BESSY

Berliner Elektronenspeicherring-Gesellschaft
für Synchrotronstrahlung m.b.H.

# Basics of Superconducting RF

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Gemeinschaft

# Overview

- **What is the theoretical behavior of superconducting RF cavities?**

- **Short introduction to RF cavities** ***(details in Tutorials 2a/b)***
- **Need some „tools“ to characterize their performance/losses**
  - Figures of Merit:  Surface resistance, Q-factor, shunt impedance ...
- **RF losses for normal and superconductors: theoretical behavior**
- **Use the Figures of Merit to understand the impact of losses on RF cavities**
- **Cavity losses: measured behavior, how to improve them**

- **Fundamental field limits of superconducting cavities** *(practical limits in Tutorial 4b)*

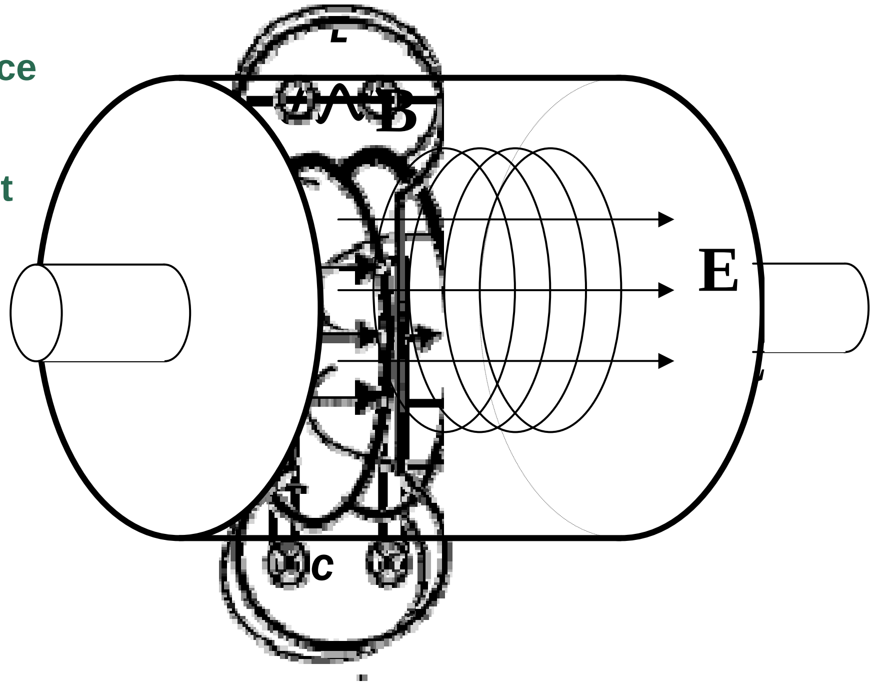
- **Note:  Throughout will calculate examples**
  - Always use a 1.5 GHz pillbox cavity
  - Superconductor:  always bulk niobium *(Other materials/thin films to Tutorial 6a/b)*
  - Some equations, most you can forget again.  Important ones are marked in yellow

# Making a cavity

- For acceleration we require an oscillating RF field
- Simplest form is an LC circuit
- Let L = 0.1 mH, C = 0.01 µF → f = 160 kHz
- To increase the frequency, lower L, eventually only have a single wire
- To reach even lower values must add inductances in parallel
- Eventually have we have a solid wall
- Shorten „wires“ even further to reduce inductance
- → Pillbox cavity, „simplest form“
- Add beam tubes to let the particles enter and exit

- Magnetic field concentrated in the cavity wall, losses will be here.

Based on Feynman‘s Lect. on Physics.



# Cavity Modes

- **Fields in the cavity are solutions to the wave equation**

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = 0$$

- **Subject to the boundary conditions**

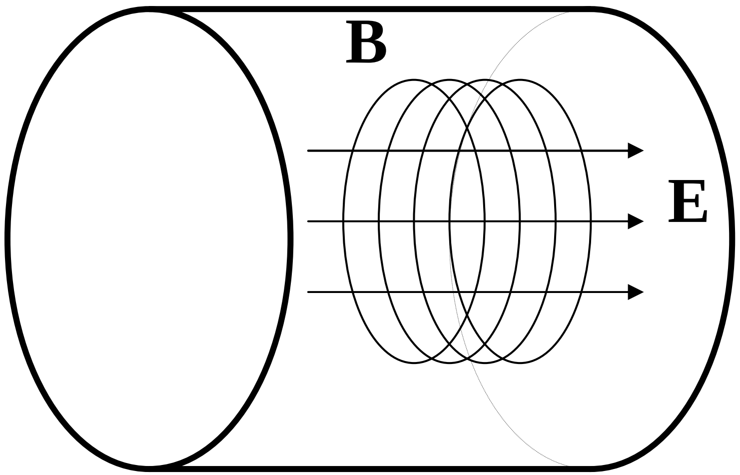
$$\hat{n}\times\mathbf{E} = 0, \quad \hat{n}\cdot\mathbf{H} = 0$$

- **Solutions are two families of modes with different eigenfrequencies**
  - TE modes have only transverse electric fields
  - TM modes have only transverse magnetic fields (but longitudinal component for **E**)
- **TM modes are needed for acceleration. Choose the one with the lowest frequency ($TM_{010}$)**
- **For pillbox (no beam tubes) solution is:**

$$E_z = E_0 J_0\left(\frac{2.405\rho}{R}\right)e^{-i\omega t}$$

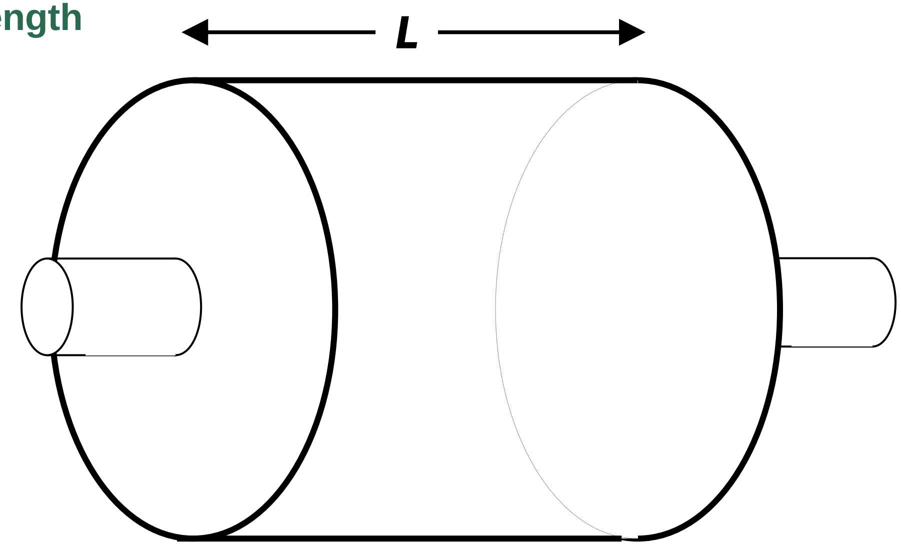
$$H_\phi = -i\frac{E_0}{\eta}J_1\left(\frac{2.405\rho}{R}\right)e^{-i\omega t}$$

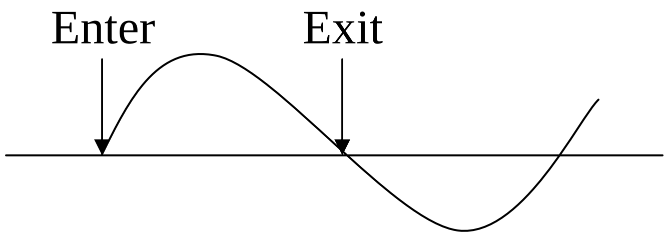
$$\omega_{010} = \frac{2.405c}{R} \qquad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$




- **Note that the frequency scales inversely with the linear dimension of the cavity (call this „*a*“)**

- **Optimizing Cavity Length**




$$T_{\text{transit}} = \frac{L_{\text{acc}}}{\beta c} = \frac{T_{RF}}{2} \quad \Rightarrow L_{\text{acc}} = \frac{\beta c T_{RF}}{2}$$

***E.g.: For 1.5 GHz cavity and speed of light electrons (ß = 1), $L_{acc}$ = 10 cm***

# Figure of Merit: Accelerating Voltage and peak fields

- **How much energy gain can we expect?**
- **Integrate the E-field at the particle position as it traverses the cavity:**

$$V_c = \left| \int_0^d E_z(\rho = 0, z) e^{i\omega_0 z/c} dz \right|$$ (assume speed of light electrons)

$$L_{acc} = \frac{cT_{RF}}{2}$$

- **For the pillbox cavity this is**

$$V_c = E_0 \left| \int_0^L \exp\left(\frac{i\omega_0 z}{c}\right) dz \right| = LE_0 \frac{\sin\left(\frac{\omega_0 L}{2c}\right)}{\frac{\omega_0 L}{2c}} = \frac{2}{\pi} E_0 L$$

