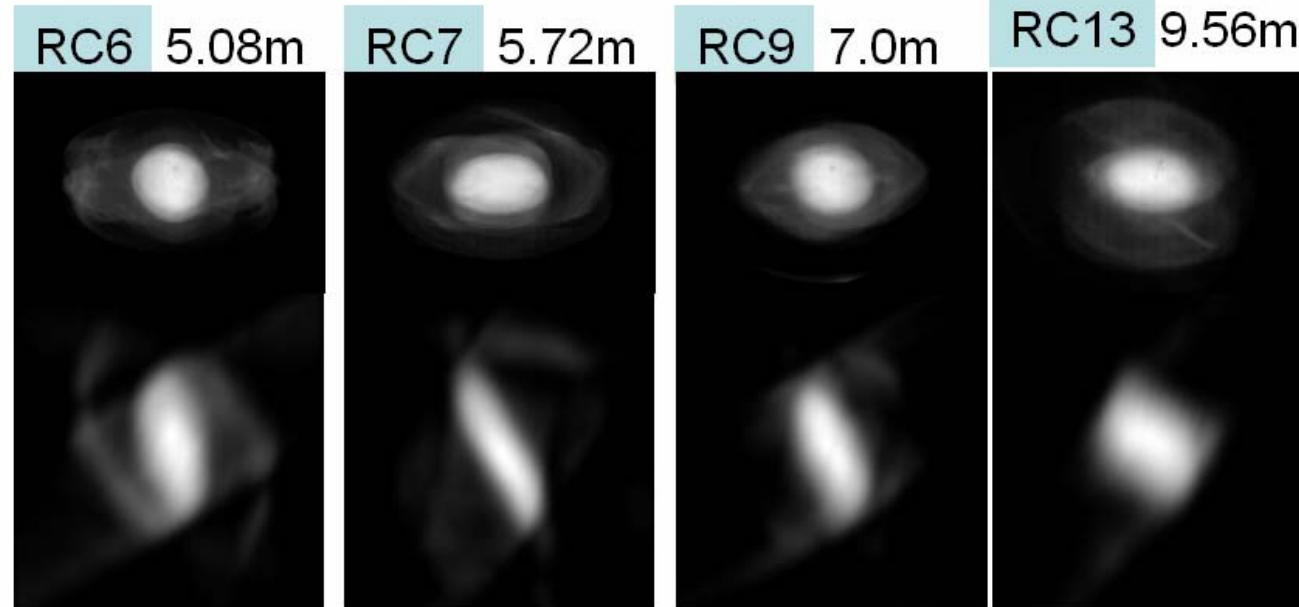




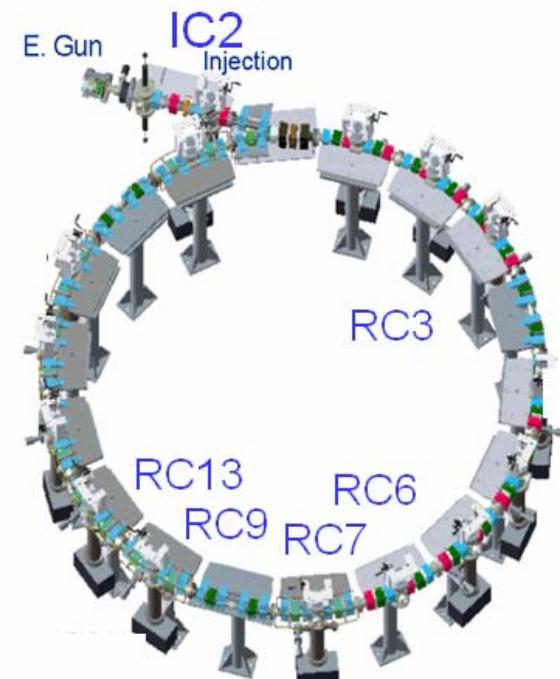
# Experiment 1: Single Beamlet Transport



