

RF EXCITATION PARAMETERS IN RESONANT EXTRACTION*

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Abstract

Ion beam resonant slow extraction from synchrotron is extensively used in experimental nuclear and particle physics, material radiation science, and deep-seated malignant tumours radiotherapy. In this paper, the ion motion of resonant extraction under RF excitation are discussed. The expression for sweeping frequency waveforms used in tracking code for resonant extraction are presented.

INTRODUCTION

The transverse excitation or RF-Knockout resonant extraction has been developed in the last twenty years as a popular method in slow beam extraction from synchrotron [1]. It also promotes the application of heavy ion radiotherapy due to the advantages of quick response on beam start and cut-off, and simple operation in controlling beam at therapy terminals [1]. The domestic heavy-ion synchrotron in operation e.g., CSRm and HIMM take this method to realize the slow extraction for experimental research and radiotherapy [2].

The process of RF transverse excitation is described as below. During the storage beam being accelerated to the energy for extraction, the betatron tune of synchrotron is adjusted to be closer to the third order resonance and synchronously introduced sextuple fields help to produce three separatrices in phase space and confine the stored beam inside the stable region that is larger than transversal beam emittance. The RF excitation heats the circulating stored beam at horizontal plane, so that the transverse emittance blows up rapid until some ions escape away the stable region along the separatrices and jump into the gap of electrostatic septum for extraction. To suppress the beam loss along the synchrotron, the electrostatic spectrum is required that further limits the dynamics aperture and deflects lost beam away from close orbit for those that jump into the septum gap. The extraction beam intensity is controlled by the RF excitation parameters. This method features fixed stable area in phase space or unchanged sextuple fields and synchrotron lattice parameters during the RF excitation process at extraction plateau. In additional, a orbit bump system upstream the electron spectrum and longitudinal RF capture are helpful to improve the extraction efficiency.

ION MOTION UNDER RF EXCITATION

The ion motion equation under sextuple field and RF transverse excitation is written as:

$$\begin{aligned} X''(\mu) + X(\mu) + \frac{1}{2}\beta_x^{5/2}k_2X^2(\mu) \\ = \beta_x^{3/2}\frac{\beta c}{B\rho}\sum_{n\geq 0}E_x\delta(\mu - 2\pi Q_x m - \mu_k) \end{aligned} \quad (1)$$

in which μ_k and μ are the Betatron phases of RF exciter and ions respectively, Q_x is the horizontal working point, k_2 is the field strength of sextuple, β_x is Twiss parameter, m is any integer greater than 0, $E_x = E_{x0} \sin(2\pi f_k t)$ represents the transverse excitation strength of RF electric field, δ at the right denotes Dirac function. If we replace the variable μ with revolution turn number n , i.e. $d\mu/dn \approx \Delta\mu/1 = 2\pi Q_x$ then equation (1) is rewritten as

$$\begin{aligned} X''(n) + (2\pi Q_x)^2(X(n) + \frac{1}{2}\beta_x^{5/2}k_2X^2(n)) \\ = (2\pi Q_x E_x)\frac{\beta_x^{3/2}\beta c}{B\rho}\sum_{n,m}\cos(2\pi\frac{f_k}{f_{rev}}n)\delta(n-m) \end{aligned} \quad (2)$$

with f_k is the RF excitation frequency, f_{rev} is the revolution frequency of ion beam in synchrotron. The homogeneous solution of equation (2) can be expressed as

$$X(n) = a \cos(2\pi q_x n + b) \quad (3)$$

where q_x is fraction part of the horizontal working point Q_x , a and b represent any constants. The summation part at the right side of equation (2) is further written as

$$\sum_n \cos(2\pi \frac{f_k}{f_{rev}} n)$$

In comparison with the right side of equation (3), we find that the horizontal excitation works only when the RF frequency satisfies the following relationship

$$\frac{f_k}{f_{rev}} = Integer \pm q_x \quad (4)$$

This is required by applying RF excitation to the circulating ion beam.

The betatron amplitude of 10 ions under influence of sextuple and RF excitation is shown in Fig. 1, in which, sextuple field is applied after 500 revolution turns, and RF excitation starts up at the 1000th turn.

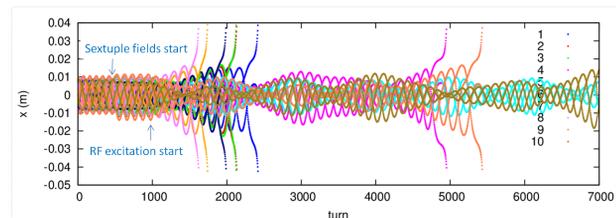


Figure 1: The betatron amplitude under influence of sextuple fields and RF excitation.

Because the betatron tune of ion also depends on momentum spread, betatron amplitude, and the magnetic field deviation and ripple etc., thus the circulated beam has a

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certain tune spread δq_x . According to equation (4), therefore, the extraction of more ions require RF exciter cover a frequency width

$$\Delta f_{ks} \geq f_{rev} \delta q_x \quad (5)$$

Generally, this is realized by white noise band or sweeping frequency back and forth around the resonance point.

