General aspects of superconductivity

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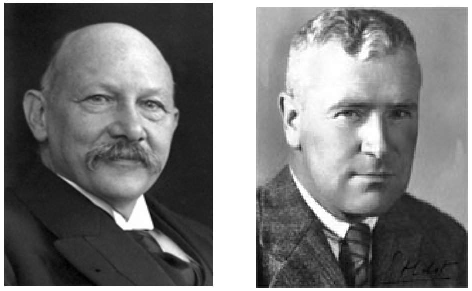
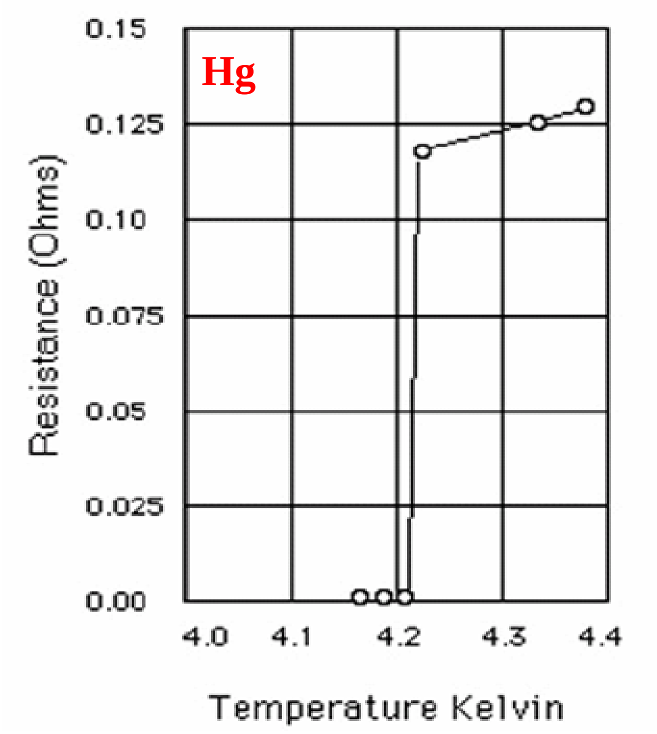
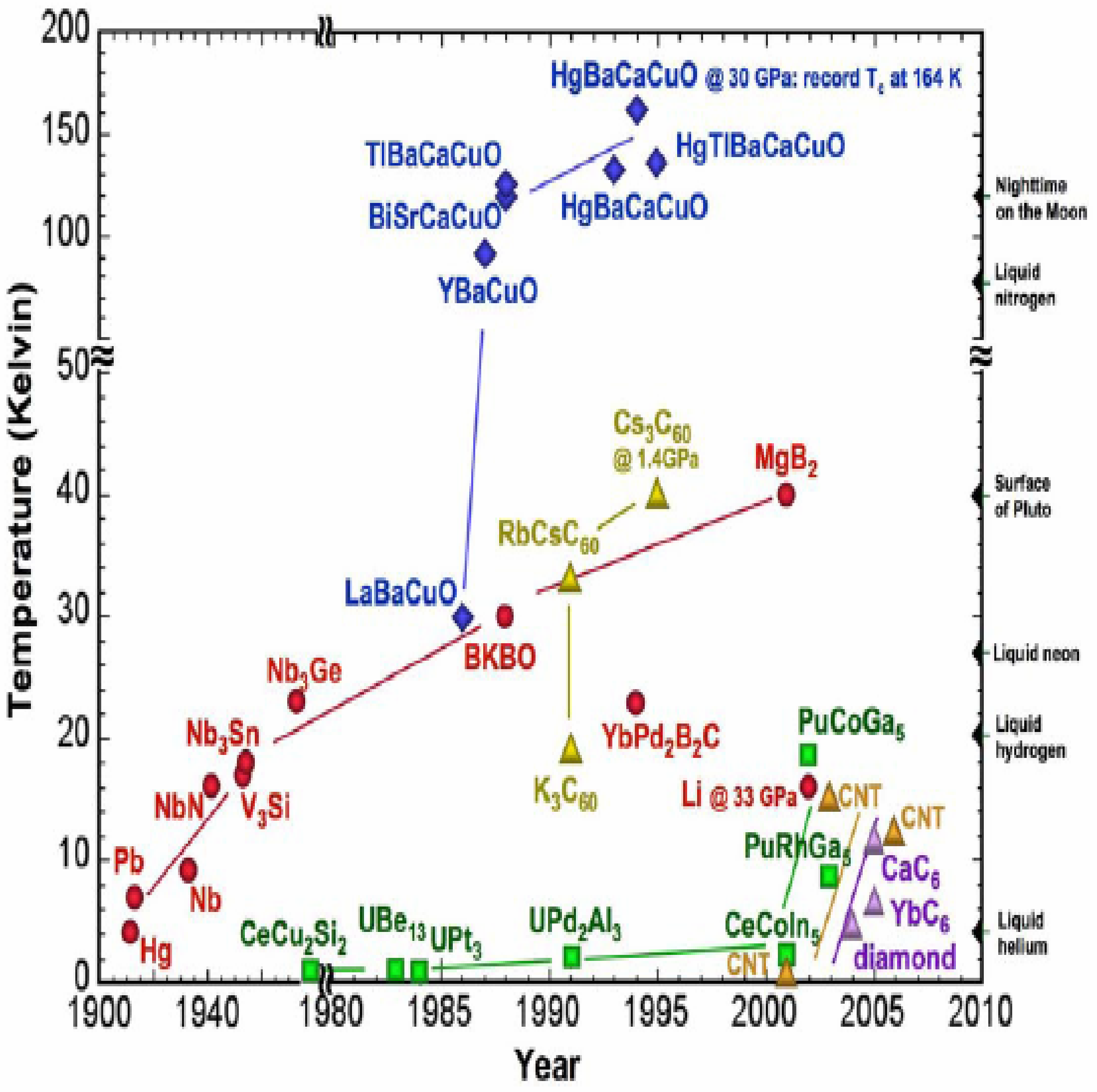
History

Discovered in 1911 by Heike Kamerlingh Onnes and Giles Holst after Onnes was able to liquify helium in 1908. Nobel prize in 1913

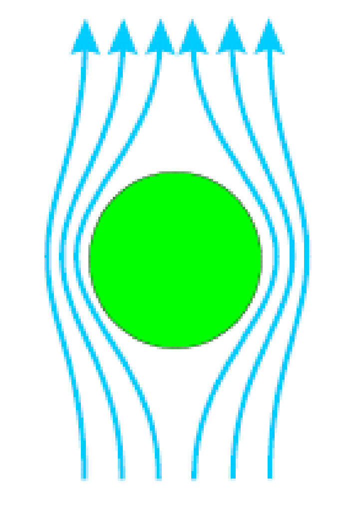






Meissner effect and critical field

Complete magnetic shielding by circulating surface supercurrents

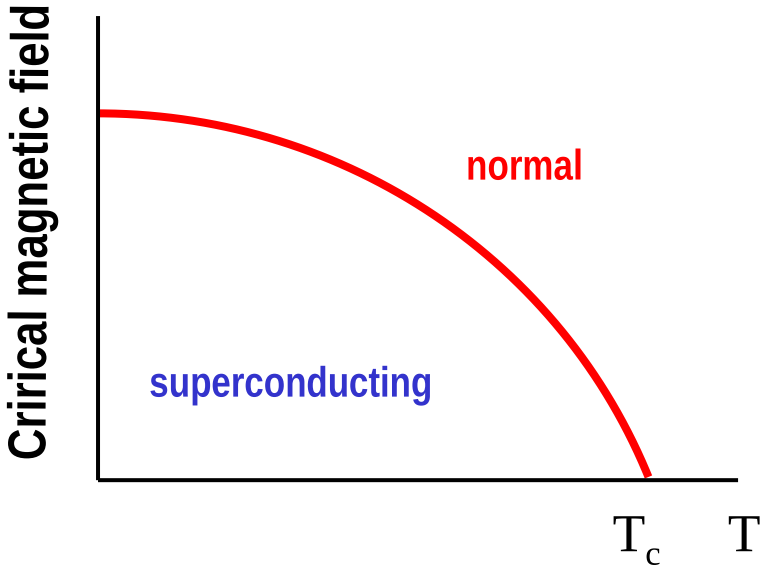


Zero resistivity at $T < T_c$ results from a phase transition to the new superconducting state

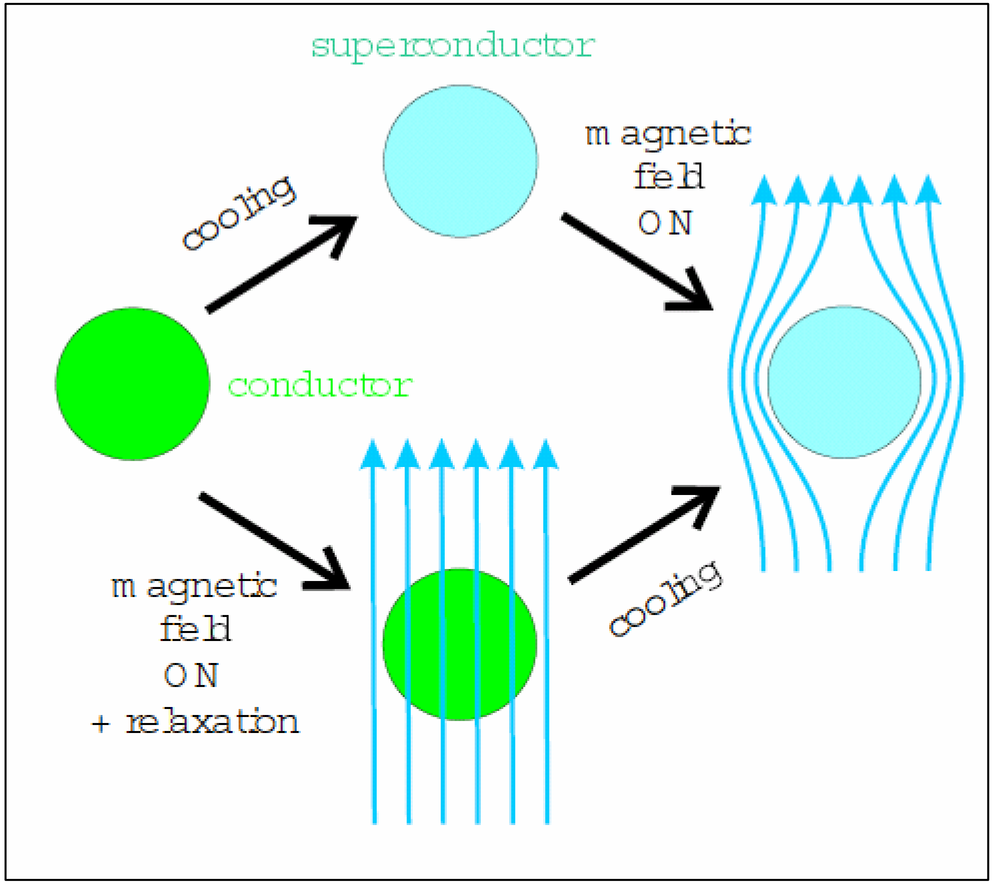
Key experimental facts

1. Magnetic field is expelled from a superconductor (Meissner effect, 1933).
2. Superconductivity is destroyed by magnetic field $H > H_c(T)$
3. Thermodynamic critical magnetic field $H_c(T)$.
4. Empirical formula:

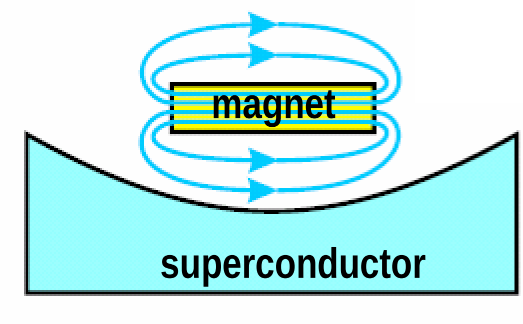
$$H_c(T) = H_c(0)[1 - (T/T_c)^2]$$



The key difference between superconductors and perfect normal conductors



Normal metal: not a phase transition but the infinite relaxation time constant $\tau = L/R$ (ideal skin effect)

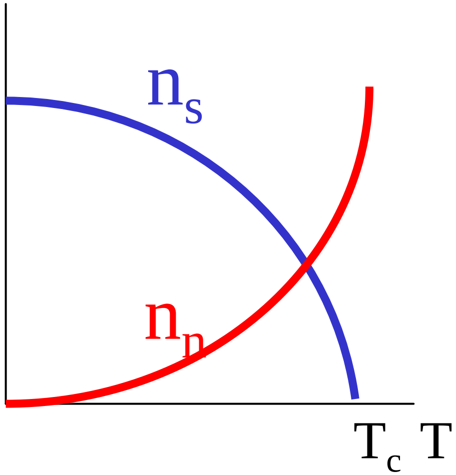


Levitation of a magnet over a superconductor

- a superconductor expels dc magnetic flux
- behavior of good normal metals and superconductors is similar in ac magnetic fields

London equations (1935)

- Two-fluid model: coexisting SC and N "liquids" with the densities $n_s(T) + n_n(T) = n$.
- Electric field E accelerates only the SC component, the N component is short circuited.
- Second Newton law for the SC component: $m d\mathbf{v}_s/dt = e\mathbf{E}$ yields the first London equation:



$$d\mathbf{J}_s/dt = (e^2 n_s/m)\mathbf{E} \quad \longleftrightarrow \quad \mathbf{J} = \sigma\mathbf{E}$$