XX' Reconstruction



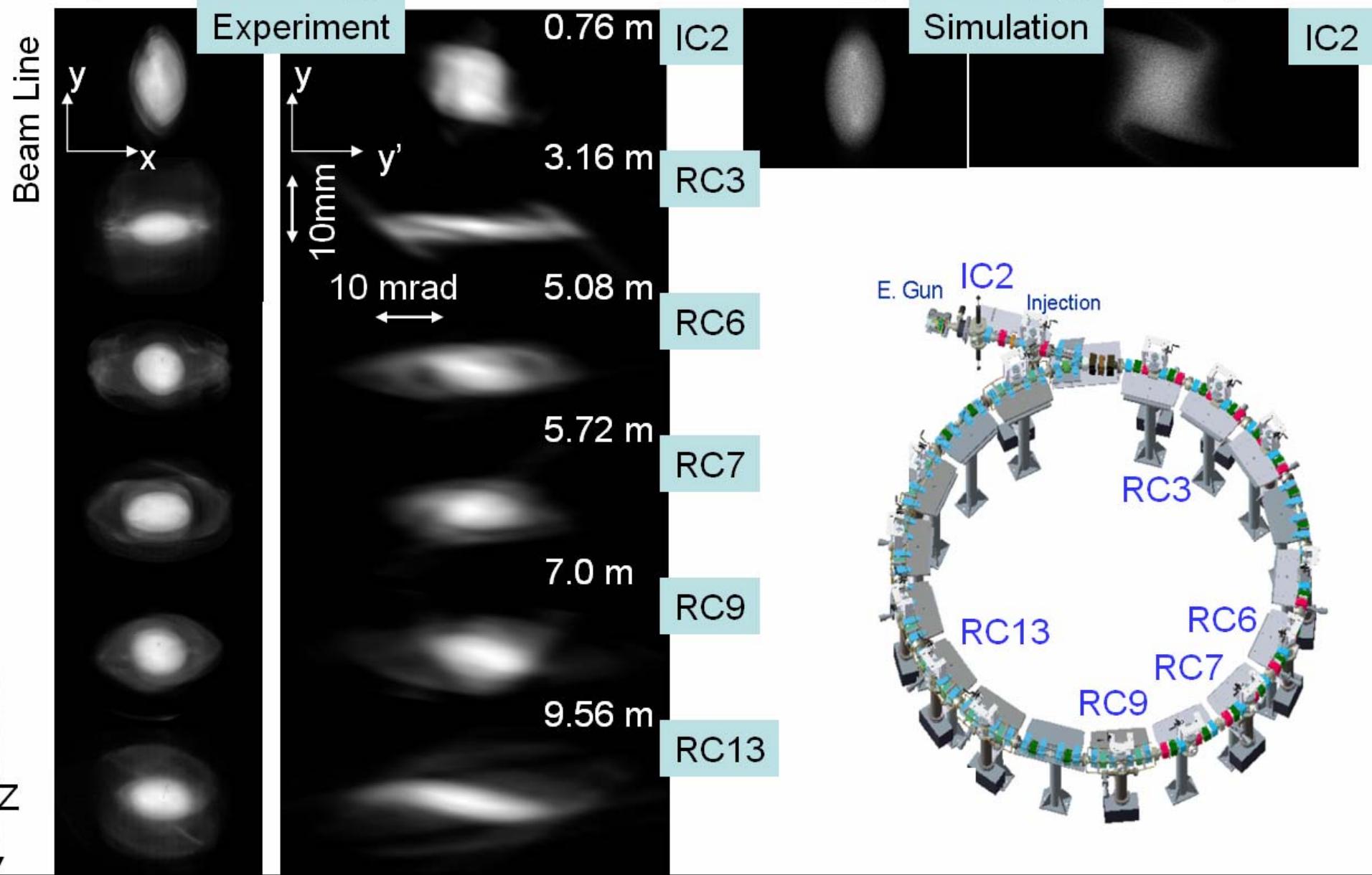
Haber (THPAS031) - poster



# Experiment 1: Single Beamlet Transport

