

# MULTI-OBJECTIVE OPTIMIZATION OF DYNAMIC APERTURE AT OFF-AXIS INJECTION LATTICE OF HEPS

J. Wu<sup>†</sup>, Y. Zhang, Y. M. Peng, IHEP, Beijing, China

## Abstract

The off-axis injection scheme is also considered at HEPS. A large beta insertion section is need in the case, which breaks the symmetry of the machine. We introduce two designs of the injection section. There exist clear difference for the dynamic aperture between the two designs. The low-order nonlinear resonance driving terms is calculated and compared. The results show a correlation between momentum acceptance and the off-momentum driving terms. We enlarge the dynamic aperture by tuning the on-momentum and off-momentum non-linear driving terms with a multi-objective optimization code.

## INTRODUCTION

Both on-axis injection scheme and off-axis injection scheme are considered in High Energy Photon Source (HEPS). Technology of off-axis injection is matured and it is well demonstrated in existing light sources. For ultra-low emittance storage ring, dedicated effort is required to reach the required dynamic aperture, but maybe at the expense of larger emittance.

Different lattices are adopted in different injection schemes. Compared with the standard 48-cell lattice, which is used for on-axis injection scheme, large-beta section is inserted in off-axis injection scheme that breaks the symmetry. As a result, dynamic aperture (DA) and momentum acceptance (MA) decrease a lot, especially MA, which falls from 3% to 2%. This result will be mentioned later.

Minimizing of the resonance driving terms (RDTs) is widely used for optimizing the dynamic aperture. This correction method is described in [1]. In consideration of the large reduction in momentum acceptance, we adopt both on-momentum and off-momentum resonance driving terms to carry out the optimization.

In this paper, we first introduce the lattice that includes the large-beta section and compare the DA and MA of two schemes. Then we give a short description for the resonance driving terms theory and analysis the 3<sup>rd</sup> and 4<sup>th</sup> order (Hamiltonian) terms with this method. According to the analytical results, we try to choose the objection functions to do the optimization.

## LATTICE

To meet the demands of off-axis injection, standard 48-cell structure should be inserted large-beta ( $\beta_x=90.86\text{m}$ ) linear section. As a result, the lattice includes: standard cell with 44 H7BA structure, injection cells with a high-beta section for injection, and the opposite two cells. See Figure 1 and Table 1.

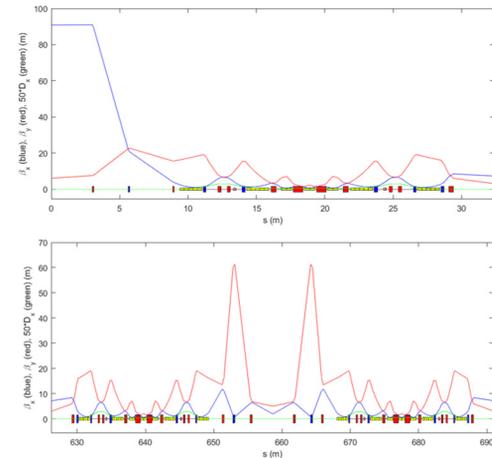


Figure 1: Half injection cell (above) and opposite cell for RF (below) [2].

Table 1: Main Parameters of the Lattice

Parameter	Value
Circumference(m)	1317.2783
Emittance(pm·rad)	60.2248
$Q_x, Q_y$	111.2839/41.1428
$\xi_x, \xi_y$	-2.4469/-2.3962
$\beta_x, \beta_y$ (m)	90.8581/5.9937
Damping time(ms)	16.68/24.97/16.61
$U_0$ (Mev)	2.11

After inserting the high-beta section, we used this design lattice to do the particle tracking (1000 turns) and compared the DA result with the same work in the standard 48-cell lattice. Results are given below (See Figure 2).

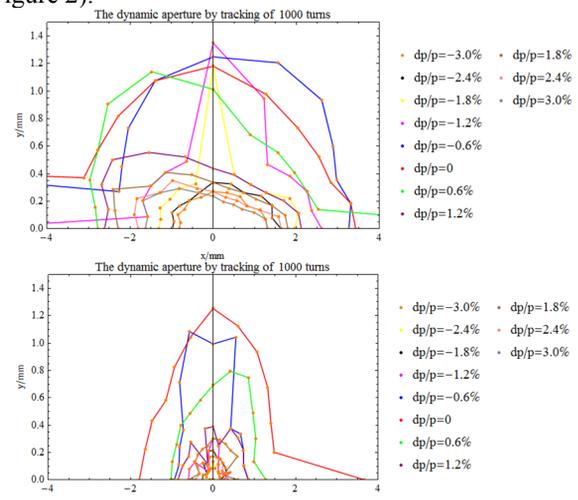


Figure 2: Dynamic aperture for HEPS on the x-y plane at the injection point for different off-energy values.