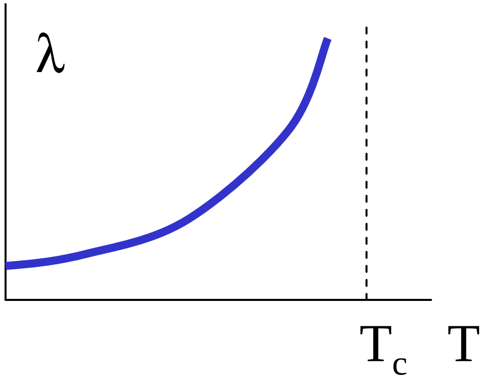
(ballistic electron flow in SC)

(viscous electron flow in metals)

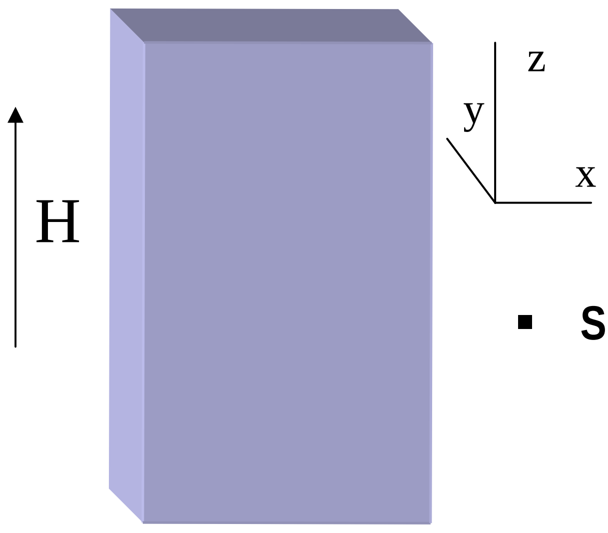
- Using the Maxwell equations, $\nabla\times\mathbf{E} = -\mu_0\partial_t\mathbf{H}$ and $\nabla\times\mathbf{H} = \mathbf{J}_s$ we obtain the second London equation:

$$\lambda^2\nabla\mathbf{H} - \mathbf{H} = 0$$

- London penetration depth: $\boxed{\lambda = \left(\frac{m}{e^2 n_s(T)\mu_0}\right)^{1/2}}$

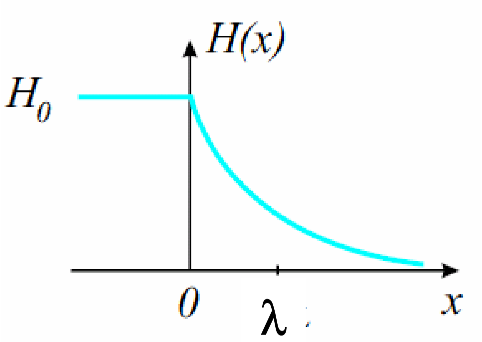


London equation explains the Meissner effect



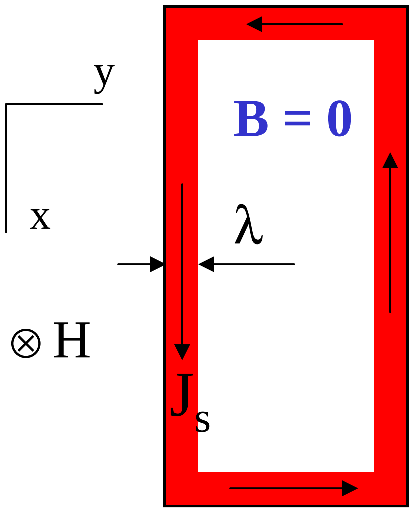
- magnetic field penetration into a slab:

$$\lambda^2 H'' - H = 0$$



- Screening surface current density $J_s(y)$:

$$H(y) = H_0 e^{-y/\lambda}, \qquad J_s(y) = \frac{H_0}{\lambda} e^{-y/\lambda}$$



- Supercurrents completely screen the external field H_0
- Meissner effect: no magnetic induction B in the bulk.
- Surface current density cannot exceed the depairing current density J_d:

$$J_d = \frac{H_c(T)}{\lambda(T)} \cong J_0\left(1-\frac{T^2}{T_c^2}\right)^{3/2}$$

Superconducting current, order parameter and phase coherence

- All superconducting electrons are paired in a coherent quantum state described by the macroscopic complex wave function $\Psi = (n_s/2)^{1/2}\exp(i\theta)$
- The same phase θ for all superconducting electrons.
- Phase gradient $\nabla\theta$ results in a superconducting current $J = -(e\hbar n_s/m)\nabla\theta$!

- Phase gradient in a magnetic field (see Feynman's lectures, vol. 2)

$$\nabla\theta \to \nabla\theta + \frac{q}{\hbar}\vec{A}, \qquad \boxed{q = 2e} \longleftarrow$$

Cooper pairs!

- Superconducting current density

$$\boxed{\vec{J}_s = -\frac{1}{\lambda^2\mu_0}\left(\frac{\phi_0}{2\pi}\nabla\theta + \vec{A}\right), \qquad \phi_0 = \frac{\pi\hbar}{|e|}} \longleftarrow$$

Magnetic flux quantum

Diamagnetic minus

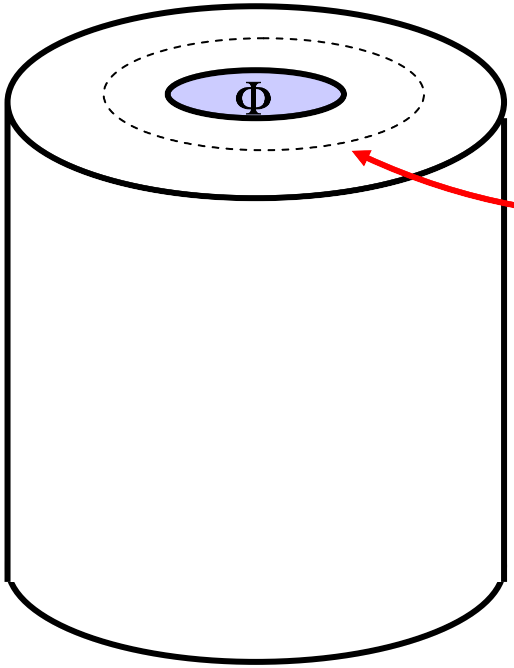
What is the phase coherence?

Incoherent (normal) crowd:
each electron for itself

Phase-coherent (superconducting) condensate
of electrons

Magnetic flux quantization

What magnetic flux $\Phi = \int BdS$ can be trapped in a hollow cylinder?



Integrate J_s along the contour l in the bulk, where J_s =0:

$$\frac{1}{\lambda^2\mu_0}\oint\left(\frac{\phi_0}{2\pi}\nabla\theta+\vec{A}\right)d\vec{l}=0$$

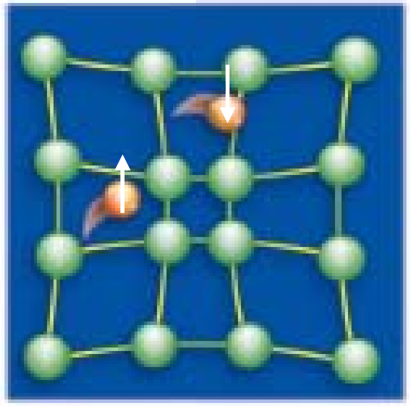
Use the Gauss theorem: $\Phi = \int\nabla\times AdS = \oint Adl$, and the fact that the wave function $\Psi = (n_s/2)^{1/2}\exp(i\theta)$ must be single valued, $\oint\nabla\theta dl = \pm 2\pi n$, n = 0, ±1, ±2 … . Hence,

$$\boxed{\Phi = \pm n\phi_0, \qquad \phi_0 = \pi\hbar/|e| = 2.07\times10^{-15}\,V\,s}$$

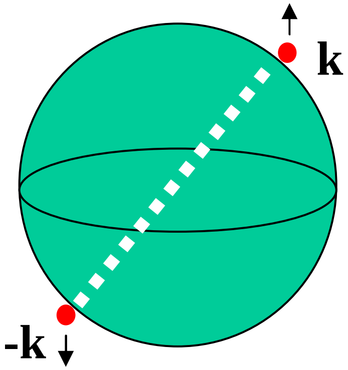
- Quantized flux (London, 1950; Deaver and Fairbank, 1961) is a trademark of magnetic behavior of superconductors (magnetic vortices, SQUID interferometers, etc.)

Cooper pairs and the BCS theory of superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957).
Nobel prize in 1972



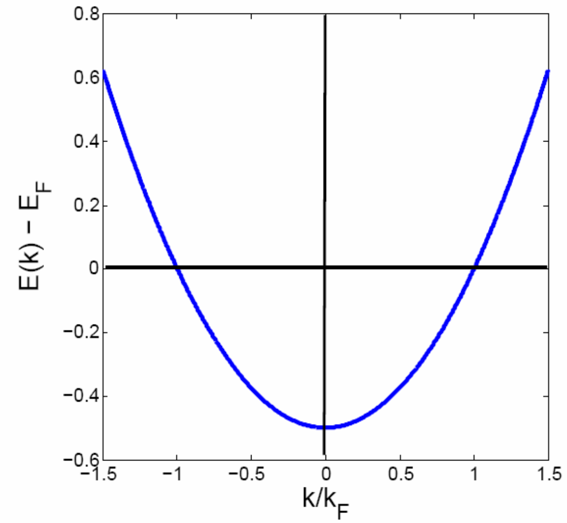
- Attraction between electrons with antiparallel momenta k and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair



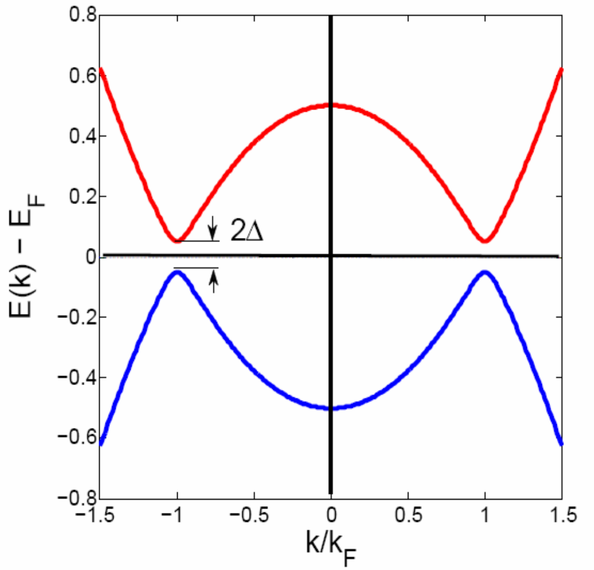
Cooper pair on the Fermi surface

The strong overlap of many Cooper pairs results in the macroscopic phase coherence

BCS theory (cont)



Normal state for T > Tc



Superconducting state for T < Tc

- Superconducting gap Δ on the Fermi surface
- Critical temperature: $T_c \approx 1.13 T_D \exp(-1/\gamma)$, $\gamma \approx VN_F = 0.1\text{-}1$ is a dimensionless coupling constant between electrons and phonons

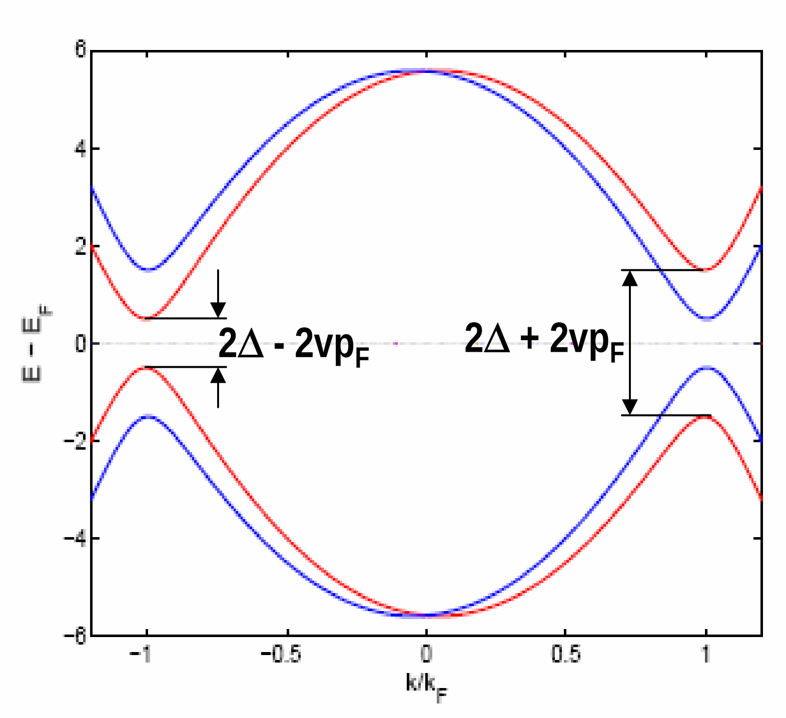
$$2\Delta = 3.52 k_B T_c, \quad T_c \ll T_D \sim 300\text{K}$$

- For T=0, all electrons are bound in the Cooper pairs
- For T<< T_c, a small fraction of electrons are unbound due to thermal dissociation of the Cooper pairs

$$n_r(T) = n_0(\pi T/2\Delta)^{1/2}\exp(-\Delta/T)$$

This normal fraction defines the small BCS surface resistance

Effect of current on thermal activation



Rocking “tilted” electron spectrum in the current-carrying rf state J = J$_0$cosωt

$$E(p) = \pm\sqrt{\Delta^2 + (p^2/2m - E_F)^2} \pm \boxed{\vec{p}_F \vec{v}_s(t)}$$

Superfluid velocity $v_s(t) = J/n_s e$

- Reduction of the gap $\Delta(v_s) = \Delta - p_F|v_s|$ in the electron spectrum increases the density of thermally-activated normal electrons $n_r(J)$, thus increasing R$_s$