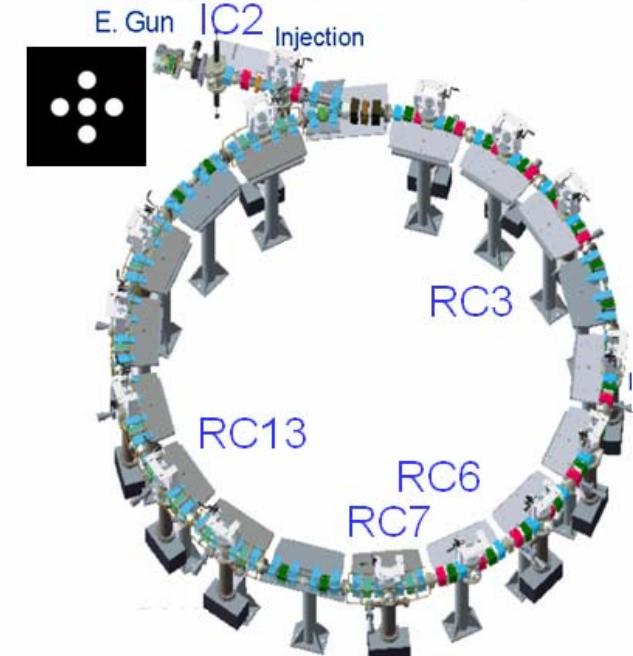
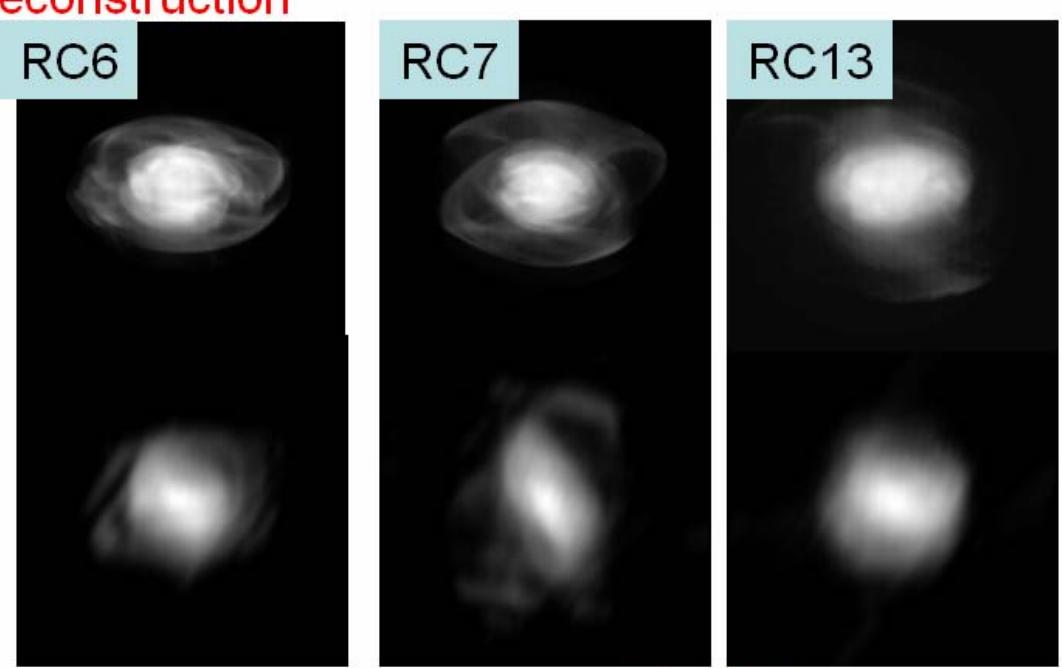
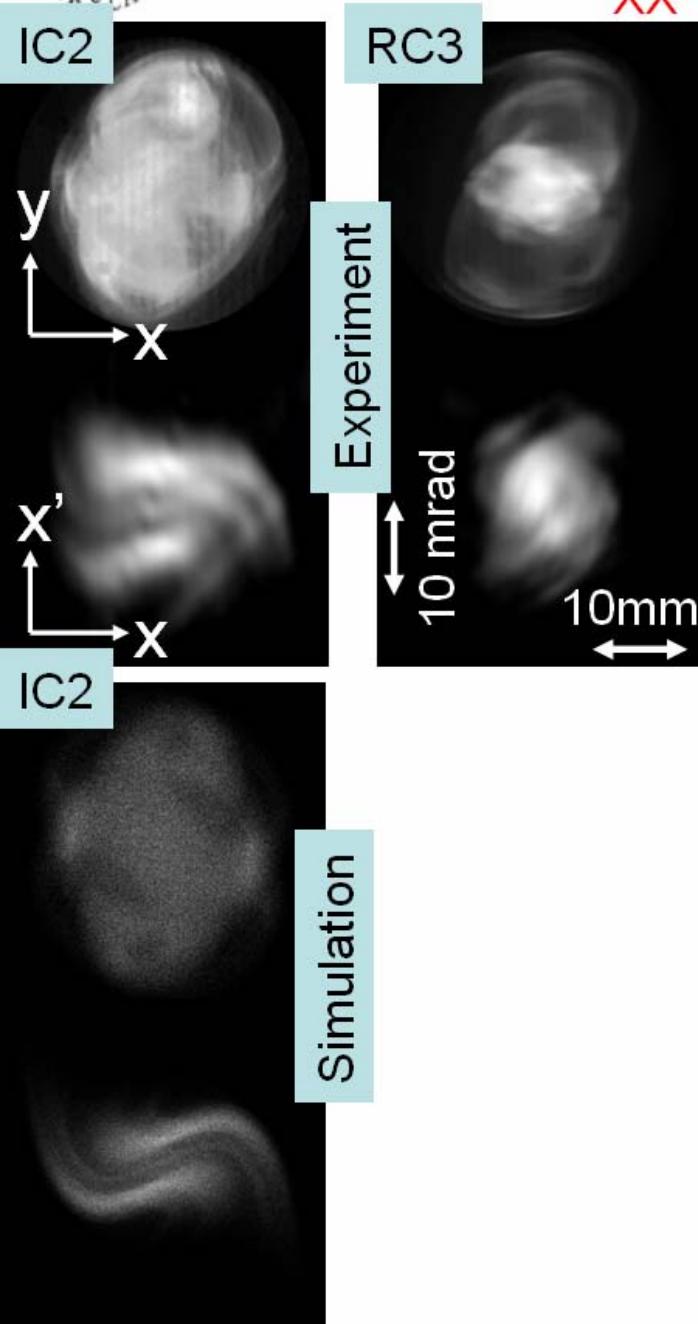
YY' Reconstruction

