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FRINGE FIELD OVERLAP MODEL FOR QUADRUPOLES

Huan Jia, Institute of Modern Physics, CAS, Lanzhou, China
Michael Plum, Oak Ridge National Laboratory, Oak Ridge, USA

Abstract

In the large aperture-to-length ratio quadrupoles, there will be long fringe field. When putting three of this type quadrupoles next to each other, the field will overlap and change the beam dynamics of "hard-edge" model. By numeric integration, we find that the transfer matrix difference is quite significant at the first part of MEBT of CADS Injector II. By re-exploring the "hard-edge" model, the traditional definition of quadrupole's effective length and effective gradient are found to be just rough approximations and not right in this condition. The finding explains the good prediction of beam dynamics model of MEBT by emittance measurements with different current settings of the triplet quadrupoles, and may be also helpful in explaining some discrepancies in beam lines around the world

INTRODUCTION

At the MEBT of CADS Injector II, quadrupoles have very large bore aperture to bore length ratio, i.e. 54 mm/52 mm for QL80 and 54 mm/ 74 mm for QL100 [1]. At the same time, due to the strong focusing properties at MEBT, the distance between quadrupoles is quite near. The distance between the first three adjacent quadrupoles is 180 mm, which is smaller than the sum of quadrupole bore length and 3 times of apertures.

Thus, the fringe field is quite significant for the quadrupoles at MEBT, and the overlap effect between adjacent quadrupoles is also significant. This effect is analysed by comparison of beam properties after tracking through both the hard-edge model and fieldmap overlap model.

In the past, people try to treat the field overlap problem by multiplying some factors for the three quadrupoles [2]. But our finding is that such a method is not right, because it is not the effective length or effective gradient making effect. It is the total transfer matrix integration making change, and the change is different in horizontal and vertical plane.

The emittance measurement at MEBT of CADS Injector II shows good agreement to multi particle tracking simulation, with 1-D fringe field overlap model [3]. The new finding in the paper explains the agreement between simulation and measurement.

HARD EDGE MODEL

The hard edge model has been prompted for the quadrupoles for many years. The "hard edge" means that quadrupole's gradient is a square-like waveform, with two step-function "edges" in both sides, as shown in Fig. 1.

The transfer matrix of "hard edge" model is that:

$$R_{xx} = \begin{bmatrix} \cos(k\Delta s) & \frac{\sin(k\Delta s)}{k} \\ -k\sin(k\Delta s) & \cos(k\Delta s) \end{bmatrix}. \quad (1)$$

$$R_{yy} = \begin{bmatrix} \cosh(k\Delta s) & \frac{\sinh(k\Delta s)}{k} \\ k\sinh(k\Delta s) & \cosh(k\Delta s) \end{bmatrix}. \quad (2)$$

Where $k = \sqrt{\frac{G}{B\rho}}$ is the focusing strength, $G = \frac{\partial B_y}{\partial x}$ is the quadrupole gradient. $B\rho$ is the magnetic rigidity.

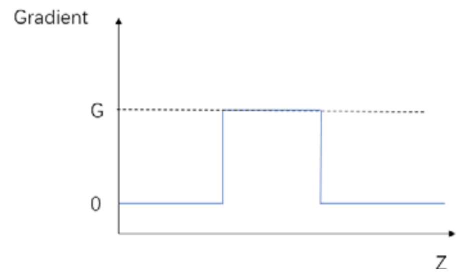

Figure 1: "hard edge" gradient along the quadrupole.

In reality, there is no quadrupole that is "hard edge". The model is right for "thick" quadrupoles, which means that quadrupole has small aperture-to-length ratio. But for "thin" quadrupoles, which means that quadrupole has large aperture-to-length ratio, the model is not right and need to be re-investigated.

For example, because of the large aperture-to-length ratio, the QL80s in MEBT of CADS Injector II can be regard as "thin" quadrupoles, as shown by field gradient simulation and measurements in Fig. 2.