- Critical pairbreaking velocity:

$$v_c = \frac{\Delta}{p_F}$$

Clean limit

Problems with the London electrodynamics

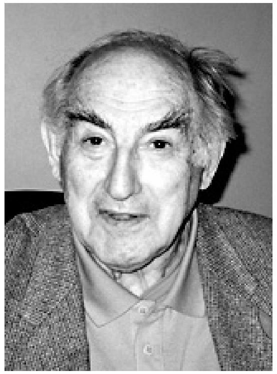
- the linear London equations

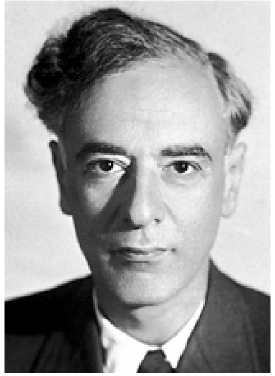
$$\boxed{\frac{\partial \vec{J}_s}{\partial t} = -\frac{\vec{E}}{\lambda^2 \mu_0}, \qquad \lambda^2 \nabla^2 \vec{H} - \vec{H} = 0}$$

along with the Maxwell equations describe the electrdynamics of SC at all T if:

- J_s is much smaller than the depairing current density J_d
- the superfluid density n_s is unaffected by current

- Generalization of the London equations to nonlinear problems
- Phenomenological Ginzburg-Landau theory (1950, Nobel prize 2003) was developed before the microscopic BCS theory (1957).
- GL theory is one of the most widely used theories





GL free energy

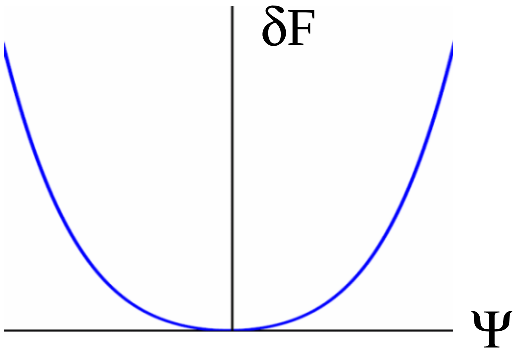
- Complex superconducting order parameter $\Psi = (n_s/2)^{1/2}\exp(i\theta)$

- For $T \approx T_c$, Ψ is small so the free energy can be expanded in the Taylor series in Ψ:

$$\boxed{F = F_n + \int dV \left[\alpha(T)|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{\hbar^2}{2m^*}\left|\left(\nabla + \frac{2\pi i\vec{A}}{\phi_0}\right)\Psi\right|^2 + \frac{\mu_0 H^2}{2} \right]}$$

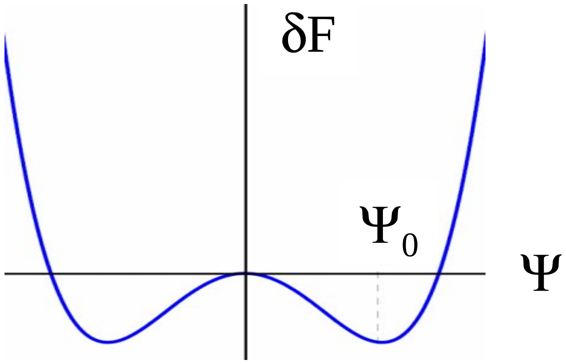
nonlinear inhomogeneity magnetic

- The coefficient $\alpha(T) = \alpha_0(T - T_c)/T_c$ changes sign at T_c



Normal state

$T > T_c$, $\Psi = 0$



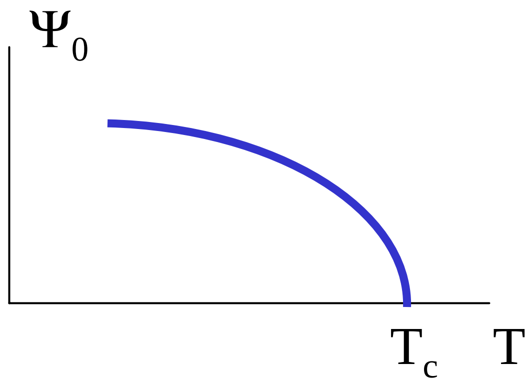
Superconducting state

$T < T_c$, $\Psi_0 = (|\alpha|/\beta)^{1/2}$

Equilibrium order parameter and H_c

- Spontaneous order parameter $\Psi_0 = [n_s/2]^{1/2}$ below T_c:

$$\boxed{\Psi_0 = \sqrt{\frac{\alpha_0 (T_c - T)}{\beta T_c}}}$$

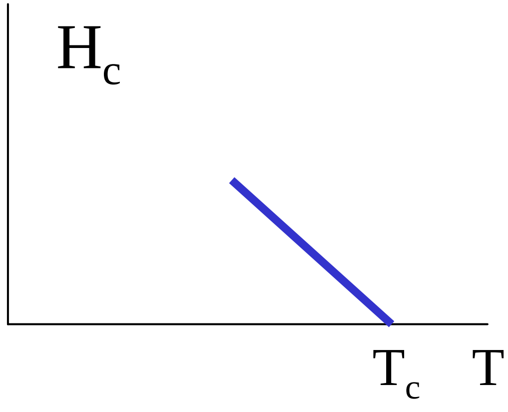


- Energy gain defines the thermodynamic critical field $H_{c:}$

$$\frac{F_n - F_s}{V} = \frac{\alpha^2(T)}{2\beta} = \frac{\mu_0 H_c^2(T)}{2}$$

- Linear temperature dependence of $H_c(T)$ near T_c:

$$\boxed{H_c(T) = \frac{\alpha_0}{\sqrt{\beta\mu_0}} \frac{(T_c - T)}{T_c}}$$



in accordance with the empirical relation $H_c(T) = H_0 [1 - (T/T_c)^2]$

GL equations for nonuniform Ψ(r) and A(r)

- Energy minimization conditions $\delta F/\delta\Psi^* = 0$ and $\delta F/\delta A = 0$ yield the GL equations for the dimensionless order parameter $\psi = \Psi/\Psi_0$

$$\boxed{\begin{gathered}\xi^2\left(\nabla + \frac{2\pi i}{\phi_0}\vec{A}\right)^2\psi + \psi - \psi\,|\psi\,|^2 = 0, \\ \nabla\times\nabla\times\vec{A} = \vec{J}_s = -\frac{|\psi\,|^2}{\lambda^2}\left(\frac{\phi_0}{2\pi}\nabla\theta + \vec{A}\right)\end{gathered}}$$

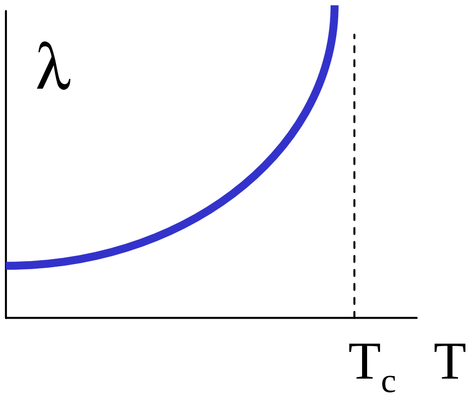
- Two coupled complex nonlinear PDE for the pair wave function $\psi(r)$ and the magnetic vector-potential $A(r)$, $(B=\nabla\times A)$.
- Two fundamental lengths ξ and λ
- Boundary condition between a superconductor and vacuum $J_s = 0$:

$$\boxed{\left(\nabla + \frac{2\pi i}{\phi_0}\vec{A}\right)\psi\vec{n} = 0}$$

Fundamental lengths λ and ξ and the GL parameter κ = λ/ξ

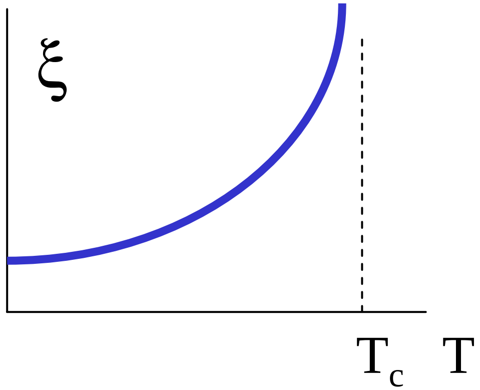
- Magnetic London penetration depth:

$$\boxed{\lambda(T)=\left(\frac{m\beta}{2e^2\mu_0\alpha_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c-T}}}$$



- Coherence length – a new scale of spatial variation of the superfluid density $n_s(r)$ or superconducting gap Δ(r):

$$\boxed{\xi(T)=\left(\frac{\hbar^2}{4m\alpha_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c-T}}}$$



- The GL parameter κ = λ/ξ is independent of T.
- Critical field $H_c(T)$ in terms of λ and ξ:

$$B_c(T)=\frac{\phi_0}{2\sqrt{2}\pi\xi(T)\lambda(T)}$$

Depairing current density

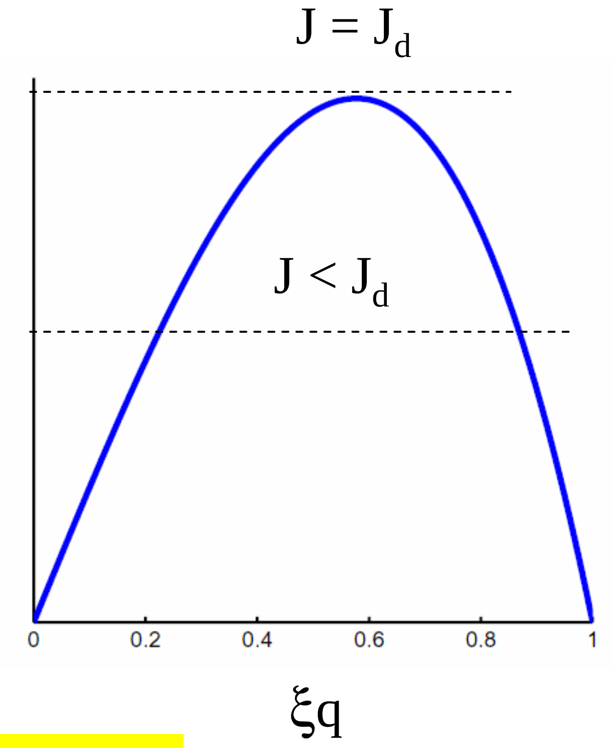
- What maximum current density J can a superconductor carry?
- Consider a current-carrying state with $\psi = \psi_0 \exp(-iqx)$, in a thin filament, where q is proportional to the velocity of the Cooper pairs. The GL equations give:

$$\psi_0^2 = 1 - \xi^2 q^2, \qquad J = \frac{\psi_0^2 \phi_0 q}{2\pi\lambda^2\mu_0}$$

- Current density as a function of q:

$$J = \frac{\phi_0 q}{2\pi\lambda^2\mu_0}(1 - \xi^2 q^2) \longleftarrow$$

Suppression of n_s by current

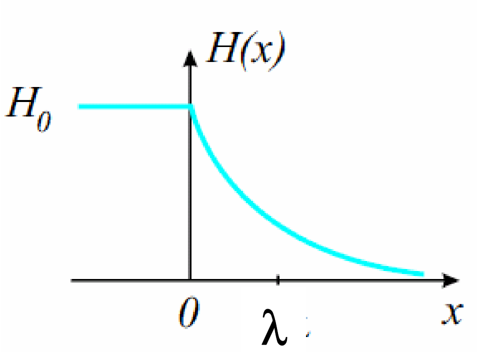


- Maximum J at $\xi q = 1/\sqrt{3}$ yields the depairing current density:

$$J_d = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\lambda^2\xi} \cong 0.54\frac{H_c}{\lambda} \propto \left(1 - \frac{T}{T_c}\right)^{3/2}$$

Paibreaking field instability of the Meissner state

- Meissner state can only exist below the superheating field $H < H_s$
- Periodic vortex instability of the Meissner state as the current density $J_s = H_s/\lambda$ at the surface reaches $\approx J_d$

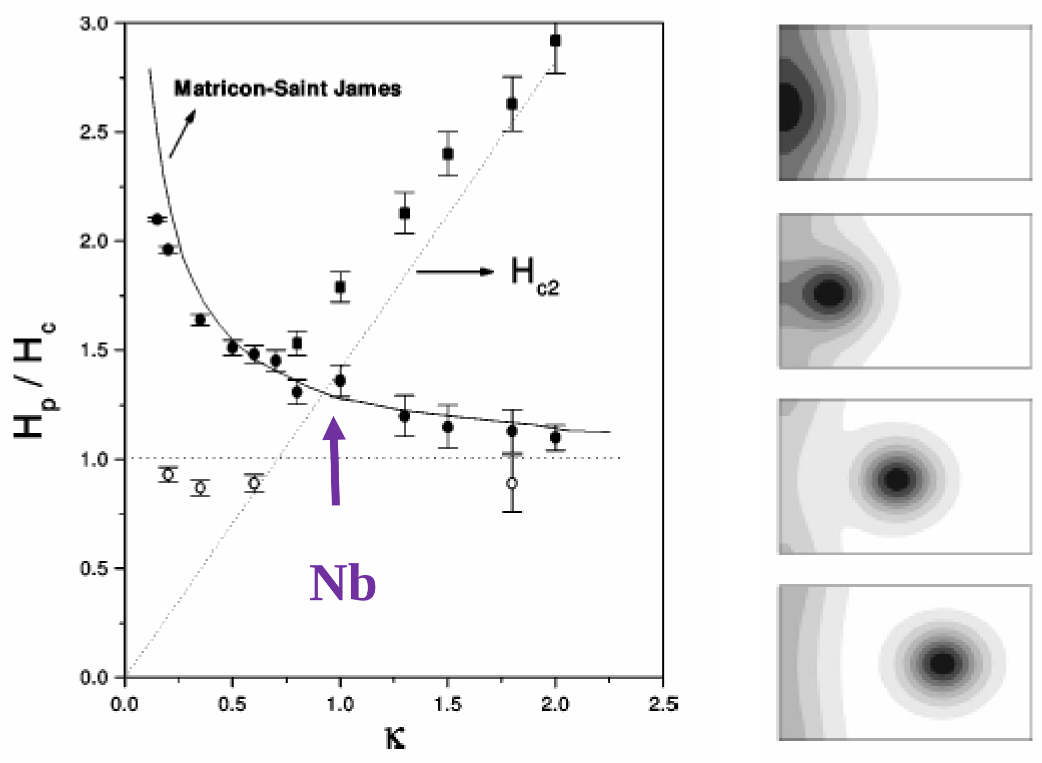


- GL calculations of the superheating field H_s (Matricon and Saint-James, 1967)

$$B_s \approx 1.2 B_c, \qquad \kappa \cong 1,$$

$$B_s \approx 0.745 B_c, \qquad \kappa >> 1$$

- B_s decreases as the surface gets dirtier and κ increases.