- **We can define the accelerating field as** $E_{acc} = \frac{V_c}{L} = \frac{2}{\pi} E_0$

- **Important for the cavity performance is the ratio of the peak fields to the accelerating field.**
- **Ideally these should be relatively small to limit losses and other trouble at high fields**

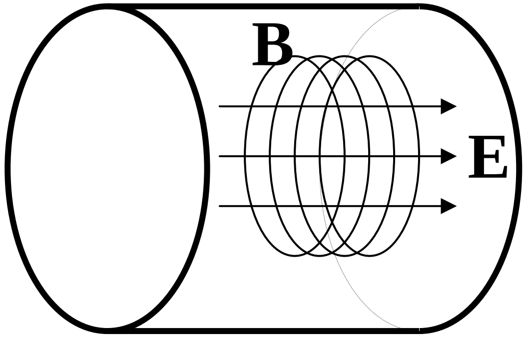
$$E_z = E_0 J_0\left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

ore like 2

$$H_\phi = -i \frac{E_0}{\eta} J_1\left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

$L_{acc}$ ly more like 3600

# Power dissipation in a cavity




- **Tangential magnetic fields exist at the cavity wall**

→ **By Maxwell's equation** $\vec{\nabla}\times\vec{B}=\mu\vec{J}+\mu\varepsilon\frac{\partial\vec{E}}{\partial t}$ **currents must flow.**

- **Current density is proportional to the magnetic field**
- **If the material is lossy, this will lead to power dissipation**
  - (one reason why one may want a low ratio of magnetic field to accelerating field)
- **By Ohm's law one can define a surface resistance such that the power dissipated per unit area is given by:**

$$\frac{dP_c}{ds}=\frac{1}{2}R_s|\mathbf{H}|^2$$

- **The total power dissipated in the cavity is given by the integral over the surface:**

$$P_c=\frac{1}{2}R_s\int_S|\mathbf{H}|^2\,ds$$

- **How does this compare to the energy stored in the cavity?**
- **Define the cavity quality as:**

$$Q_0 = 2\pi \frac{\text{Energy stored in the cavity}}{\text{Energy dissipated in one RF cycle}} \approx 2\pi \times \text{Number of cylces to dissipate the stored energy}$$

- **(Note: Easy quantity to measure. Just fill the cavity with energy, switch off and count the number of cycles it takes to dissipate the energy)**

- **The stored energy is:** $U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2\, dv$
- **Hence** $$Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2\, dv}{R_s \int_S |\mathbf{H}|^2\, ds}$$
- **Note that** $$\frac{\int_V |\mathbf{H}|^2\, dv}{\int_S |\mathbf{H}|^2\, ds} \propto a \propto \frac{1}{\omega_0}$$
- **And hence** $$Q_0 = \frac{G}{R_s}$$

$$\text{For a pillbox}: G = \frac{453\Omega \frac{d}{R}}{1 + \frac{d}{R}} = 257\Omega$$

**where *G* is the geometry factor which only depends on the cavity shape!**

# Figure of Merit: Shunt Impedance

- **The cavity quality: useful value for the performance of the cavity, measures how lossy the cavity material is**
- **But really we want to know how much power is dissipated to accelerate the charges.**
- **Hence one defines a *shunt impedance:***

$$R_a = \frac{V_c^2}{P_{diss}}$$

- **The higher the shunt impedance, the more acceleration we get per watt of dissipation**

- **A very useful quantity is generated by dividing by the quality factor:**

$$\frac{R_a}{Q_0} = \frac{V_c^2}{\omega_0 U} \Rightarrow P_{diss} = \frac{V_c^2}{\frac{R_a}{Q_0} \times Q_0} \Rightarrow P_{diss} = \frac{V_c^2}{\frac{R_a}{Q_0} \times G} R_s$$

Operating parameter, given by Accelerator ($V_c^2$)

Cavity material ($R_s$)

Cavity geometry ($\frac{R_a}{Q_0} \times G$)

- **Why? Because**

$$\left.\begin{aligned} V_c^2 &\propto a^2 \propto \frac{1}{\omega_0^2} \\ U &\propto a^3 \propto \frac{1}{\omega_0^3} \end{aligned}\right\} \Rightarrow \frac{V_c^2}{U} \propto \omega_0 \Rightarrow \frac{R}{Q_0}$$

**Depends only on the cavity shape but not its size (frequency) or material!**

- **Pillbox:**

$$\frac{R}{Q_0} = 150\Omega \frac{d}{R} = 196\Omega$$

- **Lets calculate one example: Want to operate a 1.5 GHz Pillbox at 1 MV**
- 

For copper cavities, power dissipation is a huge constraint
→ Cavity design is driven by this fact

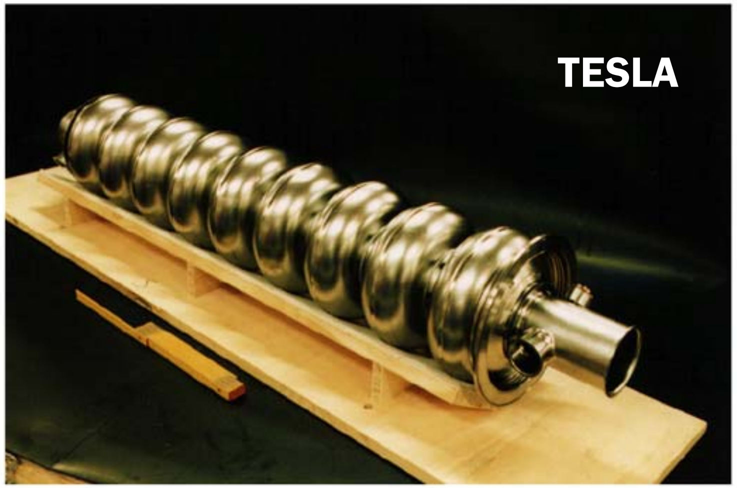
***For a clock pendulum (1 sec): 815 years!***

For SC cavities, power dissipation is minimal
→ decouples the cavity design from the dynamic losses
→ free to adapt design to specific application

$$E_{acc} = \frac{}{L_{acc}} = 10 \frac{}{\mathrm{m}} \qquad \Rightarrow E_{pk} = \frac{}{2} E_{acc} = 15.7 \frac{}{\mathrm{m}}$$

$$H_{pk} = 2430 \frac{\mathrm{A}}{\mathrm{m}} E_{acc} = 24300 \frac{\mathrm{A}}{\mathrm{m}}, B_{pk} = 31\mathrm{mT}$$

# Difference between NC and SC cavities


- **NLC design developed to reduce power dissipation to a minimum**
- **But many other areas are impacted in a negative way**
  - E.g., → Strong wakefields are created → impact beam dynamics
  - Small size → extremely tight tolerances
- **TELSA design**
- **Power dissipation less critical**
  - → Choose design that relaxes wakefields
  - Still: heat is deposited in LHe → cost issue that must be understood

# What comes now

- Clearly, cavity losses strongly impact the design/operation of the cavity
- Will analyze the behavior of normal-conducting and superconducting RF losses
- Look at scaling laws: frequency, temperature, material purity …

- Then turn to the real world, look at deviation from the ideal
  - Residual losses
  - Trapped magnetic flux
  - The Q-disease

- How far can we push a superconducting cavity?
- Theoretical Limit of superconductors

# Calculating RF losses in a conductor

- For simplicity, use the nearly-free electron model
- Losses given by Ohm's law $\mathbf{j} = \sigma\mathbf{E} = \frac{n_n e^2 \tau}{m}\mathbf{E}, \qquad \tau = \text{scattering time}$
- The electrons have a time $\tau$ between scattering events to gain energy $\Delta\mathbf{v} = \frac{-e\mathbf{E}\tau}{m}$

- In a cavity, the magnetic field drives an oscillating current in the wall
  - → Start with Maxwell's equations

