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USING A BESSEL LIGHT BEAM AS AN ULTRA-SHORT PERIOD HELICAL UNDULATOR

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Abstract

An undulator is a critical component to produce synchrotron radiation and free electron laser. When a Bessel light beam carrying the orbit angular momentum co-propagates with an electron beam, a net transverse deflection force will be subjected to the latter one. As a result of dephasing effect, the deflection force will oscillate and act as an undulator. For such a laser based undulator, the period length can reach sub-millimeter level, which will greatly reduce the electron energy for the required X-ray production.

INTRODUCTION

A magnetostatic undulator is in periodic structures of dipole magnets [1, 2]. The static magnetic field of the undulator is perpendicular to the electron beam trajectory, and periodically changes its directions, which causes an electron beam bunch to follow an undulating trajectory, hence the energy radiations. The radiation brightness from an undulator at the resonance wavelength is N^2 times higher than that from a single bending magnet, where N is total period number of the undulator.

The radiation wavelength can be calculated by Eqs. (1) and (2) [2, 3]:

$$\lambda_{\mathrm{rad}} = \frac{\lambda_{\mathrm{u}}}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right), \quad (1)$$

$$K = \frac{eB\lambda_{\mathrm{u}}}{2\pi m_e c} = 0.934B(T)\,\lambda_{\mathrm{u}}(\mathrm{cm}), \quad (2)$$

where, λ_u is period length of the undulator; B is peak magnetic field of the undulator; λ_{rad} is the radiation wavelength; θ is the radiation angle; γ, e and m_e are the Lorentz factor, charge and rest mass of electron, respectively; and c is the speed of light.

An important direction of the undualtor improvement is to decrease the period length. The shorter is the period length of undulator, the lower is the electron energy required for a desired X-ray, hence a great reduction of the facility scale and cost.

For a practical configuration, K value of an undulator should be in order of 1. To this end, the shorter the period length, the higher the peak magnetic field should be. This prevents the undulator period from being ultra-short. In-vacuum undulator was developed for short period approach [4, 5]. In this type of undulator, the permanent magnets are installed in a vacuum tank, thus the undulator pole gap can be much smaller, the peak field can be increased, and the period length can be reduced.

The discovery of increasing remanent field and coercivity of the permanent magnet at low temperatures provides the possibility of building cryogenic permanent magnet undulator (CPMU) [6, 7] with a shorter period, a little bit, though, at the cost of an additional liquid nitrogen cryogenic system. The superconducting technology helps to build undulators of even shorter periods [8, 9]. However, even for the state-of-the-arts technology, the period length of a magnetostatic undulator is beyond 1 mm [10]. It is possible for an RF undulator [11, 12] to achieve period length of shorter than 1 mm. However, for lacking of high power THz source, it is still hard to realize a millimeter-period undulator.

Optical undulator has been proposed for compact FEL purpose for years. When an intense and long enough laser pulse counter-propagates with the electron beam, the laser may act as an undulator [13–16]. The period length of optical undulator is in micron range. This requires that the electron beam is orders of magnitude brighter than the existing electron source for FEL generation. Laser plasma undulator [16, 17] was just proposed to build sub-millimeter period undulator, but the fact that the electrons do not pass through free space may prevent its use in storage rings.

In the following sections we will show that a Bessel light beam can be used for undulating relativistic electrons when it is co-propagating as shown in Figure 1. With the advent of high power laser, it is possible to achieve K value of around 1 for Bessel light beam undulator (BLU) with period length approaching sub-millimeter.

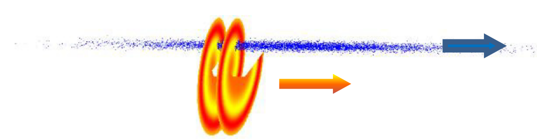

Figure 1: Sketch of Bessel light beam (red-orange vortex) interaction with electron beam (blue dot).