Hernandez and Dominguez, PRB 65, 144529 (2002)

Relation of H_s to the pairbreaking velocity

- Estimate H_s at T=0 from the condition that the superfluid velocity reaches the pairbreaking $v_c = \Delta/p_F$ at the surface

$$J_s = en\Delta / p_F = H_s / \lambda$$

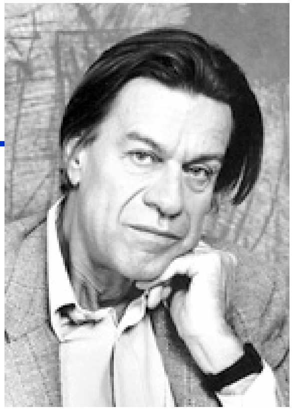
- Substitute here the BCS expressions for the coherence length ξ, London penetration depth λ, and the thermodynamic critical field $H_c = B_c/\mu_0$:

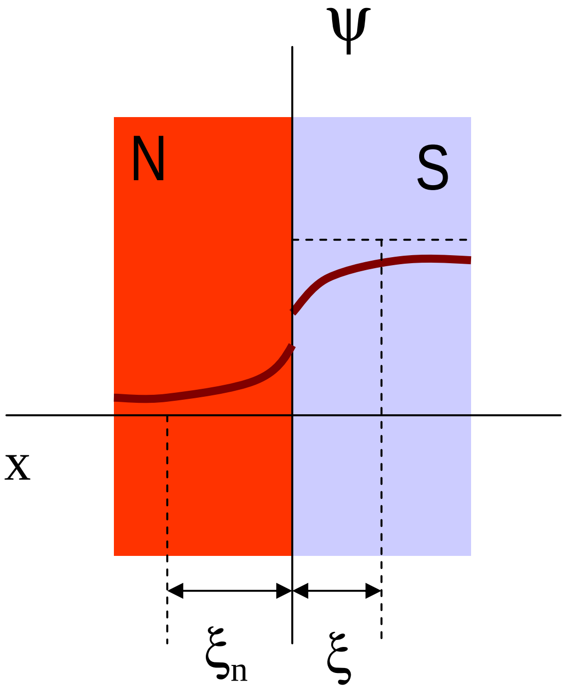
$$\xi = \frac{\hbar v_F}{\pi\Delta}, \qquad \lambda = \left(\frac{m}{ne^2\mu_0}\right)^{1/2}, \qquad B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}$$

- Hence, we estimate the superheating field at T = 0 in the clean limit and $\kappa >> 1$:

$$B_s = \frac{2^{3/2}}{\pi} B_c \approx 0.9 B_c$$

Proximity effect (deGennes, 1964)





- What happens if a normal metal is in contact with a superconductor?
- Induced superconductivity due to diffusion of the Cooper pairs in a metal over the proximity length ξ_n:

$$\psi(x) = \psi_0 \exp(-x/\xi_n)$$

- Suppression of ψ(x) near the surface of the S layer.
- Formulas for the proximity length:

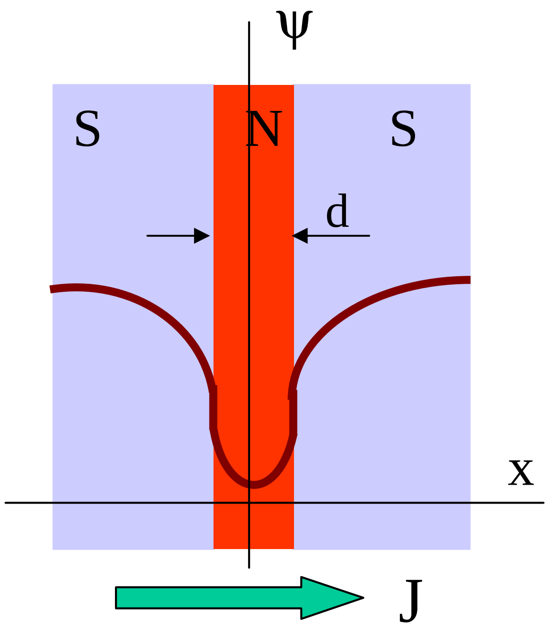
- At low T the proximity length can be greater than N thickness: N layer becomes proximity coupled

$$\xi_n = \frac{\hbar v_F}{2\pi k_B T}, \qquad \xi_n = \left(\frac{\hbar v_F l}{6\pi k_B T}\right)^{1/2}$$

clean metal, $l \gg \xi_n$ dirty metal, $l \ll \xi_n$

Example: would a 1μm Cu precipitate in a Nb cavity at 2K be superconducting or normal?
clean Cu: $v_F = 1.6\times10^6$ m/s, $\xi_n = 0.6$ μm (nearly SC). Dirty Cu: $\xi_n \ll 1$ μm (normal)

Critical currents of SNS contacts



- Maximum J in the middle of the N layer where ψ_m is minimum
- Take the GL expressions for J and ψ_m :

$$J \cong \frac{\psi_m^2 \phi_0 q}{2\pi \lambda^2 \mu_0},$$

$$\psi_m \approx \frac{\xi_n}{\xi} \exp\left(-\frac{d}{2\xi_n}\right)$$

where $q \sim 2\pi/\xi_n$

suppression of ψ at S-N interface

decay of ψ in the N layer

- Critical current density of a proximity coupled SNS contact:

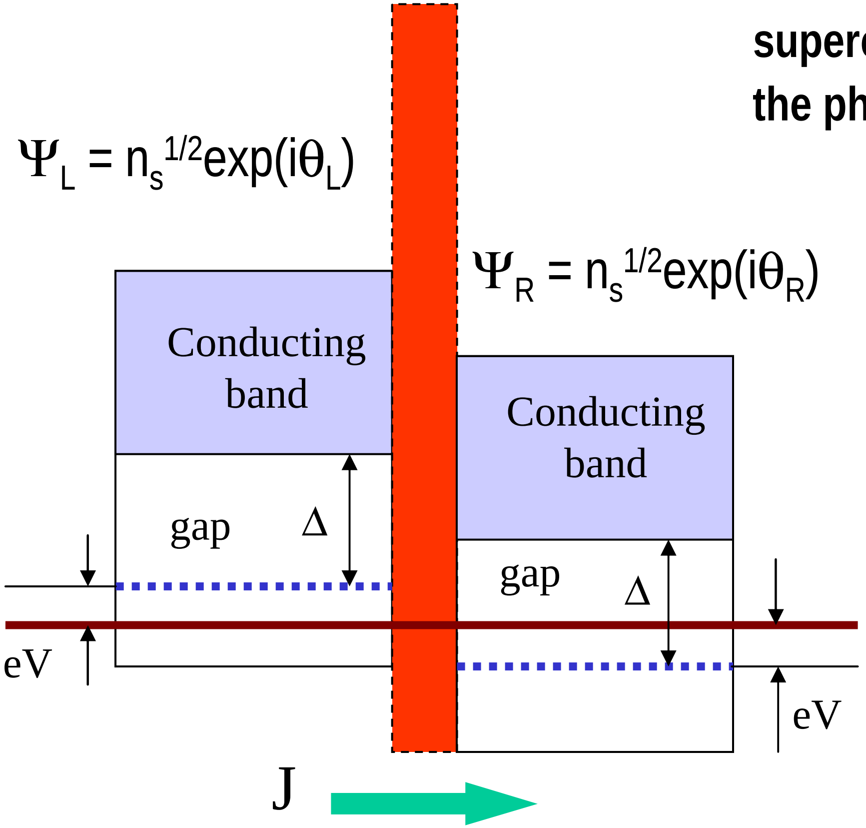
$$\boxed{J_c \approx \frac{\phi_0 \xi_n}{\mu_0 \xi^2 \lambda^2} \exp\left(-\frac{d}{\xi_n}\right)}$$

- J_c drops exponentially with d, but increases exponentially as T decreases
- $J_c \propto (T_c - T)^2$ near T_c

Weak superconductivity due to tunneling of Cooper pairs through N layer

Josephson effect (PhD thesis, 1962, Nobel prize, 1973)

- Tunneling between 2 weakly coupled superconductors strongly depends on the phase difference: $\theta = \theta_L - \theta_R$



Because of the phase coherence, each superconductor behaves as a single-level quantum-mechanical system

1. dc Josephson current

$$J = J_c \sin\theta$$

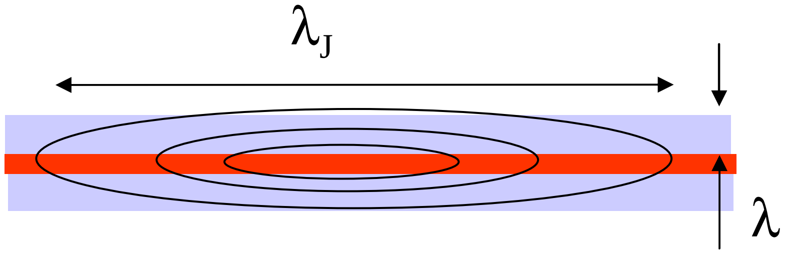
2. Josephson voltage:

$$\frac{d\theta}{dt} = \frac{2eV}{\hbar}$$

3. Oscillating Josephson current at a fixed voltage V:

$$J(t) = J_c \sin\left(\frac{2eVt}{\hbar} + \theta_0\right)$$

Josephson vortices in long junctions



Model of planar crystalline defects:
grain boundaries, etc.

Ferrell-Prange equation for the phase difference θ(x) on a long JJ

$$\tau^2\ddot{\theta} + \tau_r\dot{\theta} = \lambda_J^2\theta'' - \sin\theta$$

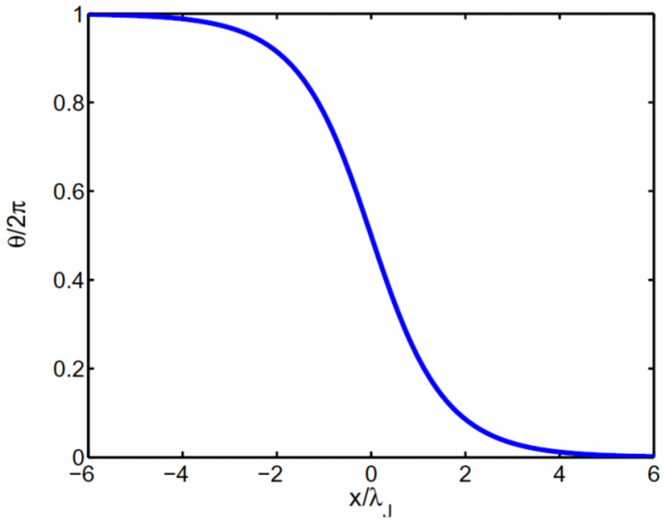
- **New length scale: Josephson magnetic penetration depth:**

$$\lambda_J = \left(\frac{\phi_0}{4\pi\mu_0\lambda J_c}\right)^{1/2}$$

Because J_c is small, λ_J is usually much greater than λ

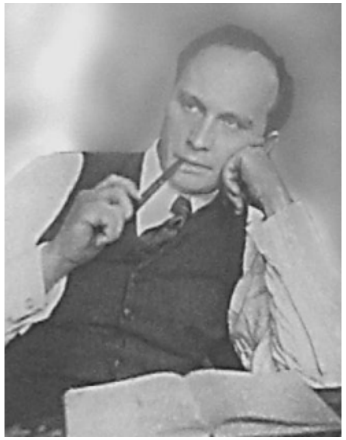
- **Josephson vortex: a long current loop along a JJ:**

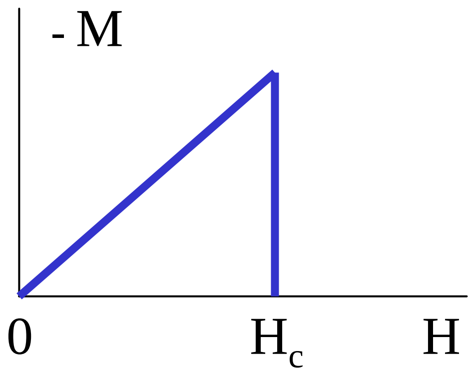
$$\theta(x) = 4\tan^{-1}\exp\left(-\frac{x}{\lambda_J}\right)$$



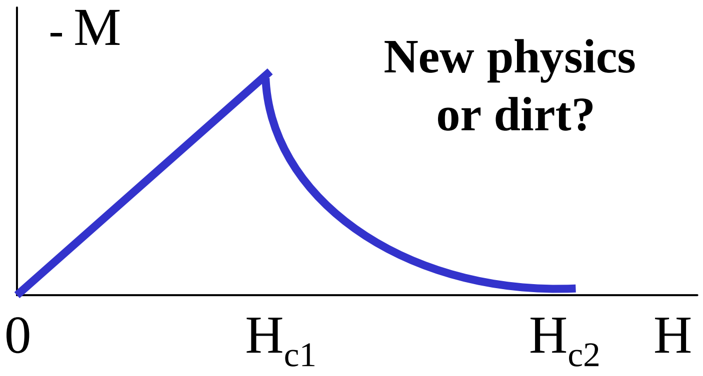
Type–I and type–II superconductors

- Measurements of magnetization M(H) have shown a partial Meissner effect in many superconducting compounds and alloys (Shubnikov, 1935).