<sup>†</sup> wujin@ihep.ac.cn

Intuitively, DA and MA decrease a lot, especially the small DA at momentum deviation  $dp/p > 1\%$ .

## RDT THEORY

The RDT correction method is described in [1]. The theory can be introduced within different equivalent theoretical frameworks, in which Lie algebra is common used.

In [3], the 3<sup>rd</sup> RDTs arising from the sextupoles (1<sup>st</sup> order to strength) can be calculated using this kind of formulas [4]:

$$h_{jklmp}^3 \propto \sum_{n=1}^{N_{sext}} (b_3 l)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} \eta_{xn}^p e^{i\{(j-k)\mu_{xn} + (l-m)\mu_{yn}\}} \quad (1)$$

where  $j, k, l, m$  and  $p$  are integers which satisfy  $j+k+l+m+p=3$ ,  $b_3 l$  is the integrated sextuple strength,  $\beta_{x,y}$  and  $\eta_x$  are the usual Twiss parameters and dispersion function and  $\mu_{x,y}$  are the horizontal and vertical phase advance.

The 4<sup>th</sup> order RDTs can be written as follow:

$$h^4 = \sum_i^{N_{oct}} f_i^4 + \frac{1}{2} \sum_{j>i=1}^{N_{sext}} [f_i^3, f_j^3] \quad (2)$$

the contributions to which arising from the octupoles (1<sup>st</sup> order to strength) are similar to  $h_{jklmp}^3$ :

$$h_{jklmp}^4 \propto \sum_{n=1}^{N_{oct}} (b_4 l)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} \eta_{xn}^p e^{i\{(j-k)\mu_{xn} + (l-m)\mu_{yn}\}} \quad (3)$$

where  $j, k, l, m$  and  $p$  are integers which satisfy  $j+k+l+m+p=4$ ,  $b_4 l$  is the integrated octupole strength.

The contributions to RDTs arising from the sextupoles (2<sup>nd</sup> order to strength) can be derived in Mathematica [5]. Explicit formulas can also be consulted in [6].

In this paper, we only consider geometric terms and tune shift with amplitude. Chromaticity related terms can be controlled with other method. The impact of these 3<sup>rd</sup> order and 4<sup>th</sup> order driving terms on the beam dynamics are summarized in Table 2 (16 terms total).

Table 2: RDT Effects on Beam Dynamics

RDT	Effect
$h_{30000}$	$3\nu_x$ resonance
$h_{21000}, h_{10110}$	$\nu_x$ resonance
$h_{10200}$	$\nu_x + 2\nu_y$ resonance
$h_{10020}$	$\nu_x - 2\nu_y$ resonance
$h_{40000}$	$4\nu_x$ resonance
$h_{31000}, h_{20110}$	$2\nu_x$ resonance
$h_{00400}$	$4\nu_x$ resonance
$h_{00310}, h_{11200}$	$2\nu_x$ resonance
$h_{20200}$	$2\nu_x + 2\nu_y$ resonance
$h_{20020}$	$2\nu_x - 2\nu_y$ resonance
$h_{22000}, h_{11110}, h_{00220}$	$\frac{\partial \nu_x}{\partial J_x}, \frac{\partial \nu_x}{\partial J_y}, \frac{\partial \nu_y}{\partial J_y}$

## ANALYSIS

Multi-objective approaches to DA optimization with RDTs had been successfully applied to the NSLS-II [7]. The results demonstrated a correlation between DA and low-order nonlinear driving terms. To give a reasonable explanation for why MA decreased, we try to find some relevance between off-momentum RDTs and off-energy DA.

Code is written in SAD [8], which is easy to use for calculating the RDTs with different momentum deviation. It is well to be reminded that the latest code to calculate the 3<sup>rd</sup> and 4<sup>th</sup> RDTs of the whole ring is efficient that time used is less than 10 seconds, and the results are proved to be consistent with those of PTC [9]. Subsequent work of optimization is also based on SAD.

We compared the on and off-momentum ( $\delta = \pm 0.02$ ) RDTs (16 terms for each  $\delta$ ) in the two versions of lattice, standard 48-cell structure (marked by CEL) and large-beta section involved structure (marked by INJ). Results shows that all on-momentum ( $\delta = 0$ ) RDTs appeared minute difference, just translation of image according to the length. We take an example with 3 terms of tune shift in Figure 3. Not surprisingly, off-momentum ( $\delta = \pm 0.02$ ) RDTs appeared huge differences, especially the terms  $h_{31000}$  and  $h_{00310}$ . We also give the pictures in Figure 4.