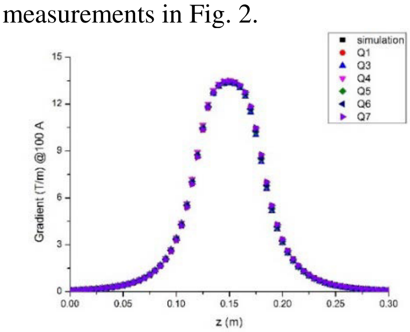

Figure 2: Simulated (black) and measured gradient along axis for six QL80 at MEBT of CADS Injector II [1].

If we treat the quadrupole with "hard edge" approximation, the effective length and effective gradient will be

$$L_{eff} = \int_0^{300} \frac{G*dL}{G_{z=150\,mm}} = 80\ mm \quad (3)$$

$$G_{eff} = G_{z=150\,mm} = 13.5\ T/m \quad (4)$$

FRINGE FIELD MODEL

For the same quadrupole field distribution, we integrate the transfer matrix by sliced pieces M_i, where each M_i is treated as "hard edge" quadrupole, then,

$$M = \prod_{i=1}^{n} M_i \quad (5)$$

Here we define the new effective length as l and effective gradient as k, thus, the new transfer matrix is [4],

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$$M=\begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\varphi & \frac{1}{k}\sin\varphi \\ -k\sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \quad (6)$$

where $\varphi = kl$, $\lambda = m - l/2$, $f = \frac{1}{Kl}$ and $K = k^2$.

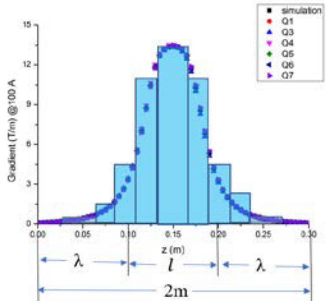

Figure 3: New definition of the effective length of quadrupole based on the numerical gradient integration.

By numeric integration of the field distribution, as shown in Fig. 3, the M can be achieved. After solving Equation (6), the new l and k will be got.

For the focusing of the quadrupole, the Equation is [5]

$$\begin{aligned} C_f - \tfrac{1}{2}LC'_f &= \cos\varphi_f + \tfrac{1}{2}\varphi_f \sin\varphi_f \\ C'_f \ell_f &= -\varphi_f \sin\varphi_f . \end{aligned} \quad (7)$$

where $k_f = \varphi_f / l_f$.

For the defocusing of the quadrupole, the Equation is

$$\begin{aligned} C_d - \tfrac{1}{2}LC'_d &= \cosh\varphi_d - \tfrac{1}{2}\varphi_d \sinh\varphi_d \\ C'_d \ell_d &= -\varphi_d \sinh\varphi_d . \end{aligned} \quad (8)$$

where $k_d = \varphi_d / l_d$.

From Equation (1) and Equation (2), for the same quadrupole, the effective length and effective gradient are different in focusing and defocusing plane. We investigate this effect by comparing the quadrupole parameters with different magnet current, as shown in Fig. 4.

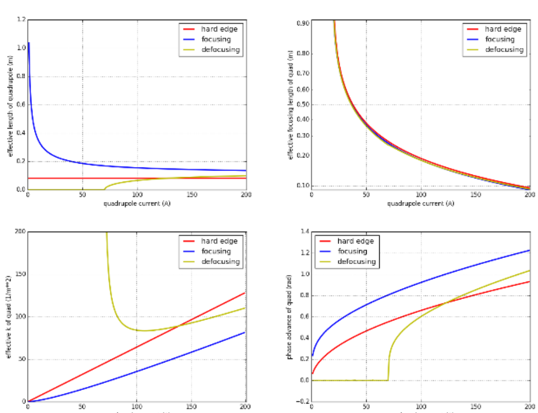

Figure 4: Effective length (left up), effective focusing length (right up), effective focusing strength (left down) and effective phase advance (right down) in focusing plane (blue), defocusing plane (green) and hard edge model (red) of the quadrupole with different quadrupole magnet currents.

Although the focusing length is same in focusing plane, defocusing plane and hard edge model, the effective length, effective gradient and effective phase advance are different.