- Space Charge Dominated Beam (19mA,  $\chi=0.85$ )





# Experiment 2: Multibeamlet Transport

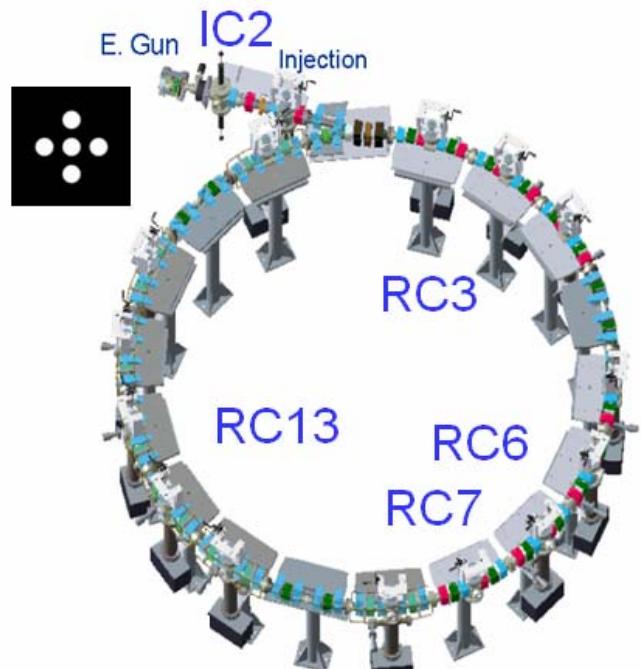
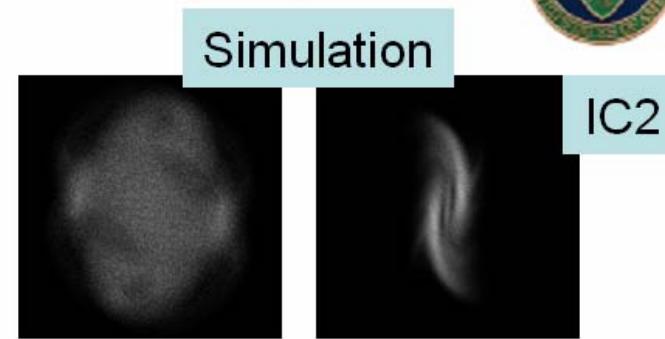
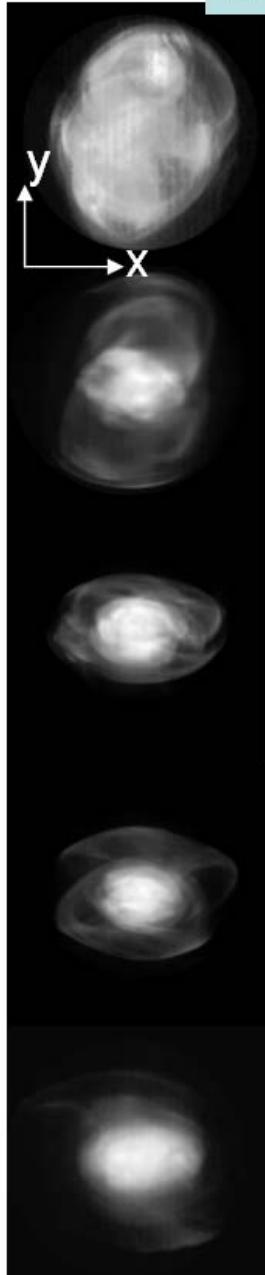