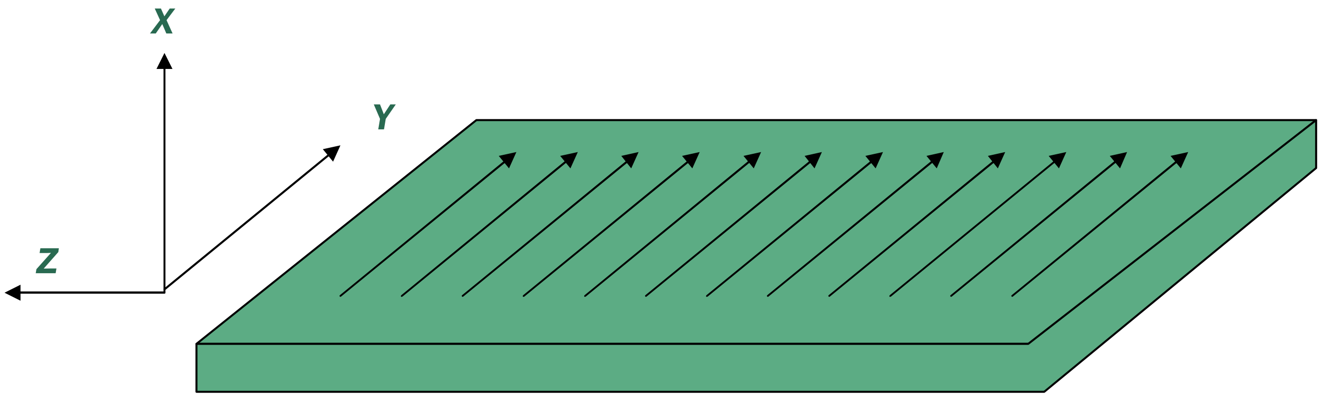
$$\nabla\times\mathbf{B} = \mu\mathbf{j} + \mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

- Combine the two and take the exp(iωt) dependence into account

$$-\nabla^2\mathbf{B} = \mu\nabla\times\mathbf{j} - \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial^2 t} = -i\mu\sigma\omega\mathbf{B} + \mu\varepsilon\omega^2\mathbf{B}$$

- Look at a typical copper RF cavity: $\sigma = 5.8\times10^7\,\frac{\text{A}}{\text{Vm}} \qquad \omega\varepsilon_0 = 0.08\,\frac{\text{A}}{\text{Vm}}\ \text{at 1.5 GHz}$

# Field near a conductor




**Consider now a uniform magnetic field (y-direction) at the surface of a conductor.**

**Solving** $\nabla^2 \mathbf{B} - i\mu\sigma\omega\mathbf{B} = 0$

**yields** $H_y = H_0 e^{-x/\delta} e^{-ix/\delta}$

**where the field decays into the conductor with over a skin depth of** $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

**Similarly, from Maxwell find that** $E_z = -\frac{(1+i)}{\sigma\delta} H_y$

**So that a small, tangential component of *E* also exists which decays into the conductor**

# Cavity losses due to the RF field

$$H_y = H_0 e^{-x/\delta} e^{-ix/\delta} \qquad E_z = -\frac{(1+i)}{\sigma\delta} H_y$$

- The losses per area are simply $P'_{\text{diss}} = \frac{1}{2} \int_{x=0}^{\infty} J_z^* E_z dx = \frac{1}{2} \int_{x=0}^{\infty} \sigma |E_z|^2 dx$

$$= \frac{1}{2\sigma\delta} H_0^2 \qquad = \frac{1}{2} R_s H_0^2 \qquad R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

- Note: Surface resistance is just the real part of the surface impedance:

$$Z_s = \frac{E_z}{H_y} = \frac{1+i}{\sigma\delta} = (1+i)\sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \exp(i\pi/4)$$

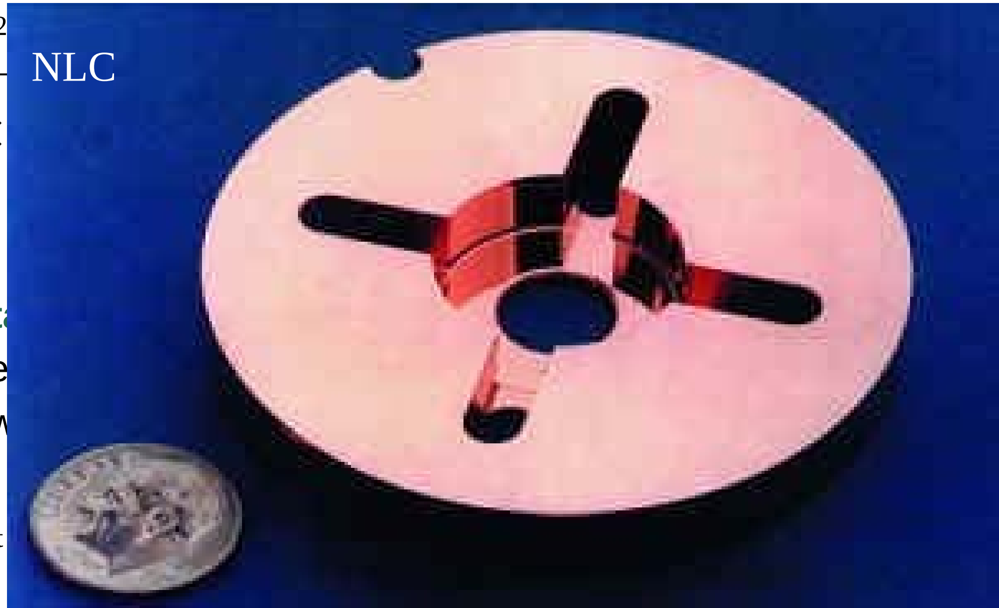
- Plug in some numbers:
- Copper: $f$ = 1.5 GHz, σ = 5.8 x $10^7$ A/Vm, $\mu_0$ = 1.26 x $10^{-6}$ Vs/Am

→ δ = 1.7 µm, $R_s$ = 10 mΩ
→ $Q_0$ = G/Rs = 25700

# Frequency scaling of the losses

- **Note that for** $R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$ **, the surface resistance scales as** $\sqrt{\omega}$

- **Recall** $P_{diss} = \frac{V_c^2}{\frac{R_a}{Q_0}\times}$




- **But consider the tot... nac):**
  - For a total voltage ... le
  - Thus the total pow...

$P_{tot}$ ... $Q_0$ ... $Q_0$

All geometry

  - Since $L_{acc} \propto \frac{1}{\omega}$
  - The total power dissipation scales as: $P_{tot} \propto \frac{1}{\sqrt{\omega}}$ ***→ For NC systems high frequencies are desirable (But beam dynamics may suffer!)***

# Losses in a superconductor: Two-fluid model

**Two fluid model, must consider both sc and nc components:**

- **Below $T_c$ superconducting cooper pairs are formed with an energy gap 2Δ**
- **The density of remaining „normal“ electrons is given by**

$$n_n \propto \exp\left(\frac{-\Delta}{k_B T}\right)$$

- **DC case: The lossless Cooper pairs short out the field**

  **→ the normal electrons are not accelerated**

  **→ the SC is lossless even for $T > 0$ K**

# Losses in a superconductor: RF case

- What's different for the RF case?

- Cooper pairs have inertia!
  → they cannot follow an AC field instantly and thus do not shield it perfectly
  → a residual field remains
  → the normal electrons are accelerated and dissipate power

- Scalings of the surface resistance:

- The faster the field oscillates the less perfect the shielding

→ We expect the surface resistance to increase with frequency

- The more normal electrons exist, the lossier the material

→ We expect the surface resistance to drop exponentially below $T_c$

- Calculate surface impedance of a superconductor
  → Must take into account the „superconducting“ electrons ($n_s$) in the 2-fluid model
- For these there is no scattering
- Thus: $\mathbf{j}_s = n_s e \mathbf{v}$

$$m\frac{\partial \mathbf{v}}{\partial t} = -e\mathbf{E} \qquad \Rightarrow \frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m}\mathbf{E}$$

First London Equation

- In an RF field with exp(iωt) dependance $\Rightarrow \mathbf{j}_s = -i\frac{n_s e^2}{m\omega}\mathbf{E}$

Acts as the AC conductivity of the superconducting fluid. „Collision time“ is the RF period.

or $\mathbf{j}_s = \frac{-i}{\omega\mu_0\lambda_L^2}\mathbf{E}$ where $\lambda_L = \frac{m}{\mu_0 n_s e^2}$ is the London penetration depth

- Total current: Just add the currents due to both „fluids“: $\mathbf{j} = \mathbf{j}_n + \mathbf{j}_s = (\sigma_n - i\sigma_s)\mathbf{E}$