Comparing the transfer matrix of fieldmap model to hard edge model, the difference will be

$$\Delta x_{ij} = \frac{(\tilde{x}_{ij} - x_{ij})}{x_{ij}} \qquad \Delta y_{ij} = \frac{(\tilde{y}_{ij} - y_{ij})}{y_{ij}} \quad (9)$$

From Fig. 5, we can see that when the magnet current is around 130 A, the difference is quite big. The reason is that in this region the transfer matrix elements are around zero, as shown in Fig. 6 and Fig. 7.

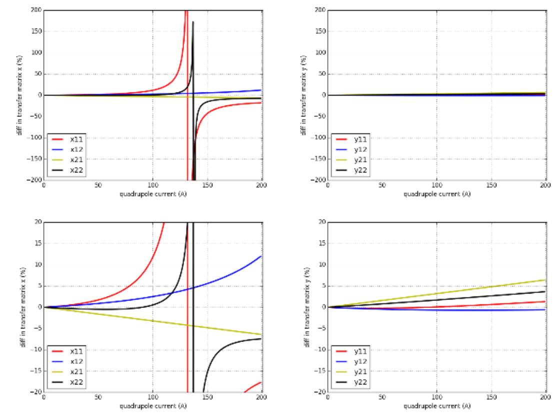

Figure 5: Transfer matrix elements difference with focusing in x plane (left up) and y plane (right up), with defocusing in x plane (left down) and y plane (right down).

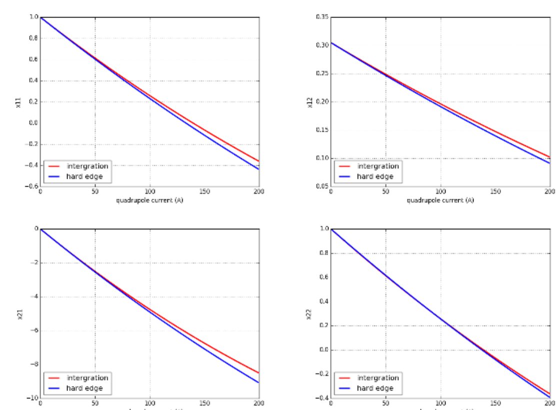

Figure 6: Transfer matrix elements of Q1 fringe field model (red) and hard edge model (blue) in x plane.

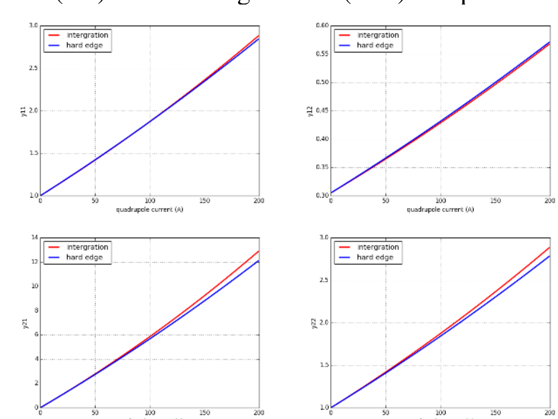

Figure 7: Transfer matrix elements of Q1 fringe field model (red) and hard edge model (blue) in y plane.

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FRIGE FIELD OVERLAP MODEL

For the first three adjacent quadrupoles of the MEBT, we add them together with current ratio of 1:-1:1, and normalize current to 1 A. The field is overlapped, as shown in the new 1D fieldmap distribution in Fig. 8. The parameters of quadrupoles are listed in Table 1.

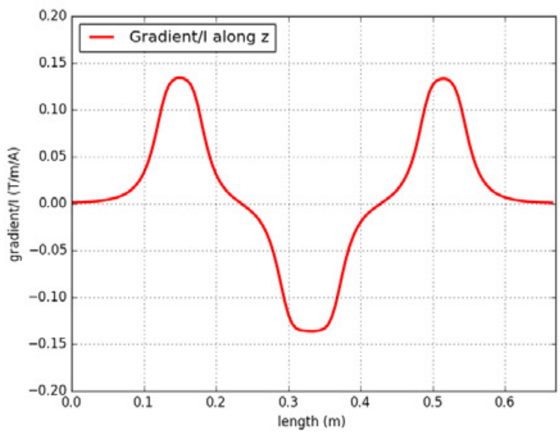

Figure 8: Field overlap distribution of Q1, Q2 and Q3.