# Experiment 2: Multibeamlet Transport



Beam Line  
↓  
Z





# Conclusions

- Extended Tomography to beams with space charge
- Simulation validated accuracy of technique
- Experimental measurements reveal evolution of beam halo and multi-beamlet merger



# Acknowledgments



Not shown  
(but thanks, too):

outside group

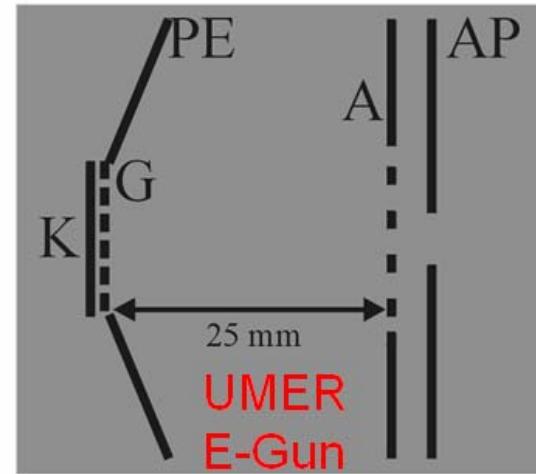
- Dr. D. Grote
- Dr. A. Friedman
- Dr. V. Yakimenko
- Dr. H. Li

UMER group

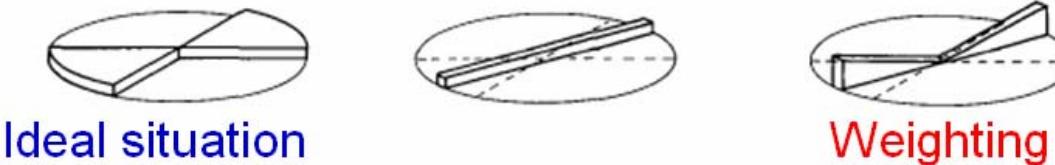
- Dr. D. Sutter
- K. Tian
- B. Beaudoin
- M. Holloway
- C. Wu



# Backup



# Filtered Backprojection Algorithm (FBA)



- A simple weighting in the frequency domain is used to take a projection and estimate a pie-shaped wedge of the object's Fourier transform.
- We multiply the value of the Fourier transform of the projection and multiply it by the width of the wedge at that frequency
- Apply inverse Fourier Transform of the filtered projections