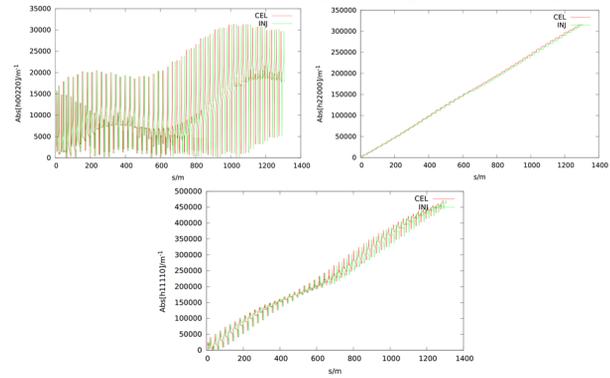


Figure 3:  $h_{22000}, h_{00220}, h_{11110}$  with  $\delta = 0$  in two versions of lattice.

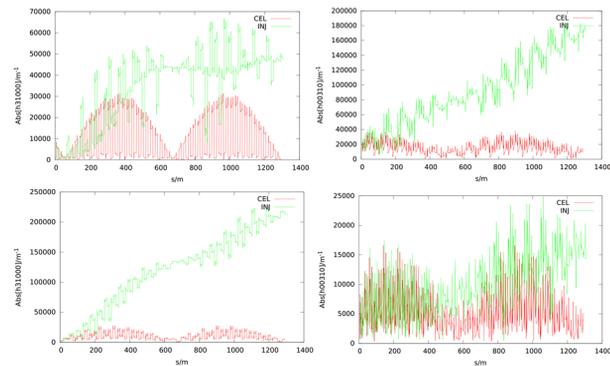


Figure 4:  $h_{31000}$  (left) and  $h_{00310}$  (right) with  $\delta = 0.02$  (up) and  $\delta = -0.02$  (down) in the two versions.

These results inspired us to set suitable objective functions in the subsequent optimization work.

## OPTIMIZATION FOR THE HIGH-BETA SECTION INCLUDED LATTICE

We use Multi-Objective optimization by Differential Evolution (MODE) [10]. In our optimization strategy, we use the geometric sum of RDTs in different energy deviation  $f_1$ , with  $f_2$  related to the chromaticities and  $f_3$  characterizing the DA area of on-momentum and off-momentum particles:

$$\begin{cases} f_1 = \sum_{\delta=0,\pm 0.02} [a \sqrt{\sum (h_{jklmp}^3)^2} + b \sqrt{\sum (h_{jklmp}^4)^2}] \\ f_2 = \xi_x, \xi_y \\ f_3 = \sum_{\delta=0,\pm 0.02} S_{DA}(\delta) \end{cases} \quad (4)$$

where  $a$  and  $b$  are the weight coefficients of 3<sup>rd</sup> and 4<sup>th</sup> RDTs according to the values,  $\xi_x, \xi_y$  contains linear and nonlinear chromaticities.

In our optimization strategy, we only changed the strength of multipoles, while the linear lattice remained with  $\nu_x=111.28$ ,  $\nu_y=41.14$ . All six families of sextupoles are used as parameters added with four families of octupoles.

Actually, suppression of 3<sup>rd</sup> RDTs can also control the 4<sup>th</sup> terms to some extent and save considerable time. After several iterations, we choose a relatively better solution. Here we give the comparison about RDTs (also  $h_{31000}$  and  $h_{00310}$ ), chromaticities and DA after optimization (See Figure5- Figure 7). It is important to mention that other terms are decreased more or less, but not presented here due to limited length.

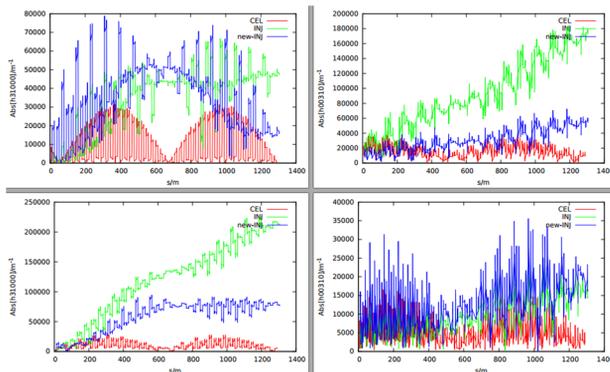


Figure 5: the change of  $h_{31000}$  (left) and  $h_{00310}$  (right) with  $\delta = 0.02$  (up) and  $\delta = -0.02$  (down) after the optimization (blue is new).