RF EXCITATION PARAMETERS

When the RF excitation is applied on the third-order resonance slow extraction, it have to sweep over a certain frequency width for a continuous extraction according to equation (5). In addition, the ion density decreases with extraction due to fixed area of stable region in phase space. For this reason, the uniform extraction needs adjustable amplitude of RF excitation throughout the extraction.

Therefore, uniform and continuous resonance extraction by RF excitation need the following adjustable parameters to the RF system:

- central frequency $f_k = (Integer \pm q_x) f_{rev}$
- sweeping frequency width Δf_{ks}
- sweeping frequency period T_{ks}
- excitation amplitude upon ion beam U_{k0}

Figure 2 shows the kick angle variation by RF excitation in three sweeping cycles when the central frequency is modulated by a sine waveform. The horizontal axis shows the normalized time of three cycles.

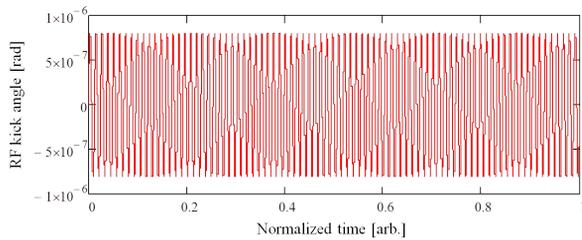


Figure 2: The kick amplitude variation over time under the modulation of sine waveform sweeping frequency.

SWEEP FREQUENCY WAVEFORM

The waveform of rising sawtooth and triangular are typically used for sweeping frequency of RF excitation. To investigate the effect of different sweeping waveform, we list the expressions of kick angle dependence upon revolution turns in cases of sine, triangular, and sawtooth waveforms and implant them into the tracking code. We assume a sweeping bandwidth $\Delta f_{ks} = \pm 0.0005 \Delta f_k$ at the following calculation in this section.

Sweeping in Sine Wave

The sweep kick expression in sine waveform implanted in tracking is expressed as

$$\begin{cases} U_{k1}(N) = U_{k0}(N) \sin(2\pi N \frac{f_k}{f_{rev}} + P_{ks1}(N)) \\ P_{ks1}(N) = \frac{1}{2} \Delta f_{ks} T_{ks} \sin(\frac{2\pi N}{f_{rev} T_{ks}}) \end{cases} \quad (6)$$

in which U_{k1} and U_{k0} denote the kick angle by RF excitation, N is revolution turns, p_{ks1} is a phase modulation term contributed by sweeping sine waveform.

Sweeping in Triangle Waveform

The implanted sweep kick expression in triangle waveform is written as

$$U_{k2}(N) = U_{k0}(N) \sin(2\pi N \frac{f_k}{f_{rev}} + P_{ks2}(N)) \quad (7)$$

in which U_{k2} denote the modulated kick angle by RF excitation, $p_{ks2}(N)$ is the modulated phase term contributed by sweeping triangle waveform.

Figure 3 shows the triangular waveform sweeping process of RF excitation central frequency over 3 sweeping cycle or $3 \cdot T_{ks}$ when the relative central frequency is showed as vertical axis and normalized time of three cycles as the horizontal coordinate.

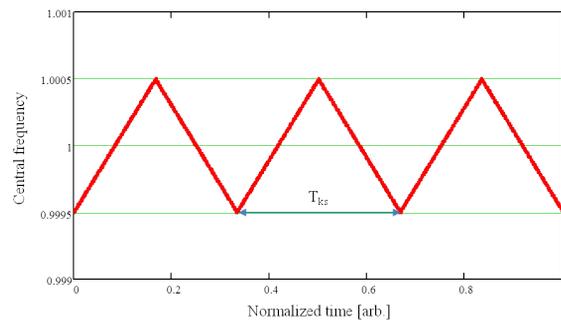


Figure 3: The relative central frequency variation over three sweeping cycles in case of triangle waveform.

The phase modulation parameter by triangle waveform sweeping frequency is expressed as

$$P_{ks2}(N) = \begin{cases} \frac{25}{4} \frac{\Delta f_{ks}}{f_{rev}^2 T_{ks}} (\text{mod}(N, T_{ks} f_{rev}) - \frac{T_{ks} f_{rev}}{4})^2, & \text{if } \text{mod}(N, T_{ks} f_{rev}) - \frac{f_{rev} - T_{ks}}{2} < 0 \\ \frac{25}{4} \frac{\Delta f_{ks}}{f_{rev}^2 T_{ks}} (\text{mod}(N, T_{ks} f_{rev}) - \frac{3T_{ks} f_{rev}}{4})^2, & \text{else} \end{cases} \quad (8)$$

The phase variation by sweeping triangle waveform over three cycles is shown in Fig. 4.

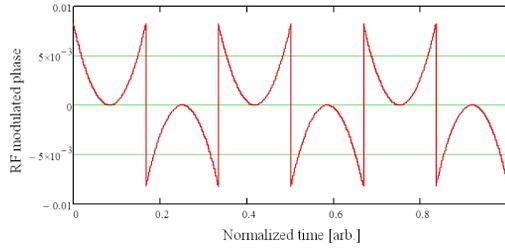


Figure 4: The phase variation over three sweeping cycles in case of triangle waveform.