# Quad Scan Tomography

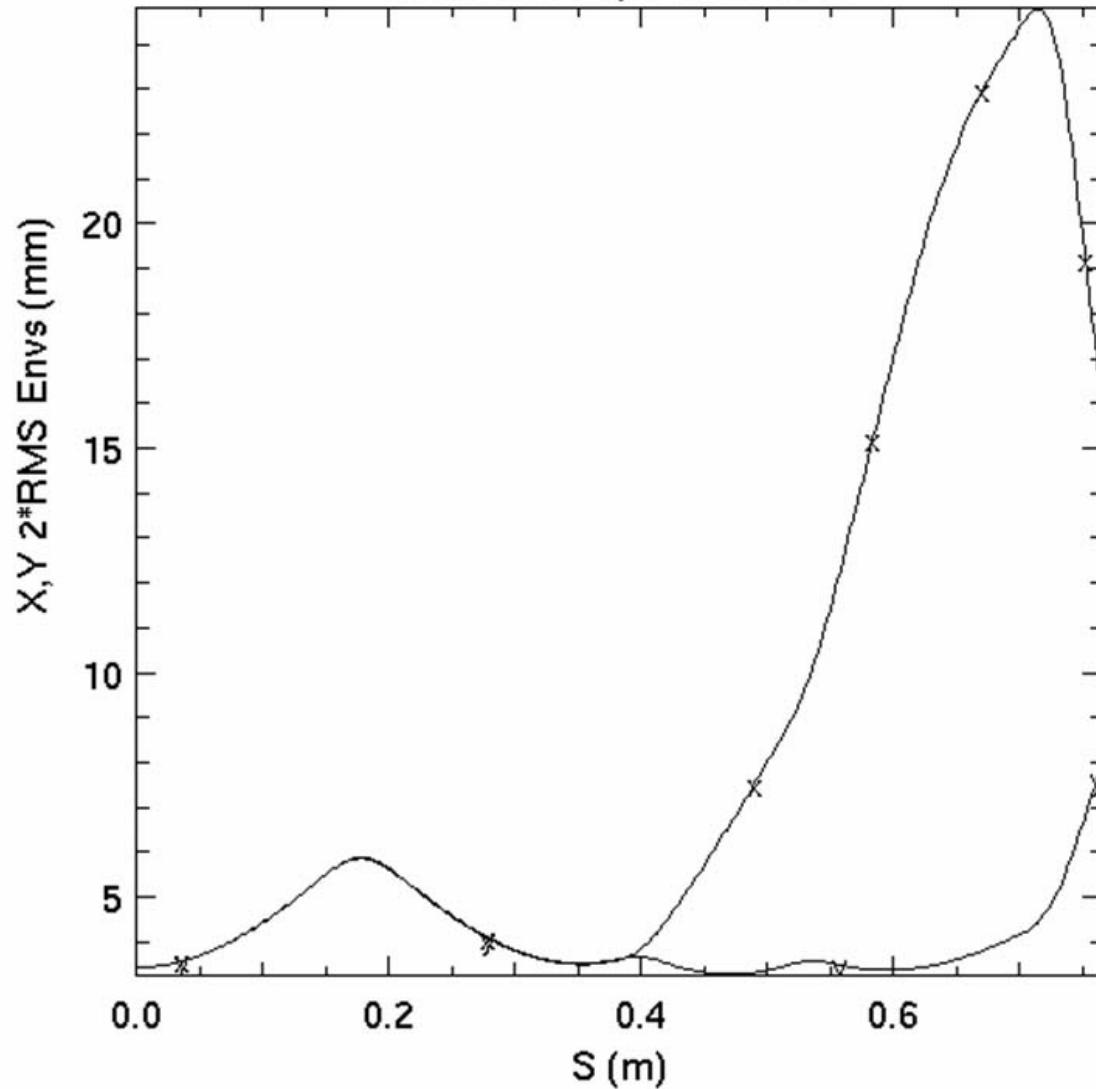


Article	Beam/Facility	Energy / Current	G. Perveance
Brunkens et al. 2000	S-DALINAC	8 MeV/ $10^{-6}$ A	$2.5 \cdot 10^{-14}$
McKee et al. 1995	Duke	44 MeV / 0.2A	$3.5 \cdot 10^{-11}$
Ohgaki et al. 2004	KU-FEL	10 MeV/ 0.1A	$1.3 \cdot 10^{-9}$
Yakimenko et al. 2003	Brookhaven	50 MeV/ 100A	$1.2 \cdot 10^{-8}$
Zhou et al. 2006	Brookhaven	60 MeV/ 266A	$1.9 \cdot 10^{-8}$
Montag et al. 2004	RHIC	54 MeV/ 330A	$3.2 \cdot 10^{-8}$
Sawamura et al. 1998	JAERI	16 MeV/ 100A	$3.4 \cdot 10^{-8}$
Geitz et al. 1999	TeslaTF	16 MeV/ 200A	$7.0 \cdot 10^{-7}$
Fraser et al. 1979	LAMPF	0.75-100 MeV/ 18mA	$10^{-5} - 10^{-8}$
Li H. PhD Dis. 2004	UMER	10 keV/ 0.007A	$1.0 \cdot 10^{-4}$

For UMER: G. Perveance  $10^{-6}$  to  $10^{-3}$



Beam Envelope from G000



Q1= -1.67A  
Q2= -1.49A  
Q3= 3.5A