Table 1: The Three Adjacent Quadrupole Parameters

Quadrupole	Aperture (mm)	Iron length (mm)	Hard edge effective length (mm)
Q1	54	52	80
Q2	54	74	100
Q3	54	52	80

The difference of the transfer matrix elements between hard edge model and fringe field overlap model is shown in Fig. 9. In y plane, the transfer matrix is significant different to the hard edge model, as shown in Fig. 10 and Fig. 11, while in x plane the difference is not very big. The difference grows as the magnet current increase. Thus, it is impossible to find "multiplying factors" for the three quadrupoles to get good approximation in both x and y plane.

The larger difference in y plane than in x plane is explained that beam envelope is bigger in y plane when the Q1 is focusing in x plane [1]. Beam will experience more fringe field effect with larger envelope.

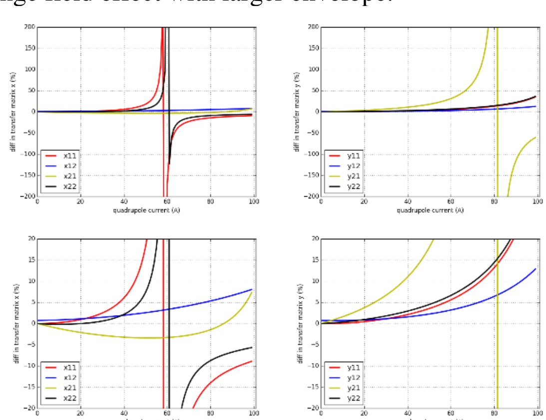

Figure 9: Transfer matrix elements difference of the first three quadrupoles with Q1 focusing in x plane (left up) and y plane (right up), with Q1 defocusing in x plane (left down) and y plane (right down).

By using the fringe field overlap model in beam tracking simulation, we have got good agreement with emittance measurement [3]. If we use the hard edge model, the emittance will have large discrepancies.

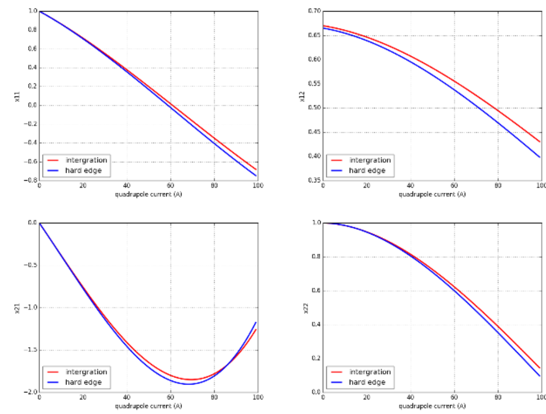

Figure 10: Transfer matrix elements of the three quadrupoles' fringe field overlap model (red) and hard edge model (blue) in x plane.

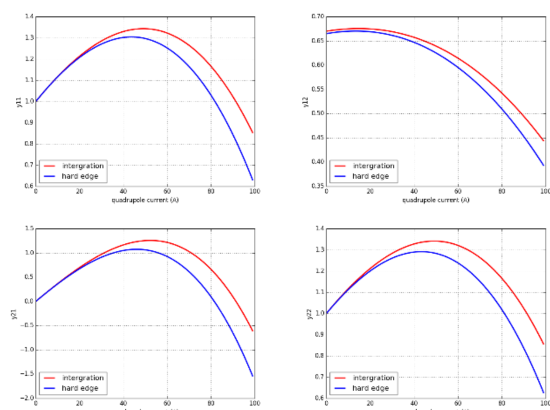

Figure 11: Transfer matrix elements of the three quadrupoles' fringe field overlap model (red) and hard edge model (blue) in y plane.

CONCLUSION

The hard edge model is not "right" considering the fieldmap distribution in "thin" quadrupoles.

Tracking in Field map shows that when the quadrupole current increase, there will be larger difference between the hard edge model and the fieldmap model

Overlap of the fieldmap enlarges the difference between the hard edge model and fieldmap model by changing the overall field distribution of nearby quadrupoles.

In the future design and tuning of beam lines with "thin" and adjacent quadrupoles, the real fieldmap distribution and their overlap should be considered directly in beam dynamics.

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