Complete Meissner effect in type-I superconductors



High-field partial Meissner effect in type-II superconductors

- **Type-I:** Meissner state $B = 0$ for $H < H_c$; normal state at $H > H_c$
- **Type-II**: Meissner state $(H < H_{c1})$, partial flux penetration $(H_{c1} < H < H_{c2})$, normal state $(H > H_{c2})$
- Lower and upper critical fields H_{c1} and H_{c2}.
- High field superconductivity with $H_{c2} \sim 100$ Tesla

Upper critical field H_{c2}

- For a uniform field H along the z-axis, the GL equation for small ψ is:

$$\boxed{\xi^2\nabla^2\psi + [1 - (2\pi Bx\ \xi / \phi_0)^2]\psi = 0}$$

- Similar to the Schrodinger equation for a harmonic oscillator:

$$\frac{\hbar^2}{2M}\nabla^2\psi + (E - \frac{M\omega^2 x^2}{2})\psi = 0: \qquad \frac{\hbar^2}{2M} \to \xi^2, \qquad E \to 1, \qquad \sqrt{M}\omega \to \frac{2^{3/2}\pi H\xi}{\phi_0}$$

- The oscillator energy spectrum $E = \hbar\omega(n + ½)$ for $n = 0$, then gives H_{c2} below which bulk superconductivity exists (surface SC can exist at even higher $H_{c3} = 1.69H_{c2}$)

$$B_{c2}(T) = \frac{\phi_0}{2\pi\xi^2(T)} = \frac{\phi_0}{2\pi\xi_0^2}\left(1 - \frac{T}{T_c}\right)$$

How can H_{c2} be higher than H_c?

$$B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}, \qquad B_{c2} = \frac{\phi_0}{2\pi\xi^2}$$

- Type-I superconductors: $B_c > B_{c2}$, or $\kappa = \lambda/\xi < 1/\sqrt{2}$: mostly simple metals
- Type-II superconductors: $B_c < B_{c2}$, or $\kappa = \lambda/\xi > 1/\sqrt{2}$: 100 (HTS), 40 (MgB_2)
- Marginal type-II superconductor: Nb, $\kappa \cong 1$.

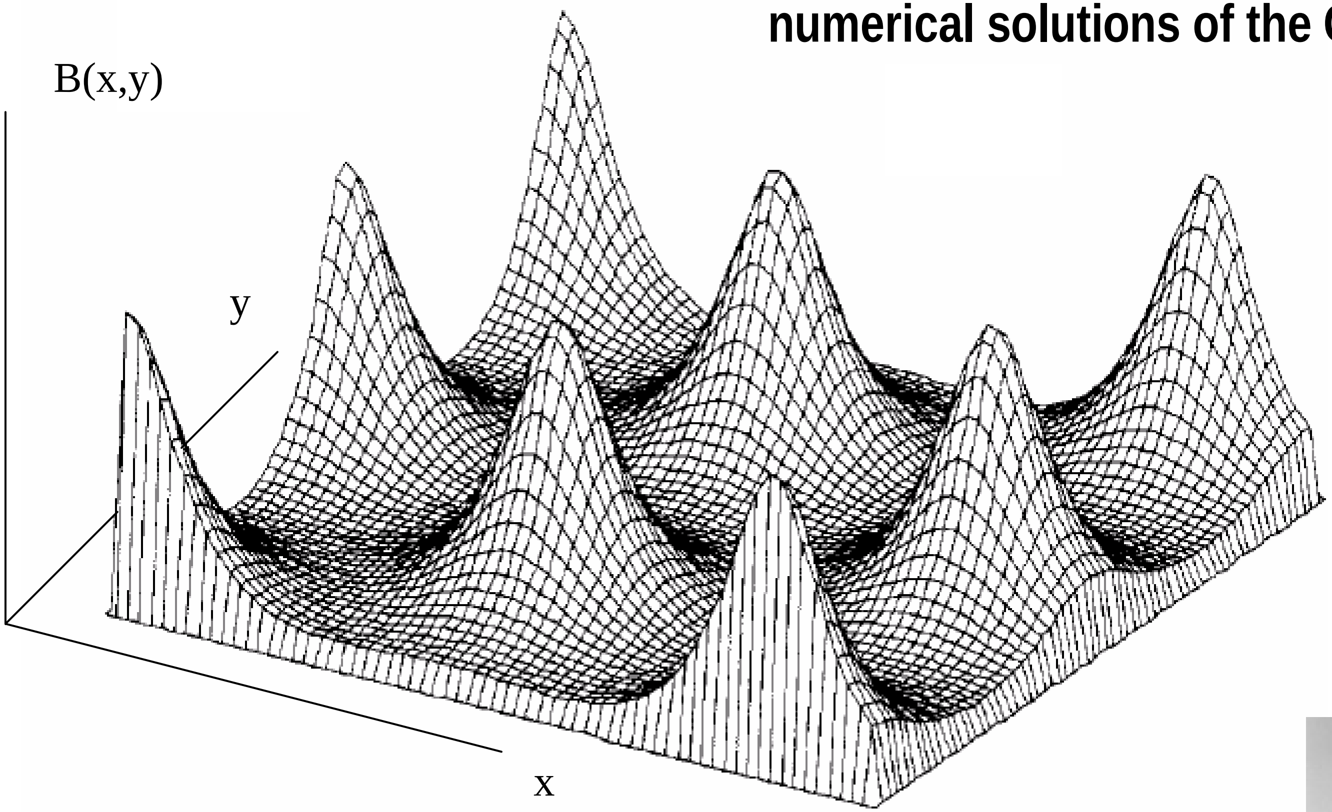
- In many type-II superconductors the GL parameter $\kappa = \lambda/\xi$ can be increased by alloying with nonmagnetic impurities.

Dirty SC with the electron mean-free path $\ell < \xi_0$: the penetration depth $\lambda \cong \lambda_0(\xi_0/\ell)^{1/2}$ increases as ℓ decreases, but the coherence length $\xi = (\xi_0\ell)^{1/2}$ decreases as ℓ decreases. Thus, H_c does not change, but H_{c2} increases proportionally to the residual resistivity ρ

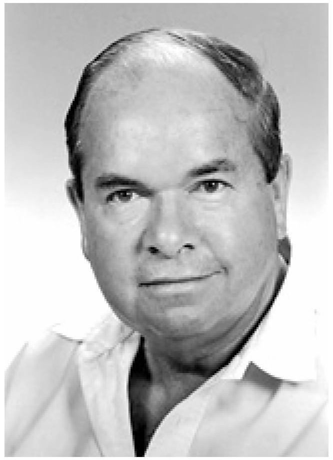
$$B_{c2} \cong \frac{\phi_0}{2\pi\xi_0 l}\left(1 - \frac{T}{T_c}\right) \propto \rho$$

Vortex lattice at $H_{c1} < H < H_{c2}$ (Abrikosov 1956, Nobel prize, 2003)

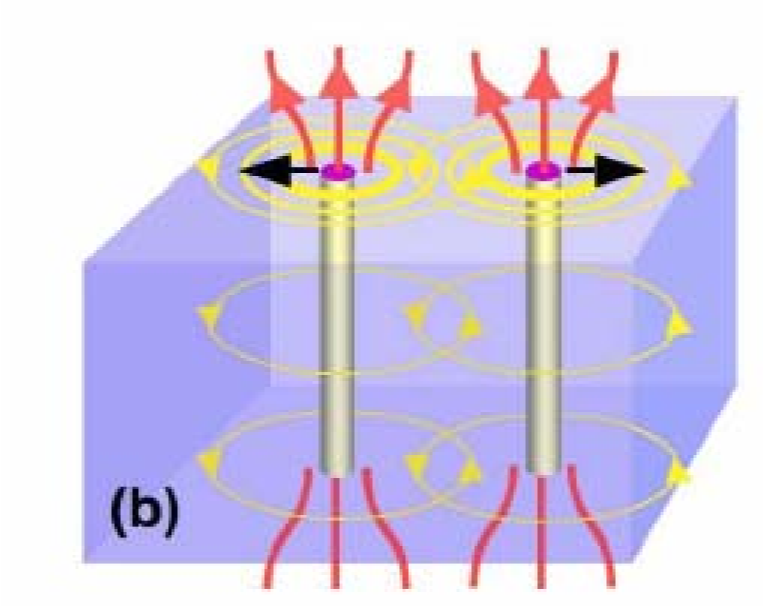
numerical solutions of the GL equations

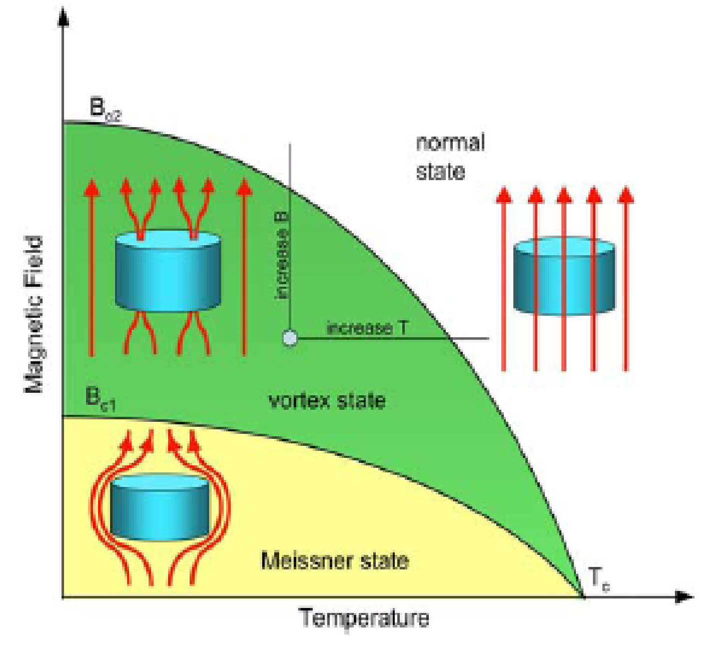


- Hexagonal lattice of vortex lines, each carrying the flux quantum ϕ_0
- Vortex density $n(B) = \phi_0/B$ defines the magnetic induction B
- Spacing between vortices: $a = (\phi_0/B)^{1/2}$



Type-II superconductors



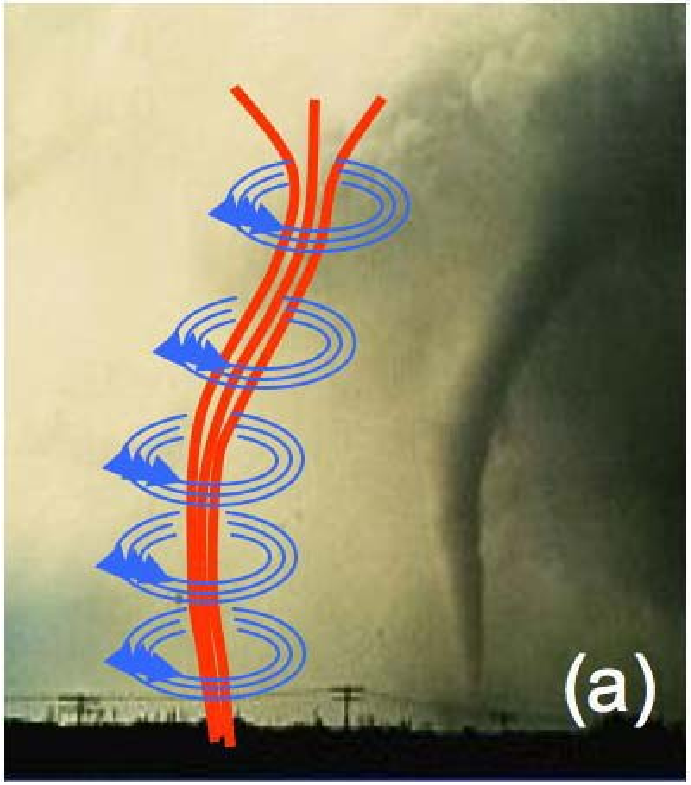


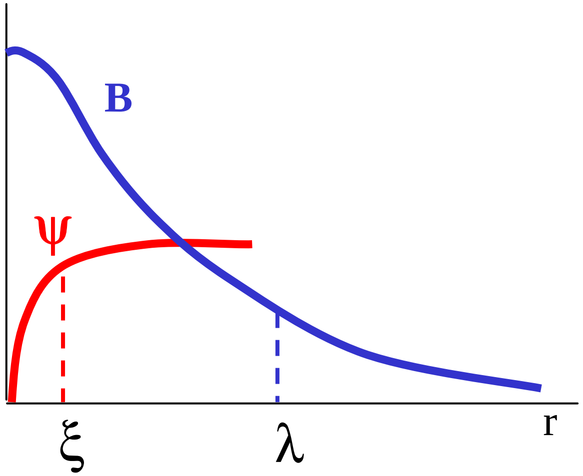
Main thermodynamic parameters of type-II superconductors:

1. Critical temperature, T_c
2. Lower critical field H_{c1}
3. Upper critical field H_{c2}

Periodic hexagonal lattice of quantized vortex filaments at $H_{c1} < H < H_{c2}$

Single vortex line



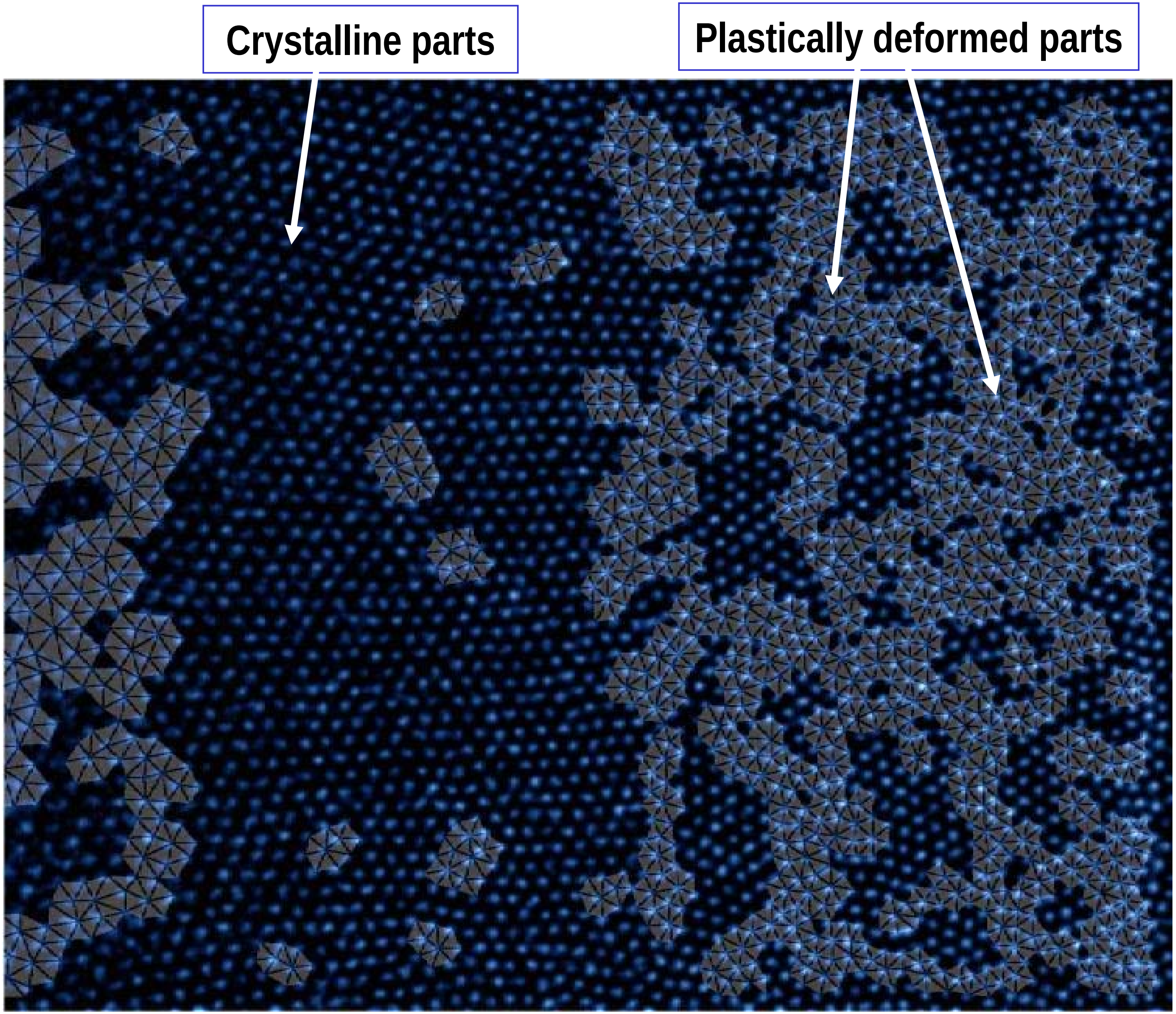


- Small core region $r < \xi$ where superconductivity is suppressed by strong circulating currents
- Region of circulating supercurrents, $r < \lambda$.
- Each vortex carries the flux quantum ϕ_0

Distributions of Δ(r) and J(r) for $r < \lambda$

$$\Delta(r) \cong \frac{r\Delta_0}{\sqrt{2\xi^2 + r^2}}, \qquad J(r) \cong \frac{\phi_0}{2\pi\mu_0\lambda^2 r}$$

Decoration image of a vortex “polycrystal”



Crystalline parts

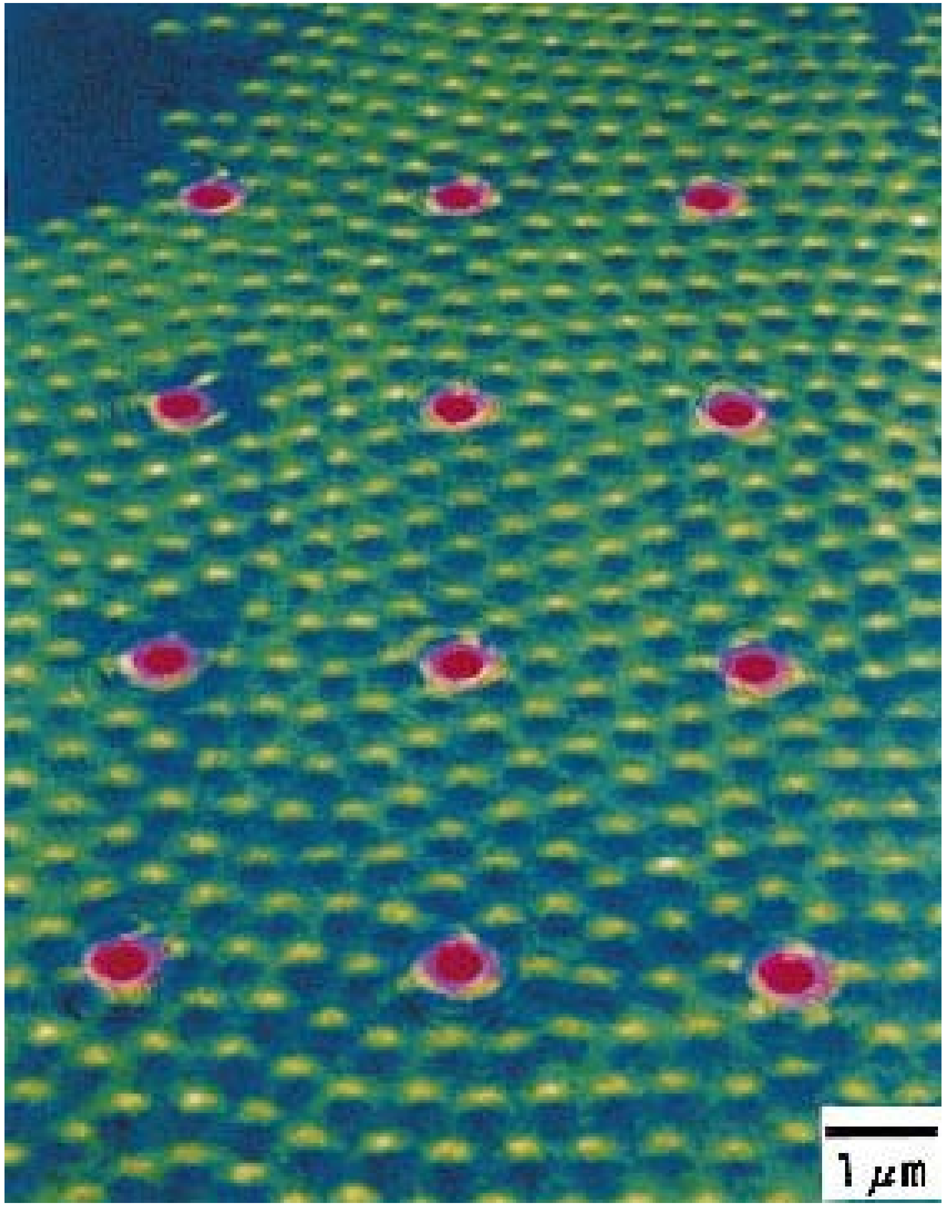
Plastically deformed parts

Magnetic decoration was introduced by Essmann and Trauble, who were the first to observe vortex lattice, 1967

Nb_2Se

Pardo et al, 1998

Weak pinning of the vortex lattice in Nb



- Lorentz electron microscopy of vortices in Nb film
  A. Tonomura et al, 1999.
- Ideal hexagonal vortex lattice between the pins (30 nm nanodots produced by FIB)
- Plastic deformation of the vortex lattice by current
- Vortex “rivers” flowing between the pins for $J > J_c$
- $J_c = 0$ if the vortex lattice melts

Why are vortices energetically favorable?

- Each vortex carries the paramagnetic flux quantum, so its thermodynamic potential G in a magnetic field H is reduced by $H\phi_0$:

$$G = \varepsilon - H\phi_0, \qquad \varepsilon = \frac{1}{2\mu_0}\int[\lambda^2(\nabla B)^2 + B^2]dS$$

Vortex self energy

Magnetic dipole in field

Kinetic energy of supercurrents

Energy of local fields

- Vortices are energetically favorable for G < 0, above the lower critical field $H_{c1} = \varepsilon/\phi_0$

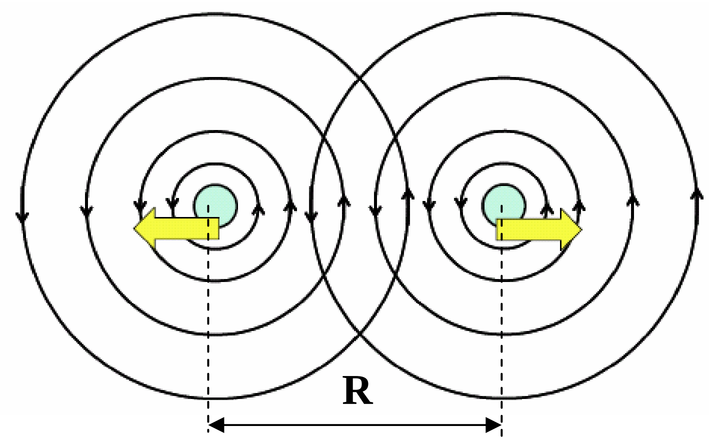
$$\varepsilon \cong \frac{\lambda^2}{2\mu_0}\left(\frac{\phi_0}{2\pi\lambda^2}\right)^2\int_\xi^\lambda \frac{2\pi r}{r^2}dr = \frac{\phi_0^2}{4\pi\mu_0\lambda^2}\ln\frac{\lambda}{\xi}$$

- Detailed calculations with the account of the vortex core structure give:

$$H_{c1} = \frac{\phi_0}{4\pi\mu_0\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right)$$

$H_{c1} \sim H_c/\kappa \sim H_{c2}/\kappa^2$, thus
$H_{c1} \ll H_c \ll H_{c2}$ for $\kappa \gg 1$

Interaction between vortices



- Energy of two vortices

$$U = \frac{\phi_0}{2}\left[H(r_1) + H(r_2)\right], \qquad H(r) = H_0 + H_{12}(R)$$

H_0 is the self-field in the core, H_{12}(R) is the field produced at the position of the other vortex:

- Interaction energy $U_i(R) = \phi_0 H_{12}(R)$ and force f = - $\partial U_i/\partial R$:

$$U = 2\varepsilon + \phi_0 H_{12}(R), \qquad U_{\text{int}} = \frac{\phi_0^2}{2\pi\mu_0\lambda^2} K_0\left(\frac{R}{\lambda}\right), \qquad f_y = -\phi_0 \frac{\partial H_{12}}{\partial R} = \phi_0 J_x$$

- Vortices repel each other, vortex and antivortex attract each other .
- General current-induced Lorentz force acting on a vortex

$$\vec{f} = \phi_0[\vec{J} \times \hat{n}]$$

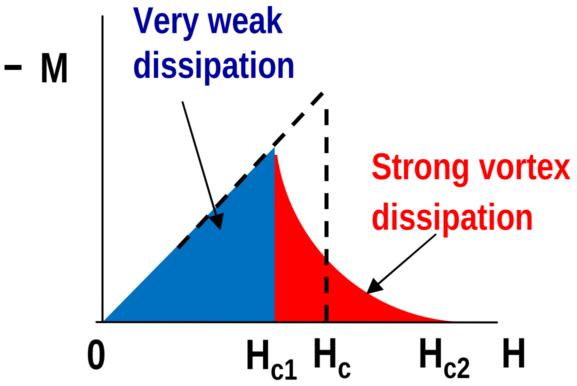
- vortex is pushed perpendicular to the local current density **J** at the vortex core

Intermediate fields, $H_{c1} << H << H_{c2}$

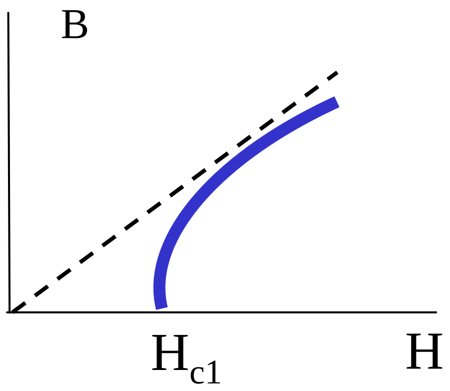
- For a << λ, and κ >> 1, the field H(B) and the magnetization M(H) are

$$H \approx \frac{B}{\mu_0} + H_{c1}\frac{\ln(B_{c2}/B)}{2\ln\kappa}, \qquad M \cong -H_{c1}\frac{\ln(H_{c2}/H)}{2\ln\kappa}$$