# Surface impedance of superconductors

- Thus, the treatment with a superconductor is the same as before, only that we have to change: $\sigma \rightarrow (\sigma_{\mathrm{n}} - i\sigma_{\mathrm{s}})$
- Impedance $Z_{\mathrm{s}} = \sqrt{\frac{\omega\mu}{\sigma}} \exp\left(i\pi/4\right) \rightarrow \sqrt{\frac{\omega\mu}{\sigma_{\mathrm{n}} - i\sigma_{\mathrm{s}}}} \exp\left(i\pi/4\right)$
- Penetration depth $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \rightarrow \frac{1}{\sqrt{\pi f \mu (\sigma_{\mathrm{n}} - i\sigma_{\mathrm{s}})}}$
- Where $H_y = H_0 \exp\left(-\frac{(1+i)}{\delta} x\right), \quad \sigma_{\mathrm{n}} = \frac{n_{\mathrm{n}} e^2 \tau}{m}$ and $\sigma_{\mathrm{s}} = \frac{n_{\mathrm{s}} e^2}{m\omega}$
- Note that 1/ω is of order 100 ps whereas for normal conducting electrons *τ* is of order few 10 fs. Also, $n_s >> n_n$ for $T << T_c$. Hence $\sigma_{\mathrm{n}} << \sigma_{\mathrm{s}}$
- As a result one finds that: $\delta \approx (1+i)\lambda_L \left(1 + i\frac{\sigma_{\mathrm{n}}}{2\sigma_{\mathrm{s}}}\right)$ $\quad H_y = H_0 e^{-x/\lambda_{\mathrm{L}}} e^{-ix\sigma_{\mathrm{n}}/2\sigma_{\mathrm{s}}\lambda_{\mathrm{L}}}$
- Again, the field decays rapidly but now over the London penetration depth

*Surface resistance of the superconductor*

- For the impedance we get: $Z_s \approx \sqrt{\frac{\omega\mu}{\sigma_s}}\left(\sigma_n / 2\sigma_s + i\right)$ $\quad X_s = \omega\mu_0\lambda_L$ $\quad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0^2\lambda_L^3$
- Lets look at some numbers:

  For niobium $\lambda_L$ = 36 nm, for Copper the penetration depth was 1.7 µm (@ 1.5 GHz)

  → The field penetrates over a much shorter distance than for a normal conductor
- At 1.5 GHz: $X_s$ = 0.43 mΩ, whereas $R_s$ is < 1µΩ

  → The superconductor is mostly reactive in line with our previous explanation of losses in a superconductor

# Frequency scaling of the surface resistance

- Note $R_s = \frac{1}{2}\sigma_n \omega^2 \mu_0^2 \lambda_L^3$

→ The surface resistance scales quadratically with frequency, also in agreement with our previous analysis

- Recall that the total dissipated power for all accelerating cavities was given by

$$P_{tot} = \text{frequency independant stuff} \times \frac{R_s}{\omega}$$

- Hence for a superconductor $P_{tot} \propto \omega$

  → Favors low-frequency cavities if cryogenic power is an issue.


# Temperature scaling of the surface resistance

$$R_s = \frac{1}{2}\sigma_n \omega^2 \mu_0^2 \lambda_L^3$$

- The surface resistance is proportional to the conductivity of the normal fluid!

  → If the normal-state resistivity is low, the superconductor is more lossy!

- Explanation: For „residual“ field not shielded by the Cooper pairs more „normal current“ flows → more dissipation

$$P_{diss} \propto \sigma_n E^2$$

- *Temperature dependance*: Below $T_c$, electrons condense into the superconducting state.
- In the previous tutorial we saw for the normal fluid:

$$n_n \propto \exp\left(-\Delta(T) / k_B T\right) \approx \exp\left(-1.86 T_c / T\right)$$

→ Conductivity is $\sigma_n \propto \ell \exp\left(-1.86 T_c / T\right)$

*Property of the superconductor*

*Property of cooling*

- Hence the SC surface resistance is given by

$$R_s \propto \omega^2 \lambda_L^3 \ell \exp\left(-1.86 T_c / T\right)$$

$$R_s \propto \omega^2 \lambda_L^3 \ell \exp\left(-1.86T_c / T\right)$$

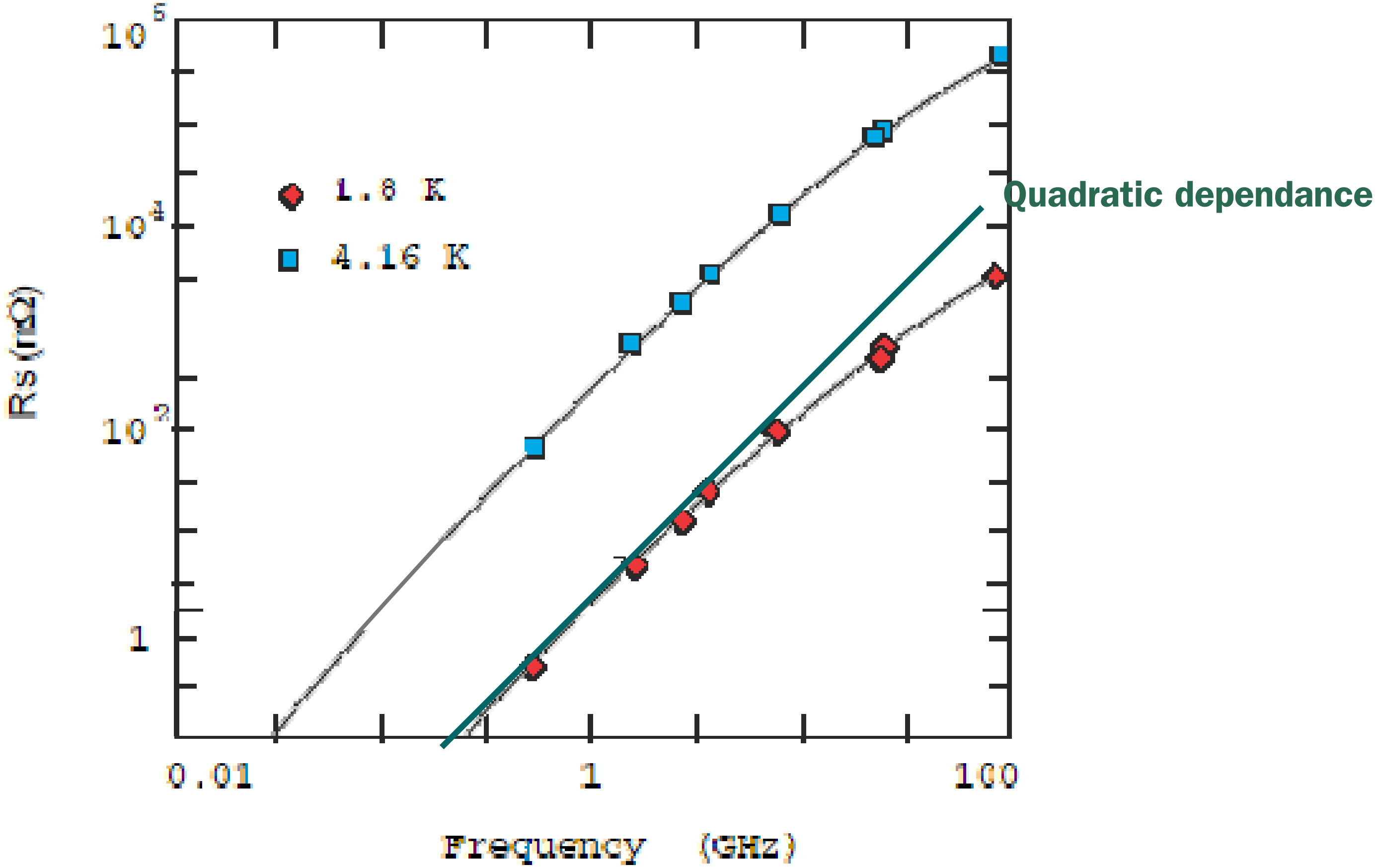
**The surface resistance**

- Increases quadratically with frequency → use low frequency cavities
- Decreases exponentially with temperature → stay well below $T_c$
- Increases with increasing purity of the material → ~~use impure materials~~

No! This statment breaks down for very impure SC + there are compelling arguments to use high-purity material (see later and turorial 4b!)