TRANSVERSE FORCE OF BESSEL LIGHT BEAM TO THE RELATIVISTIC ELECTRON

For a monochromatic Bessel light beam, in the dimensionless system *(c=1)*, the electric and magnetic fields of the wave in the Cartesian coordinate for paraxial approximation can be expressed as [18]:

$$\begin{pmatrix}\mathcal{E}_x\\ \mathcal{E}_y\\ \mathcal{E}_z\end{pmatrix} = \begin{pmatrix}\kappa_- C_{M+1} + \kappa_+ C_{M-1}\\ \kappa_- S_{M+1} - \kappa_+ S_{M-1}\\ 2S_M\end{pmatrix}, \quad (3\text{-a})$$

$$\begin{pmatrix}\mathcal{B}_x\\ \mathcal{B}_y\\ \mathcal{B}_z\end{pmatrix} = \begin{pmatrix}\kappa_- S_{M+1} + \kappa_+ S_{M-1}\\ -\kappa_- C_{M+1} + \kappa_+ C_{M-1}\\ 2C_M\end{pmatrix}, \quad (3\text{-b})$$

$$C_M = \cos(k_\parallel z - \chi\omega t + M\phi)\,J_M(k_\perp\rho), \quad (4\text{-a})$$

$$S_M = \sin(k_\parallel z - \chi\omega t + M\phi)\,J_M(k_\perp\rho), \quad (4\text{-b})$$

where, z is the light propagation direction, $\rho = \sqrt{x^2+y^2}$ is the transverse distance to the z axis, ϕ is the azimuthal phase to the z axis, M is the order of Bessel beam, and

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$$\boldsymbol{\kappa}_{\pm} = \frac{k \pm k_{\parallel}}{k_{\perp}}. \quad (5)$$

In this paper, we only consider the forward propagated wave with $\boldsymbol{\chi} = \mathbf{1}$. The wavenumbers have the following relationships,

$$\boldsymbol{k} = \sqrt{\boldsymbol{k}_{\parallel}{}^2 + \boldsymbol{k}_{\perp}{}^2}, \quad (6)$$

$$\boldsymbol{\omega} = \boldsymbol{c}\,\boldsymbol{k}, \quad (7)$$

The Lorentz force in the horizontal (x) and vertical (y) planes for a co-propagated relativistic electron ($v \cong$ c) can be derived by:

$$F_x(z,t,\rho,\phi) = -e(\boldsymbol{\mathcal{E}_x} - c\,\boldsymbol{\mathcal{B}_y}) = -2e\boldsymbol{\kappa}_{-}\boldsymbol{C}_{\boldsymbol{M+1}}(z,t,\rho,\phi), \quad (8\text{-a})$$

$$F_y(z,t,\rho,\phi) = -e(\boldsymbol{\mathcal{E}_y} + c\,\boldsymbol{\mathcal{B}_x}) = -2e\boldsymbol{\kappa}_{-}\boldsymbol{S}_{\boldsymbol{M+1}}(z,t,\rho,\phi). \quad (8\text{-b})$$

Forces in x and y planes expressed in Eq. (8) oscillate in a sin waveform with phase difference of π/2, forming a force like a helical undulator.

From Eqs. (4), (6) and (7), it can be found that the phase velocity of the Bessel beam is faster than the light speed. As the relativistic electron is in velocity of $v \cong c$, the phase slip of a relativistic electron to the light produces upconversion undulate period of the laser wavelength:

$$\lambda_{\text{u}} = \frac{1}{1-\boldsymbol{k}_{\parallel}/\boldsymbol{k}}\lambda_{\text{laser}}. \quad (9)$$

The key factor for a Bessel light to undulate the relativistic electron is that its phase velocity is faster than the light speed. The magnetic field can't be cancelled completely by the electric field when it is seen by a co-propagated relativistic electron, leaving a net periodic oscillated deflecting force. As EM field of Bessel light rotates in the transverse plane, by neglecting energy absorption from the laser to the electrons, BLU only produces circular polarized radiations.

LASER POWER REQUIREMENT

An ideal Bessel light beam has an infinitely extended transverse profile and carries infinite power, but Bessel light beams in practice cannot be ideal, with finite radius and power. A non-ideal Bessel beam in radius R has diffraction distance [19]:

$$L = R\frac{k}{k_{\perp}}, \quad (10)$$

The Bessel light beam holds N rings within radius R will be diffracted layer by layer until the innermost ring diffracts away at the end of diffraction distance.

For $k_{\perp}/k \ll 1$, the laser power is mainly determined by the EM term in right hand of Eq.(3) with $\boldsymbol{\kappa}_{+}$ coefficient. The power can be integrated approximate as:

$$P \approx 2\boldsymbol{\kappa}_{+}{}^2 \int_0^{2\pi} d\phi \int_0^R \rho J_{M-1}(\boldsymbol{k}_{\perp}\rho)^2\, d\rho \quad (11)$$

Eq. (11) is in normalized form as it is directly derived from Eq. (3). For power calculation, a factor $cB_0^2/2\mu_0$ should be multiplied, where B_0 is the normalized magnetic field for Eq. (3), μ_0 is the permeability of vacuum.