Sweeping in Rising Sawtooth Wave

The implanted kick expression in sweeping rising sawtooth waveform has the form

$$\begin{cases} U_{k3}(N) = U_{k0}(N) \sin(2\pi N \frac{f_k}{f_{rev}} + P_{ks3}(N)) \\ P_{ks3}(N) = \frac{25}{8} \frac{\Delta f_{ks}}{f_{rev}^2 T_{ks}} (\text{mod}(N, T_{ks} f_{rev}) - \frac{T_{ks} f_{rev}}{2})^2 \end{cases} \quad (9)$$

with U_{k3} denotes the kick angle by RF excitation, $p_{ks3}(N)$ the modulated phase term by sweeping rising sawtooth waveform. Figure 5 shows variation of central frequency and modulated phase in case of rising sawtooth waveform while sweeping over three cycles.

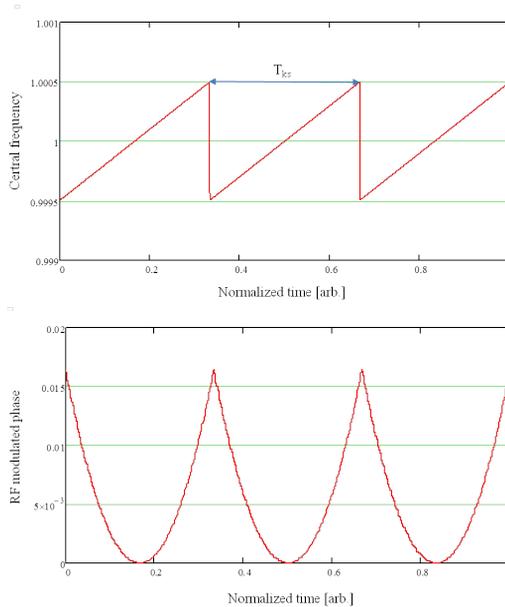


Figure 5: The dependence of central frequency and modulated phase variation on normalized time of three sweeping cycles in case of rising sawtooth waveform.

BEAM UNIFORMITY WITH DUAL FM

In order to improve the time uniformity of extracted spill, the dual modulated RF frequency method is proposed by HIMAC [3]. The dual FM waveforms of triangular and rising sawtooth are implanted into simulation code. Fig. 6 shows the dependence of central frequency and modulated

phase on normalized time in three sweeping cycles in which red and blues represent the two RF modulations with half cycle of phase shift.

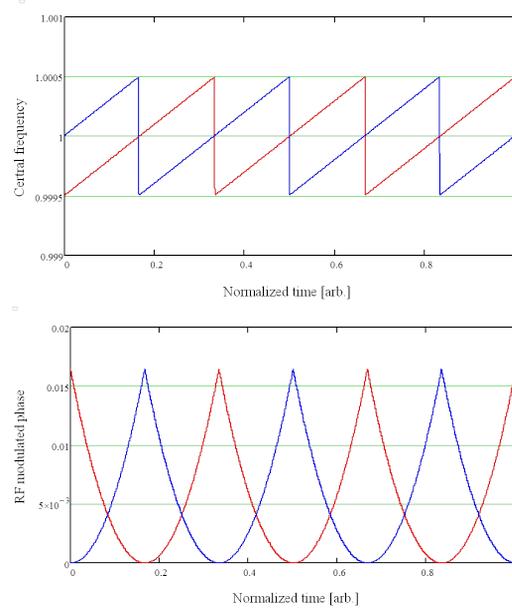


Figure 6: The variation of modulated central frequency and phase over three sweeping cycles in case of dual FM rising sawtooth waveform.

As an example of simulation, Figure 7 shows extracted intensity at the first y axis by histogram in red and the ratio of extracted ions to the total number one at the secondary vertical axis by green line in about 6 sweeping cycles. The left figure in Fig. 7 shows extraction at single RF modulation mode, while the right is the result of dual FM one but with smaller excitation amplitude. The RF sweeps in rising sawtooth waveform and starts at the 500th revolution turn. Simulation shows that the uniformity of extracted ions gets improved with the dual FM methods.

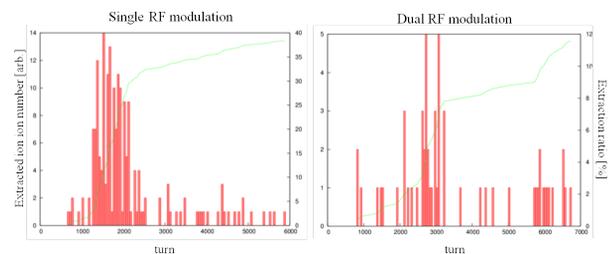


Figure 7: Comparison of extracted ion numbers and extraction ratio between single (left) and dual modulation (right).

CONCLUSION

We discussed the ion motion of resonant extraction under RF excitation. Particularly, the expression of sweeping frequency waveforms are given which can be added into other tracking code for resonant extraction. As an example of simulation, the beam uniformity with dual FM is present.

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