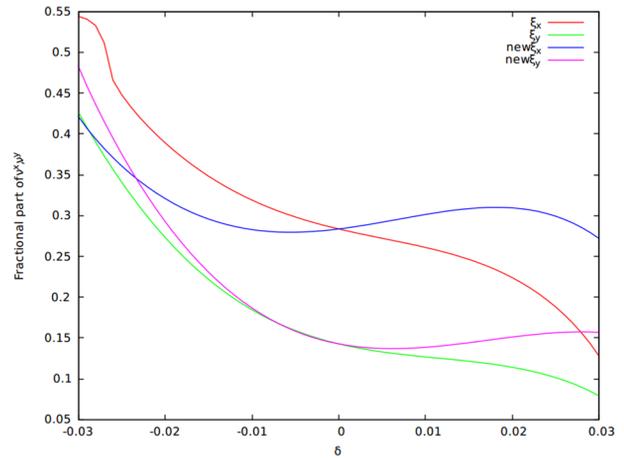


Figure 6: chromaticities turn to be smooth after optimized (blue and pink), while tunes ( $\delta = \pm 0.03$ ) are still deviating much.

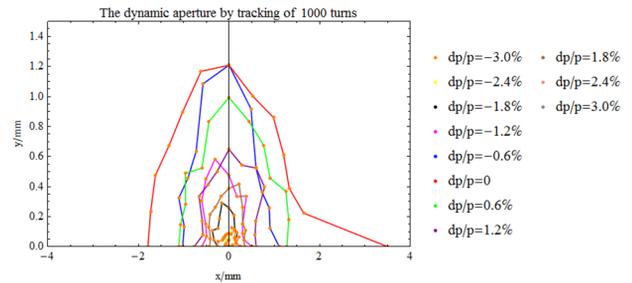


Figure7: compared with Figure 2, area of DA ( $|\delta| \geq 1.2\%$ ) increased, but not so much.

## CONCLUSION

From analysis we could see that the decreasing of DA and MA has some correlation to the off-momentum RDTs. Contrasted with the on-momentum RDTs, off-momentum terms deviated a lot.

The optimization had a little bit better result but it is not so obvious. In consideration of the symmetry of the whole ring, six families of sextupoles were not split more. Next work may use more variables and objective functions may contains more RDTs that would spend more time.

Frequency map analysis (FMA) is also useful to check the tunes and resonances. If needed, linear part of lattice could be changed. As a consequence, more work is necessary.

## ACKNOWLEDGEMENT

The author would like to thank Chenghui Yu for providing the early version of lattice, and Yi Jiao for providing the latest version and discussing some key questions.

## REFERENCES

- [1] J. Bengtsson, I.P.S. Martin, J.H. Rowland and R. Bartolini, “On-line control of the nonlinear dynamics for synchrotrons”, *Phys. Rev. ST Accel. Beams*, vol. 18, p.074002, 2015.
- [2] Y. M. Peng, talk on LERLD 2016 workshop.
- [3] J.Bengtsson, “The sextupole scheme for the Swiss Light Source (SLS): an analytic approach”, SLS Note 9/97, 1997.
- [4] I.P.S. Martin, M. Apollonio, R. Bartolini, “Online suppression of the sextupole resonance driving terms in the diamond storage ring”, in *Proc. IPAC’16*, Busan, Korea, May 2016, paper THPMR001, pp. 3381-3384.
- [5] Mathematica, <http://www.wolfram.com/mathematica/>
- [6] Chun-xi, Wang, “Explicit formulas for 2nd-order driving terms due to sextupoles and chromatic effects of quadrupoles”, Argonne National Laboratory (ANL), USA, ANL/APS/LS-330, April 2012.
- [7] L. Y. Yang, Y. J. Li, W. M. Guo, Samuel Krinsky, “Multiobjective optimization of dynamic aperture”, *Phys. Rev. ST Accel. Beams*, vol. 14, p.054001, 2011.
- [8] SAD, <http://acc-physics.kek.jp/SAD/>
- [9] PTC, <http://madx.web.cern.ch/madx/>
- [10] Y. Zhang, D. Zhou, “Application of differential evolution algorithm in future collider optimization”, in *Proc. IPAC’16*, Busan, Korea, May 2016, paper TUOBA03, pp. 1025-1027.