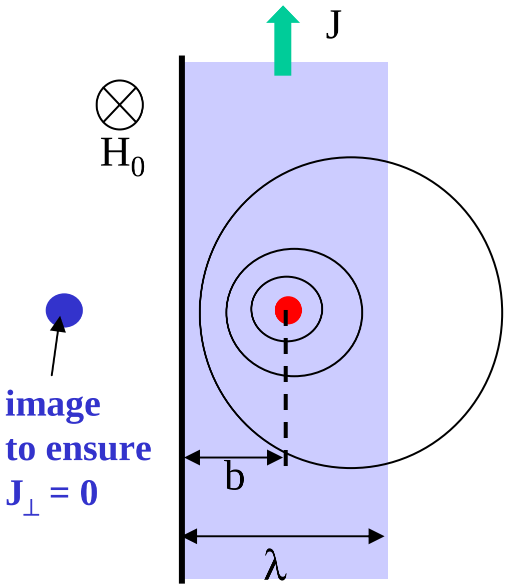
Superconductivity disappears at $B_{c2} = \phi_0/2\pi\xi^2$
because nonsuperconducting vortex cores overlap



Material	T_c (K)	$H_c(0)$ [T]	$H_{c1}(0)$ [T]	$H_{c2}(0)$ [T]	$\lambda(0)$ [nm]
Pb	7.2	0.08	na	na	48
Nb	9.2	0.2	0.17	0.4	40
Nb_3Sn	18	0.54	0.05	30	85
NbN	16.2	0.23	0.02	15	200
MgB_2	40	0.43	0.03	3.5	140
YBCO	93	1.4	0.01	100	150

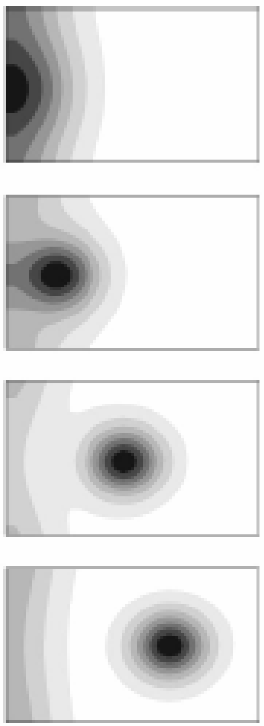


Surface barrier: How do vortices penetrate at $H > H_{c1}$?



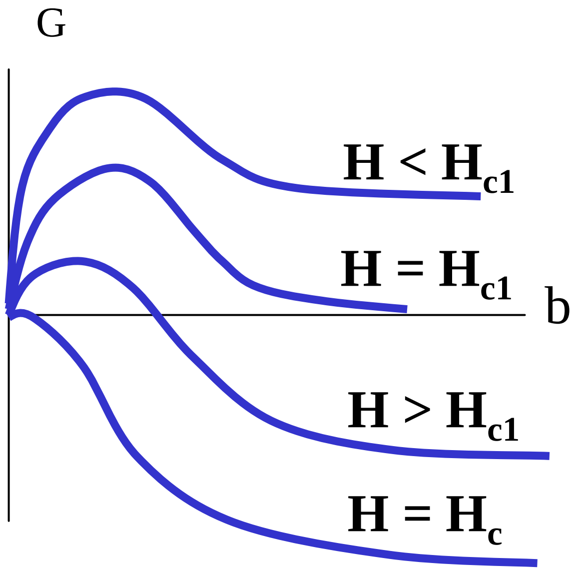
- Two forces acting on the vortex at the surface:

- Meissner currents push the vortex in the bulk
- Attraction of the vortex to its antivortex image pushes the vortex outside



Thermodynamic potential G(b) of the vortex:

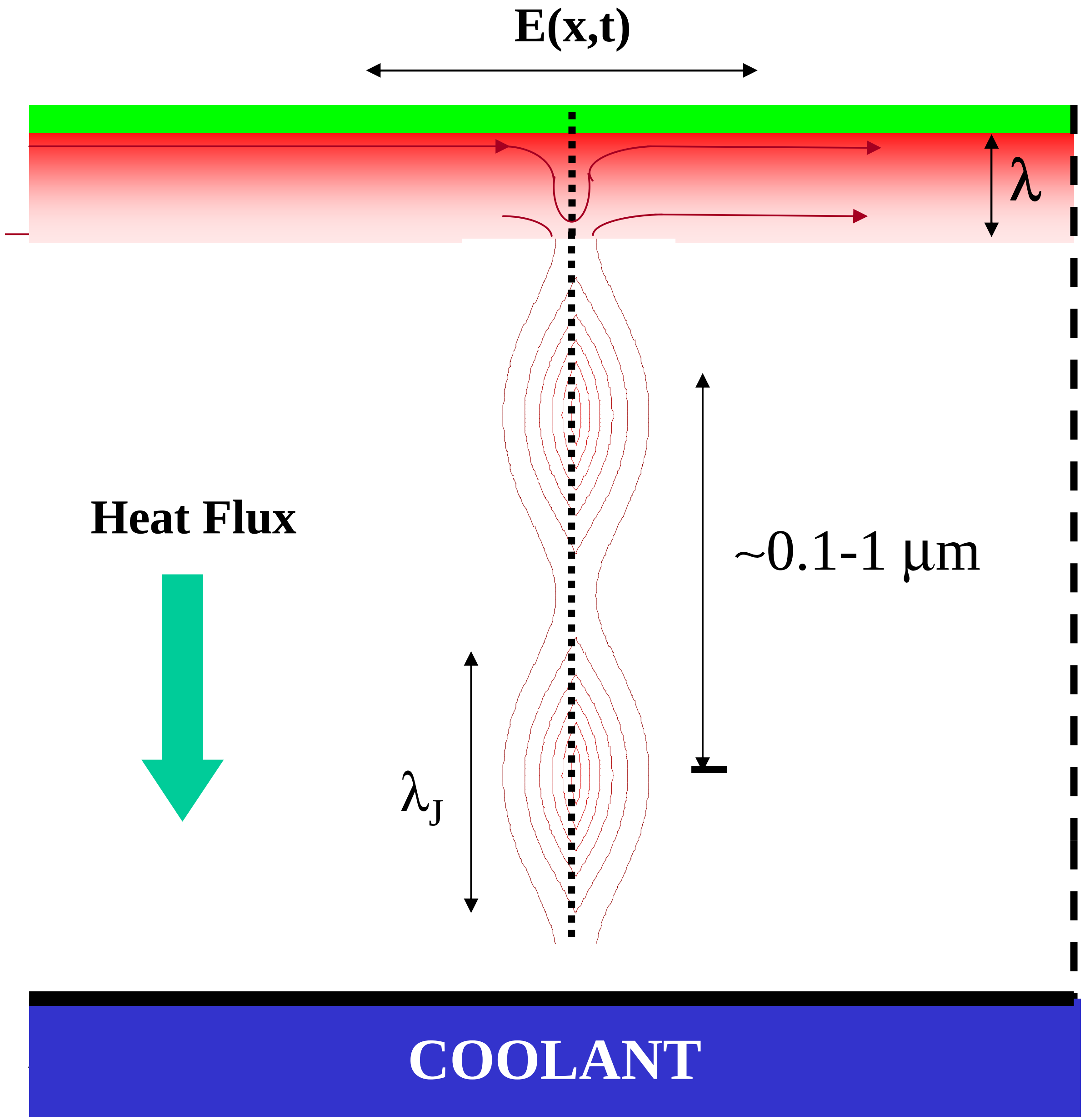
$$G(b)=\phi_0[\underset{\text{Meissner}}{H_0 e^{-b/\lambda}} - \underset{\text{Image}}{H_v(2b)} + H_{c1} - H_0]$$



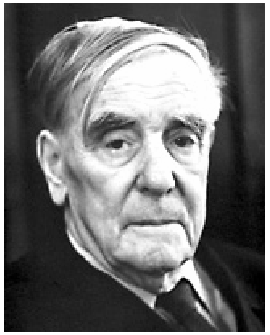
Vortices have to overcome the surface barrier even at $H > H_{c1}$ (Bean & Livingston, 1964)

Surface barrier disappears only at the overheating field $H = H_s$

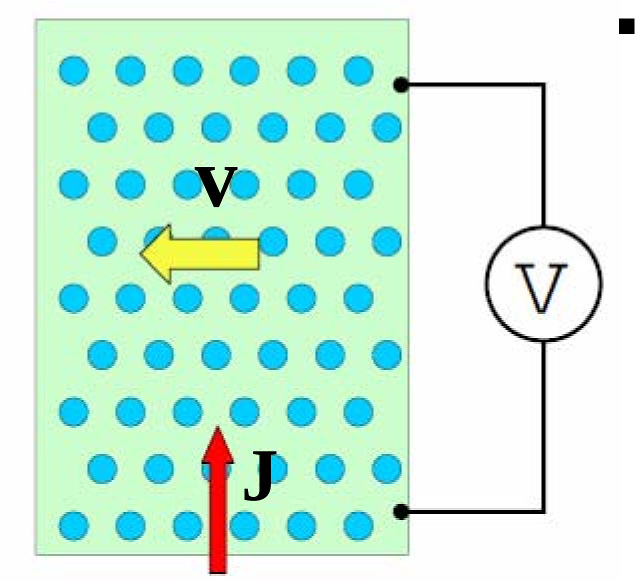
Grain boundaries as gates for penetration of the Josephson vortices



- Reduction of the surface barrier by grain boundaries
- Pento-oxides (5-10 nm)
- RF field penetration depth λ = 40 nm defines R_s
- Heat transport through cavity wall ~ 3mm and the Kapitza thermal resistance



Lorentz force and motion of vortices



- Viscous flux flow of vortices driven by the Lorentz force

$$\eta\vec{v} = \phi_0[\vec{J}\times\hat{n}], \qquad \vec{E} = [\vec{v}\times\vec{B}]$$ **Faraday law**

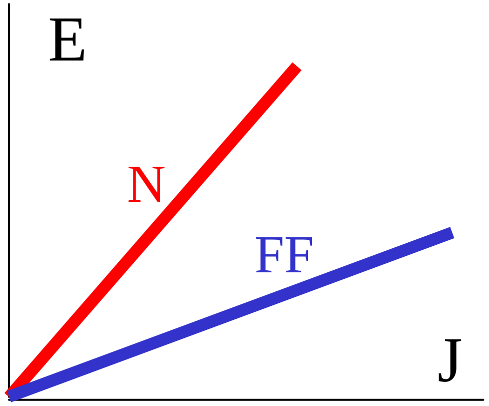
This yields the liner flux flow E-J dependence:

$$\vec{E} = \rho_f\vec{J}, \qquad \rho_f = \rho_n B/B_{c2}$$

Volume fraction of normal vortex cores

Vortex viscosity η is due to dissipation in the vortex core and can be expressed in terms of the normal state resistivity ρ_n:

$$\boxed{\eta = \phi_0 B_{c2}/\rho_n}$$



For E = 1μV/cm and B = 1T, the vortex velocity

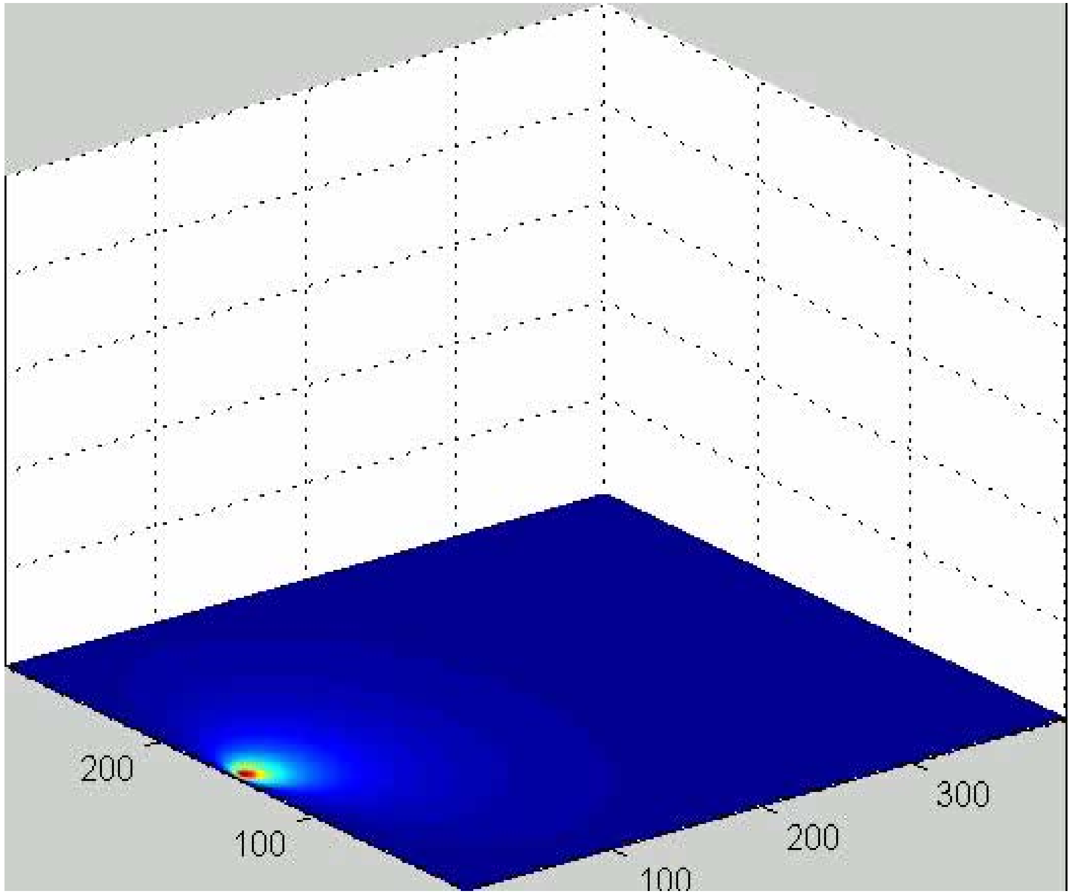
v = E/B = 0.1 mm/s

Penetration of vortices through the oscillating surface barrier

$$\eta\dot{u} = \frac{\phi_0 H_0}{\lambda} e^{-u/\lambda} \sin \omega t - \frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2\sqrt{u^2+\xi^2}}{\lambda}\right)$$

Nonlinear dynamic ODE in the high- κ London approximation

- Onset of vortex penetration
  $B_v \approx \phi_0/4\pi\lambda\xi = 0.71B_c$
- Vortex relaxation time constant:
  $\tau = \mu_0\lambda^2 B_{c2}/B_v\rho_n \approx 1.6\times10^{-12}$ s
  for Nb_3Sn, $\rho_n = 0.2\ \mu\Omega$m, B_{c2} = 23T, B_c = 0.54T, λ = 65 nm



How fast can vortices penetrate when breaking through the surface barrier?