# Frequency scaling

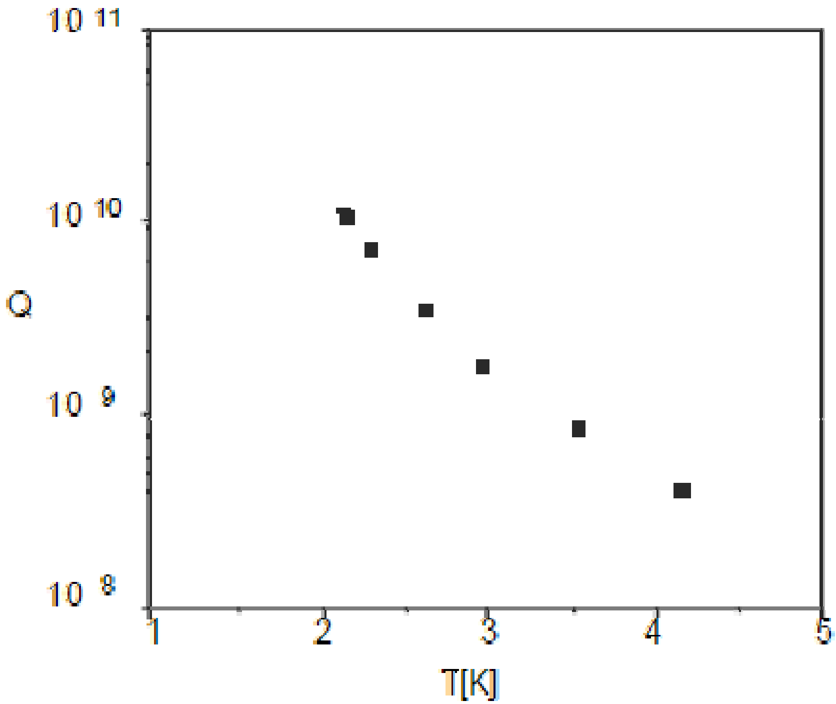
- **Measurements at 4.2 K and 1.8 K confirm the frequency dependance.**
- **Slight deviation at high frequencies due to anisotropy of niobium**




*U. Klein, Thesis, Wuppertal Univ., WUB–DI 81–2 (1981)*

# Temperature scaling

- Exponential dependance confirmed experimentally
- Measure Q factor of a cavity v. temperature
- Calculate surface resistance = $G/Q_0$




H. Padamsee et. al, Cornell

# Impact of the purity of the superconductor

$$R_s \propto_n \omega^2 \lambda_L^3 \ell \exp\left( -1.86 T_c / T \right)$$

- **Surface resistance decreases as the mean free path decreases (less pure)**
- **This is only valid as long as the coherence length is << mean free path**

$$\xi_0 << \ell$$

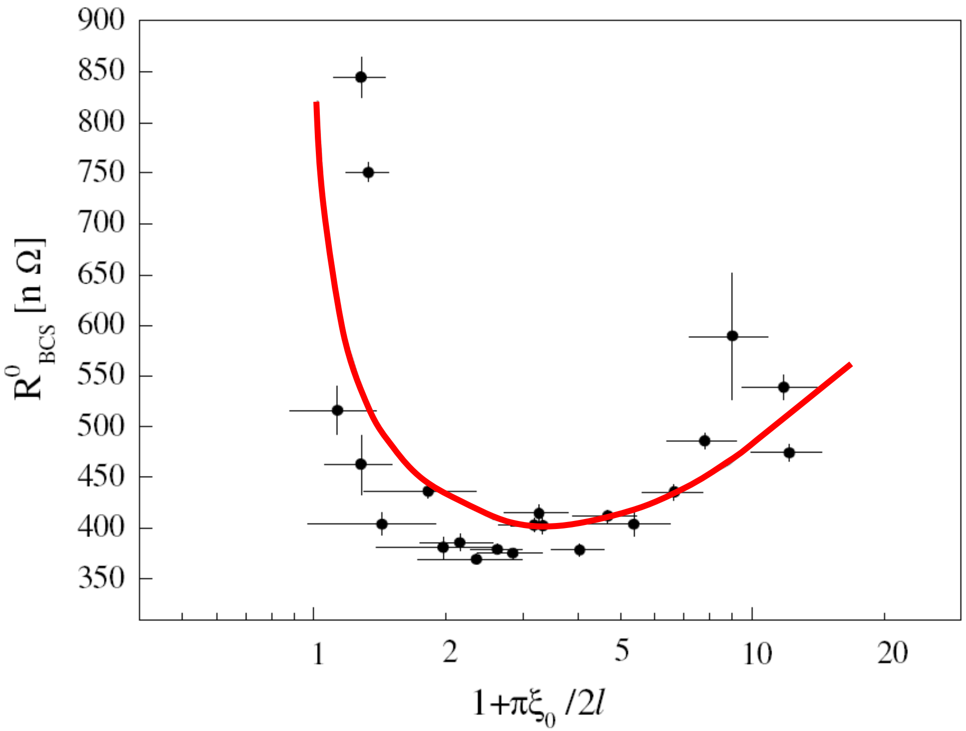
- **Otherwise the first London equation (local equation) breaks down.**
- **In that case must replace:**

$$\lambda_L \Rightarrow \Lambda_L = \lambda_L \sqrt{1 + \frac{\xi_0}{\ell}}$$

- **And thus the surface resistance increases when** $\ell \le \xi_0 = 64\text{nm}$

# Impact of purity of superconductor

- Measurements have confirmed the general dependance on purity
- Sputtered niobium on copper
- By changing the sputtering species, the mean free path was varied (see Tutorial 6 (?))



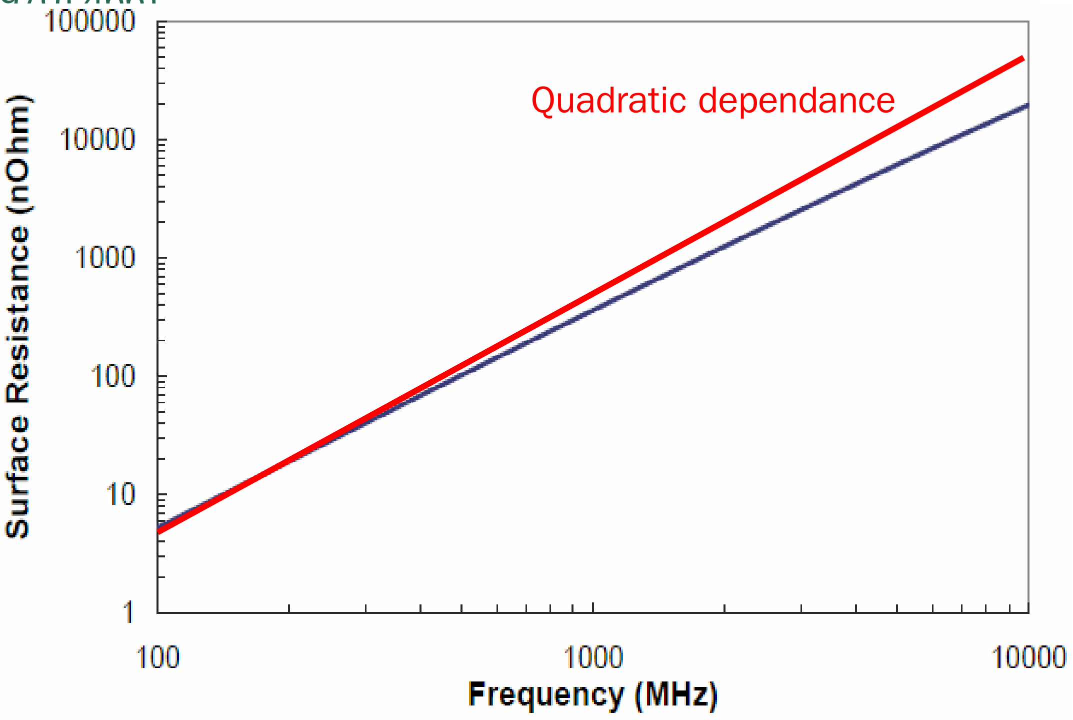

*C. Benvenuti et. al, Physica C 316 (1999)*

# Calculating the surface resistance

- **Clearly, absolute calculation of surface resistance must take into account numerous parameters.**
- **Mattis & Bardeen developed theory based on BCS: „involves many tricky integrals“ ←HSP**
- **Approximate expression for Nb:**

$$R_{\mathrm{BCS}} \approx 2\times10^{-4}\,\Omega\left(\frac{f}{1500\,\mathrm{MHz}}\right)^2 \frac{1}{T}\exp\left(\frac{-17.67}{T}\right)$$