Take a CO_2 laser in wavelength of 10.6 μm for example, to produce an undulator of 47 periods, λ_u=0.53 mm, and K_x=0.5, a laser power of 9.3 TW is needed. The laser beam parameters are listed in Table 1.

Table 1: BLU Parameters

Laser wavelength /μm	10.6
$\boldsymbol{k}_{\perp}/\boldsymbol{k}$	0.199
$\boldsymbol{k}_{\parallel}/\boldsymbol{k}$	0.98
M	2
λ_{u} / mm	0.53
R / mm	5
L / mm	25
Periods	47
Laser power / TW	9.3
Kx(K)	0.5 (0.707)

In Ref. [20] a laser carries OAM at TW level demonstrated the feasibility of building a BLU for X-ray production.

BEAM TRACKING AND PHOTON FLUX

Here we take Shanghai soft X-ray linac beam [21] as an example for tracking, whose electron energy is 840 MeV. With BLU parameters listed in Table 1, 1.2 Å hard X-ray will be produced.

To maximize the deflection force, it is nature to align the electron beam at the place where $\boldsymbol{J}_{\boldsymbol{M+1}}(\boldsymbol{k}_{\perp}\boldsymbol{\rho})$ gets its peak value, and where the deflecting force has a small range of flat top in amplitude in the radial direction, which allows a relatively large part of an electron beam to radiate monochromatically.

In the tracking, both transverse and longitudinal parts of the EM fields are counted while radiation effect is neglected. The tracking is mainly focused on finding out the electron trajectory which can be used for undulator phase error evaluation.

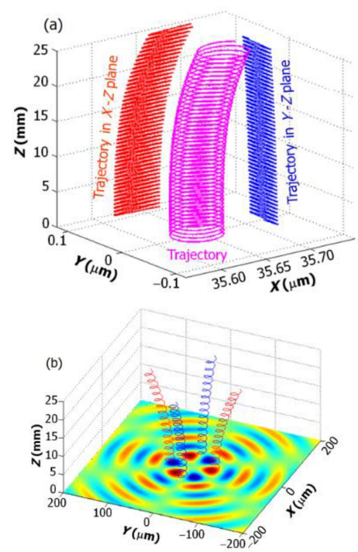

Figure 2: Tracking result of electron trajectory (a); and the defocus effect (b), with the zoomed beam trajectory for a clear view, and with the electrical field rotating anticlockwise in the bottom.

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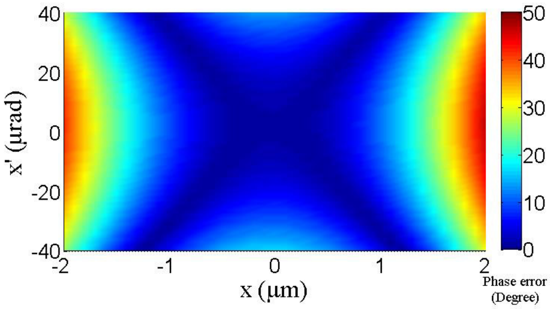

Figure 3: Radiation phase errors for electrons with different initial x-x' conditions.

In the tracking, it is found that the longitudinal electrical field causes an energy modulation of the electron beam, resulting in a deflecting effect in the radial direction. The defecting force is in axial symmetry, so the focus (for left handed helix) or defocus (for right handed helix) effects are in transverse directions as shown in Fig.2.

To evaluate how many radiations from the electrons in transverse phase space are monochromatic, the radiation phase error is calculated from electron trajectory. As shown in Figure 3, the BLU is sensitive to the transverse place where electron passes through, however it is tolerant to slope angle of the electron. It also shows that a monochromatic radiation is produced with the transverse acceptance of just around 2 microns, which requires a novel focus lattice design for the electron beam at the end of the linac [22].

DISCUSSIONS

The BLU has unique properties that other types of undulators do not have. It only undulates a part of the electrons when the laser pulse is shorter than the electron beam. Because the Bessel light beam is at the speed of light, the interaction is limited at the place where electrons and the laser overlap in the longitudinal coordinate. This property can be used for short pulse (femtosecond) X-ray production with picosecond electron beam bunch.

The undulated electrons may shift backward in longitudinal coordinate, as the interaction is located in a part of longitudinal area of the electron beam, which may cause an electron beam density modulation.

From the preliminary tracking results, it can be found that BLU may radially focus (or defocus) electrons, which is different to quadrupoles and solenoids. More research efforts are needed for applications of the effects.

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