- Maximum Lorentz force at the superheating field balanced by the viscous drag force:

$$\eta v_m \approx \phi_0 H_s / \lambda$$

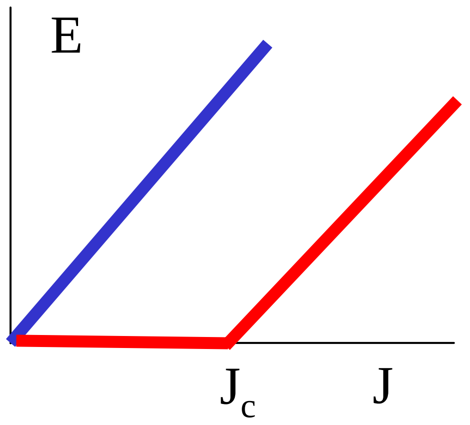
- Maximum vortex velocity:

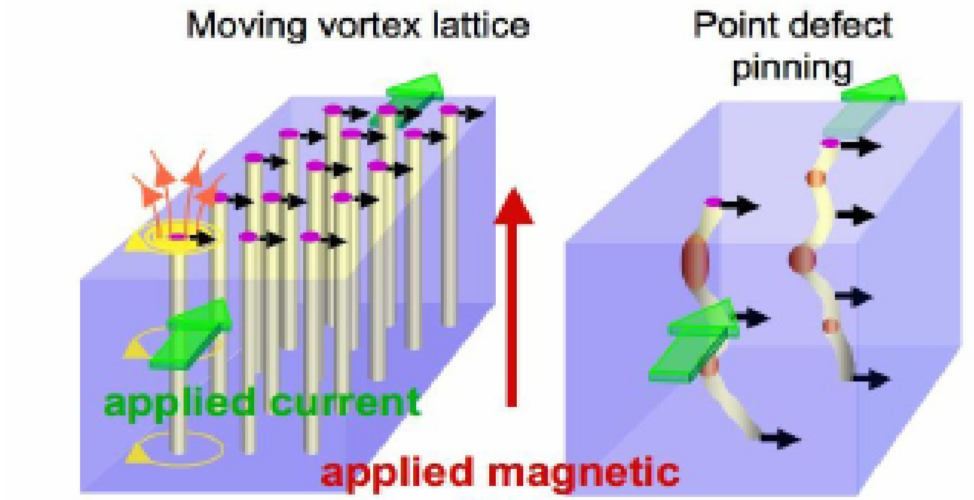
$$v_m \approx \frac{\rho_n \xi}{2\mu_0 \lambda^2}$$

- For Nb: $\lambda \approx \xi$ = 40 nm, $\rho_n = 10^{-9}$ Ωm, we obtain $v_m \sim$ 10 km/s, greater than the speed of sound !

- Strong effect of local heating

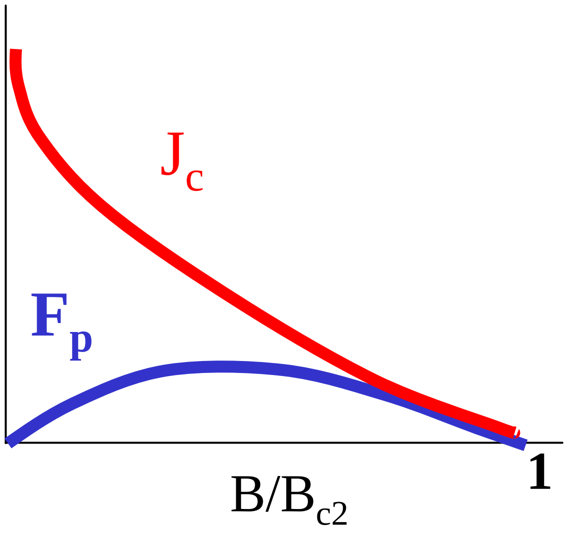
Pinning and superconductivity at $H > H_{c1}$





- Balance of the volume Lorentz and pinning forces defines the critical current density J_c

$$BJ_c(T,B) = F_p(T,B)$$

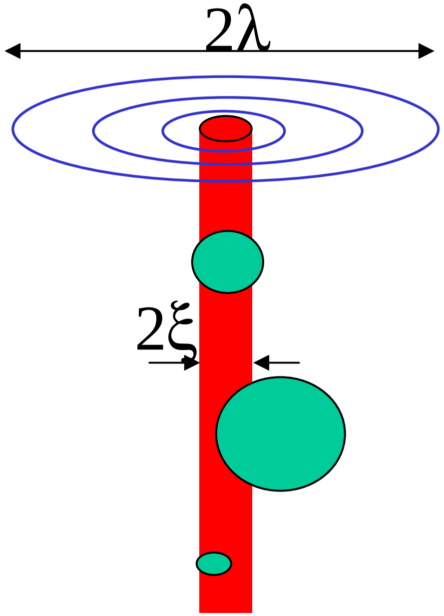


- Ideal crystals without defects have finite flux flow resistivity and partial Meissner effect

- Defects pin vortices restoring almost zero resistivity for J smaller than the critical current density J_c

- Unlike the thermodynamic quantities (T_c H_{c1}, H_{c2}), J_c is strongly sample dependent.

Core pinning



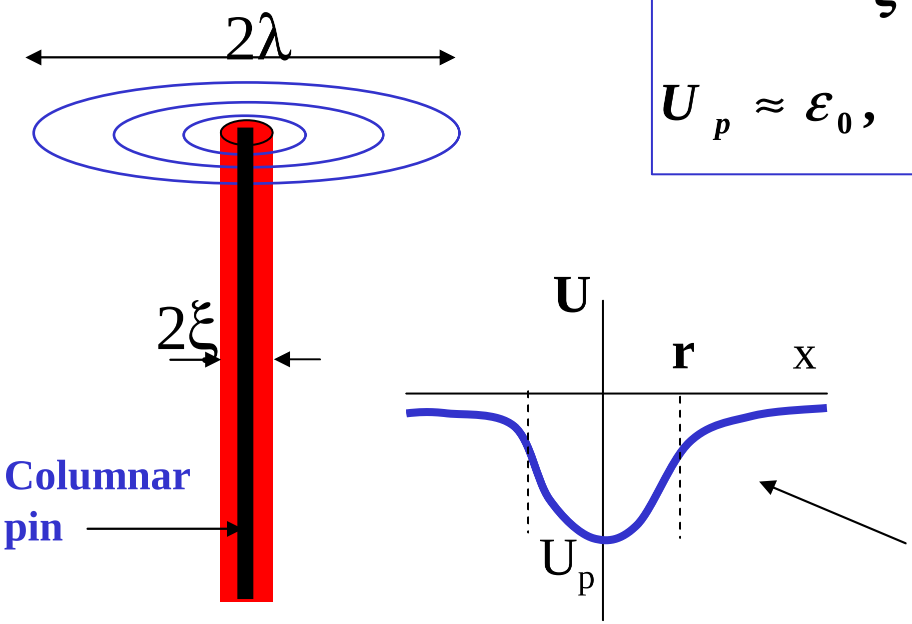
- Nonsuperconducting precipitates, voids, etc.
- Columnar defects (radiation tracks, dislocations)

- Gain of a fraction of the vortex core line energy, $\varepsilon_0 = \pi\xi^2\mu_0 H_c^2/2$, if the core sits on a defect

- Pinning energy U_p and force f_p for a columnar pin of radius r:

$$U_p \approx \varepsilon_0 \frac{r^2}{\xi^2}, \qquad f_p \approx 2\varepsilon_0 \frac{r}{\xi^2}, \qquad r << \xi,$$

$$U_p \approx \varepsilon_0, \qquad f_p \approx \frac{\varepsilon_0}{r}, \qquad r > \xi$$

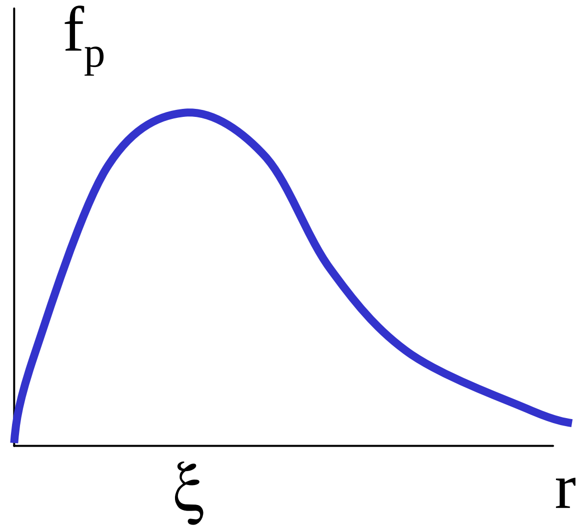


Columnar pin

- For r << ξ, only a small fraction of the core energy is used for pinning, f_p is small

- For r >> ξ, the whole ε_0 is used, but the maximum pinning force $f_p \sim \varepsilon_0/r$ is small

Optimum core pin size and maximum J_c

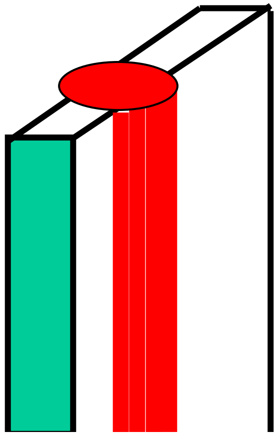


- Because $f_p(r)$ is small for both r << ξ and r << ξ, the maximum pinning force occurs at r ≅ ξ.
- The same mechanism also works for precipitates.

What is the maximum J_c for the optimum columnar pin?

- Optimum pin allows to reach the depairing current density!

$$J_{\max} \cong \frac{f_p(\xi)}{\phi_0} = \frac{\phi_0}{8\pi\mu_0\xi\lambda^2} \cong J_d$$

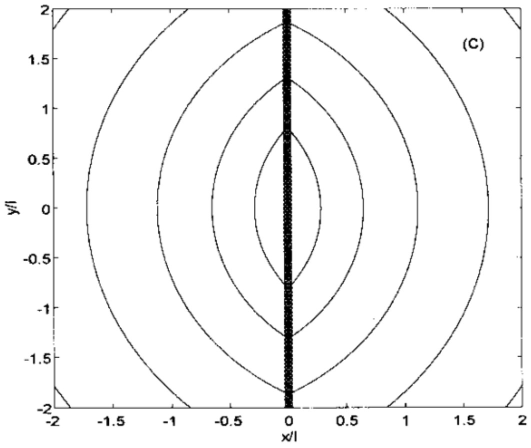


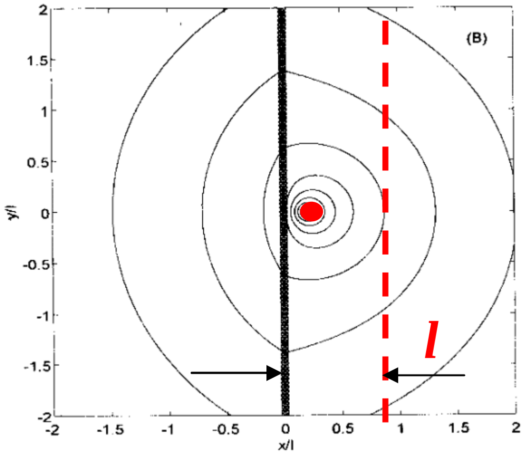
Core pinning by a planar defect of thickness ≈ ξ is also very effective

- Core pinning by small precipitates of size ≈ ξ yields smaller J_c reduced by the factor ≈ r/l_p (fraction of the vortex length taken by pins spaced by l_p)

Magnetic pinning

- Planar defects: grain boundaries in polycrystals (Nb_3Sn) or α-Ti ribbons in NbTi





- Distortion of vortex currents: attraction to an image similar to that of the vortex at the surface

- Distance l of strong interaction: $f(x)= \phi_0^2/2\pi\mu_0\lambda^2 x$

Distance l from the vortex core at which J(l) equals J_b of the defect

$$J_v(l) = \frac{\phi_0}{2\pi\mu_0\lambda^2 l} = J_b \quad \rightarrow \quad l = \frac{\phi_0}{2\pi\mu_0\lambda^2 J_b}$$

- Abrikosov vortex with normal core turns into a mixed Abrikosov vortex with Josephson core: Pinning defect can radically change the vortex core structure

- Magnetic pinning by a thin insulating defect ($d < \xi$) can result in a very high $J_c \sim J_d$!