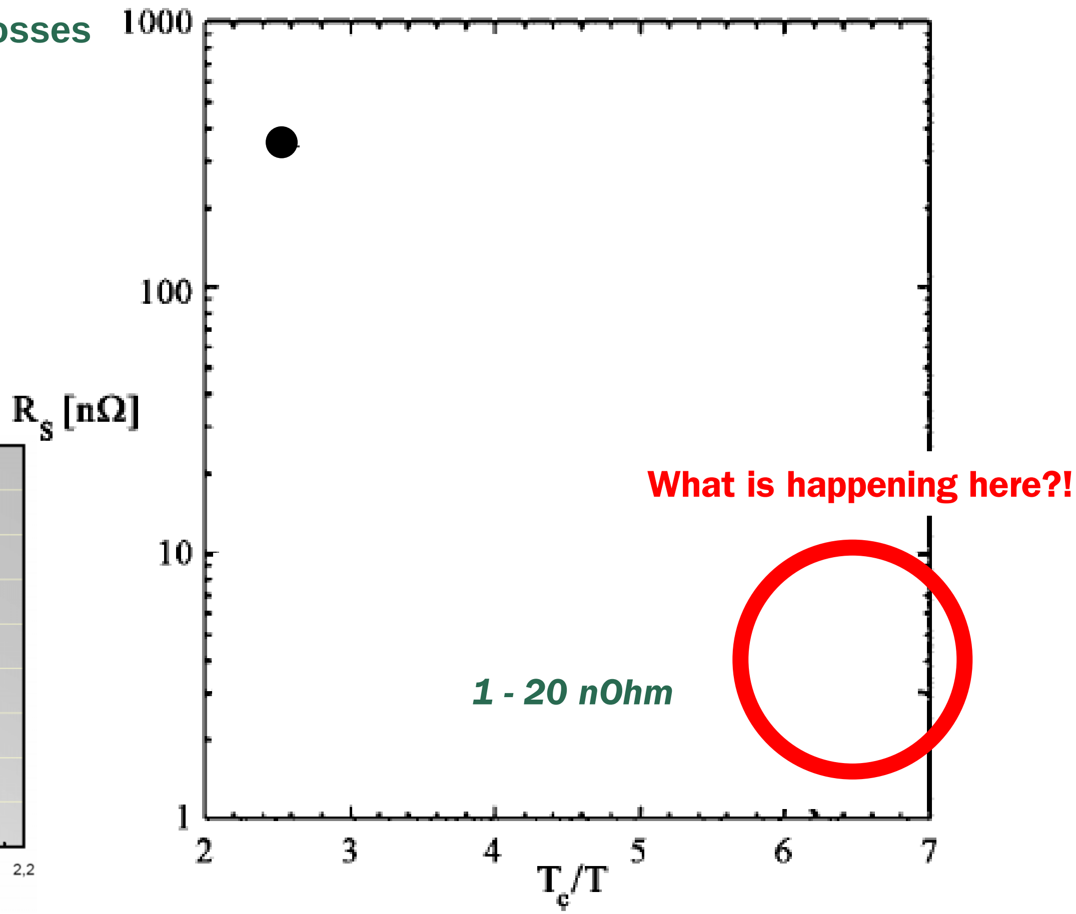
- **Program written by J. Halbritter to calculate resistance under wide range of conditions (**J. Halbritter, Zeitschrift für Physik 238 (1970) 466)
- **At Cornell: SHRIMP**
- **Must only supply a minimum number of parameters**
- **Effect of material purity included**
- **Frequency dependance calculated**

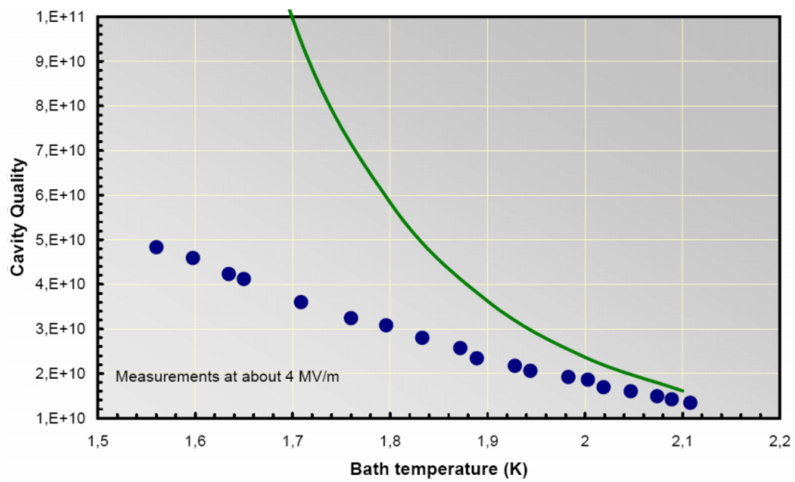



*H. Lengeler et al., IEEE Trans. Magn MAG 21 1014*

# Measured surface resistance

- Measured cavities display a behavior similar to the theoretical surface resistance
- Thus lowering the temperature further should always improve the dynamic losses
- But eventually the effect saturates
- Temperature independent term is called *Residual Resistance*




H. Padamsee, Supercond. Sci. Technol. **14** (2001) R28–R51

# Pause

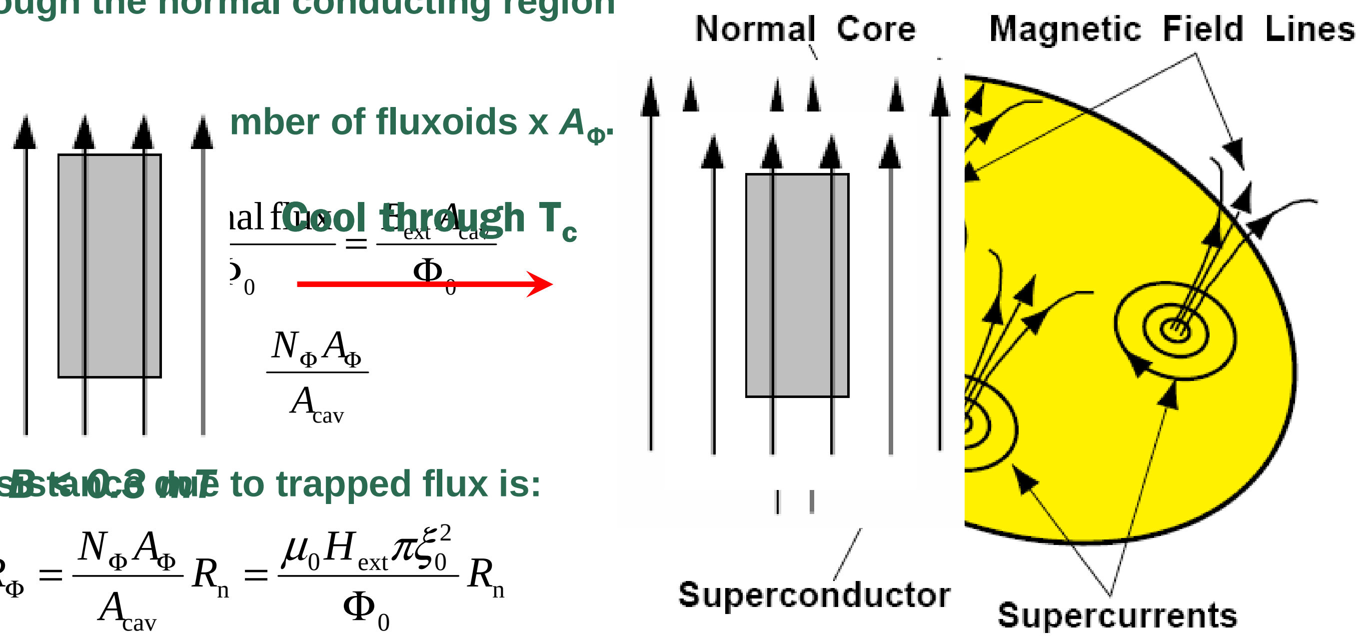
# Sources of residual resistance: Trapped flux

- **Theory: Below $H_{c1}$, a superconductor always is in the Meissner State**
- **Reality: Nb „traps" entire field if < 0.3 mT (even for high purity Nb)**
  - →Earths field (50 µT) would be completely trapped.

C. Valet et al., in *Proc. EPAC 1992, p. 1295*

- **The field penetrates the wall in „fluxoids" of flux** $\Phi_0 = \frac{\pi\hbar}{e}$ **and normal conducting area** $A_\Phi \approx \pi\xi_0^2$
- **The RF field „tugs" on the fluxoids and because of their motion a current flows through the normal conducting region**
- **Total area of the N… mber of fluxoids x $A_\Phi$.**
- **Number of fluxoid…** …al flux / $\Phi_0$ $= \frac{H_{ext} A_{cav}}{\Phi_0}$

**→ Fraction of surface…** $\frac{N_\Phi A_\Phi}{A_{cav}}$




**→ Effective surface res…** **…e to trapped flux is:**

$$R_\Phi = \frac{N_\Phi A_\Phi}{A_{cav}} R_n = \frac{\mu_0 H_{ext} \pi \xi_0^2}{\Phi_0} R_n$$

$$R_\Phi = \frac{\mu_0 H_{\text{ext}} \pi \xi_0^2}{\Phi_0} R_{\text{n}}$$

- **From the BCS theory we have:** $$H_{\text{c}2} = \frac{\Phi_0}{2\pi\mu_0\xi_0^2}$$
- **So that the contribution from trapped flux is simply:** $$R_\Phi \approx \frac{H_{\text{ext}}}{2H_{\text{c}2}} R_{\text{n}}$$
- **Note:**
  1. Normal surface resistance scales as √f
     → resistivity due to flux trapping increases with frequency
  2. Resistivity decreases with increasing critical field
     → Thin film superconductors (which have a much higher critical field) are less susceptible to trapped flux.

- **Some values: For Nb, $B_{c2}$ = 240 mT, at 1.5 GHz and 10 K, $R_n \approx$ 1.8 mΩ → $R_\Phi$ = 3.75 nΩ/μT**

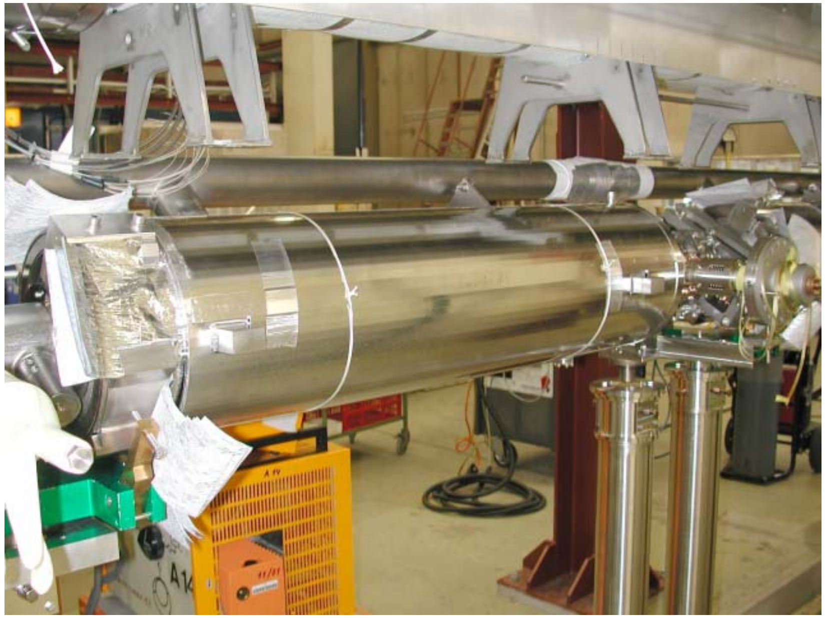
- **In general (for Nb)** $$R_\Phi \approx 3\frac{\text{n}\Omega}{\text{礣}} B_{\text{ext}} \sqrt{\frac{f}{1\text{GHz}}}$$

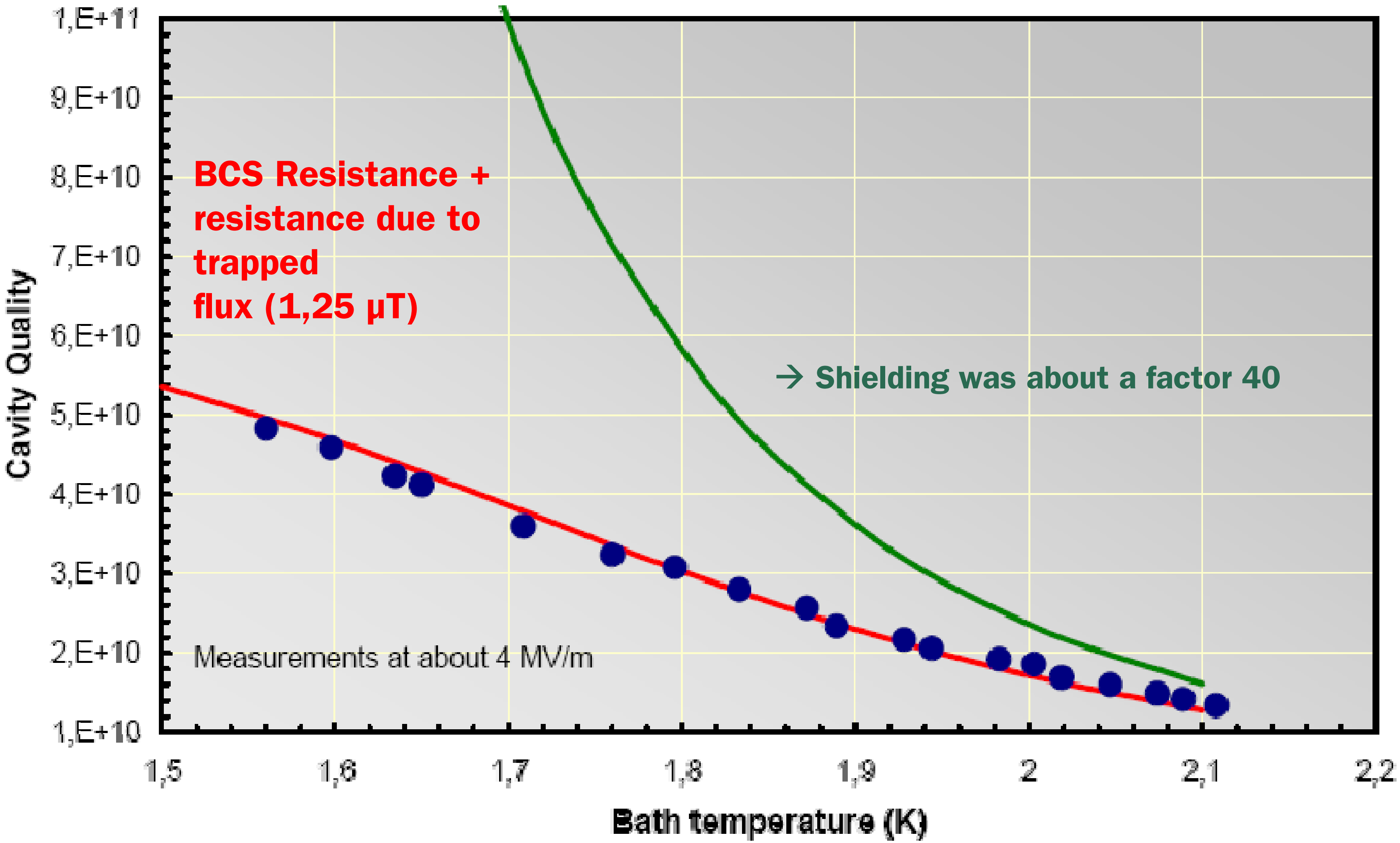
# Trapped flux due to earth's field

- Earth's field is 50 µT

  → Residual resistance (at 1.5 GHz) is = 175 nΩ
- Hence for a pillbox cavity $Q_0 < 1.5 \times 10^9$

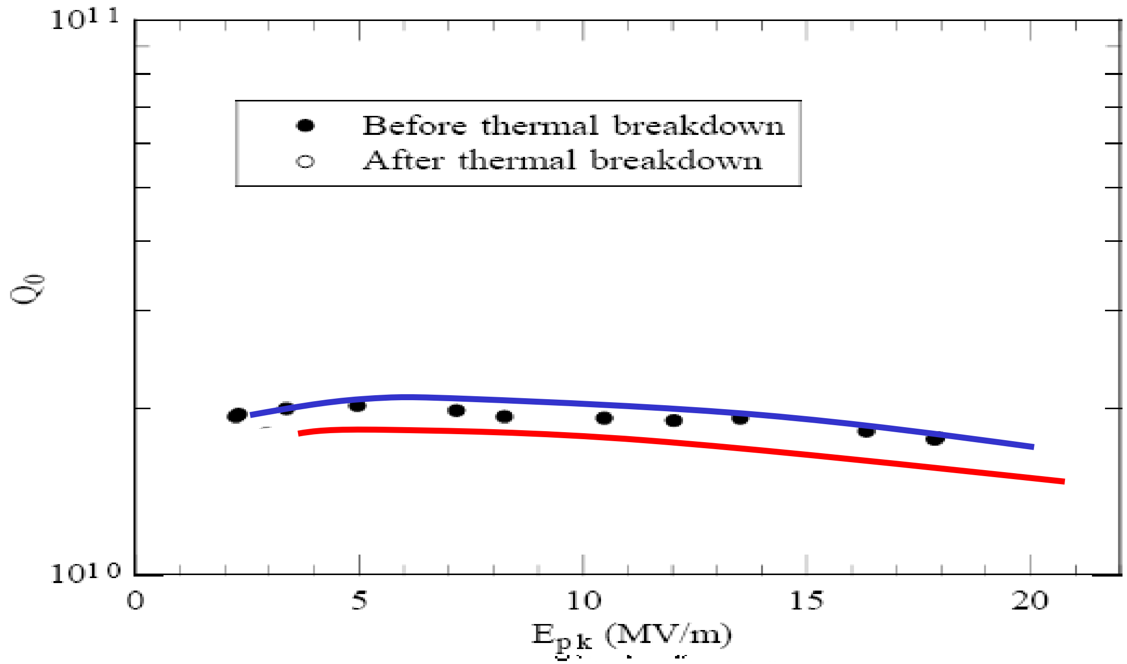
  → *To achieve Q factors in the $10^{10}$ range, the earth's field must be shielded by at least a factor 10 – 20.*
- Use µ-Metal for shielding
- \+ MAKE SURE NO MAGNETIC MATERIALS ARE NEAR THE CAVITY
- \+ Don't turn nearby magnets on until the cavity is superconducting




1,E+11
9,E+10
8,E+10
7,E+10
6,E+10
5,E+10
4,E+10
3,E+10
2,E+10
1,E+10
Cavity Quality
1,5
1,6
1,7
1,8
1,9
2
2,1
2,2
Bath temperature (K)
BCS Resistance + resistance due to trapped flux (1,25 µT)
→ Shielding was about a factor 40
Measurements at about 4 MV/m

- Sometimes, during cavity tests, one observes a quench
- Cavity is heated locally above $T_c$ due to
  - Defects on the surface
  - Electron bombardment from field emitters
  - Electron bombardment due to multipacting

  } Details are covered in Tutorial 4b
- When the heating becomes to strong it drives the cavity normal conducting
- After the quench the *Q*-factor often is reduced



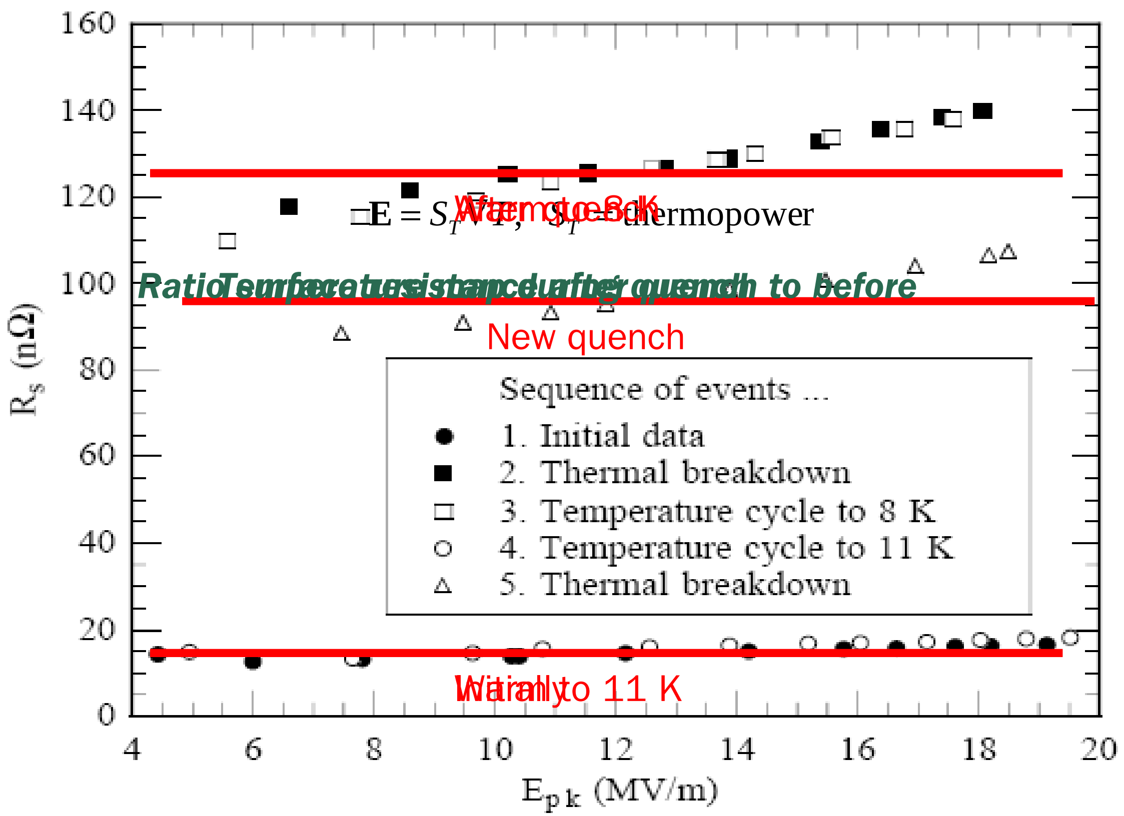
- **Perform measurements of the surface resistance in the region of the quench (with thermometry)**
- **After the quench, the surface resistance increased in this region**
- **Raising the temperature above $T_c$ eliminates the additional resistivity**

- **Explanation:**
- **This creates**

- **As the cavity**
- **Warming the**

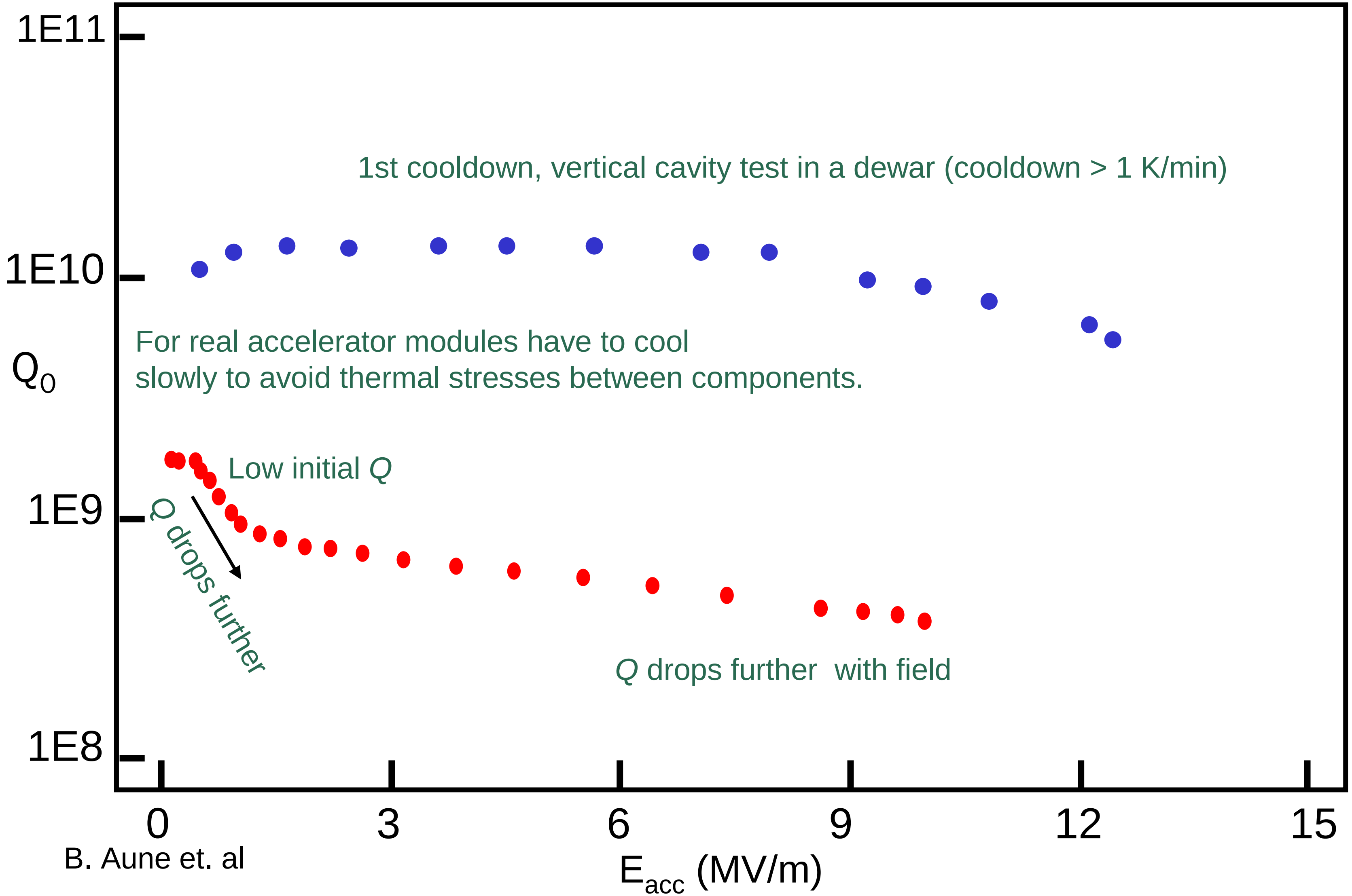
- **Typical value**
  - Temperatu
  - Electrical r
  - Thermopo
  - → current de

**→ Resistance du n)**



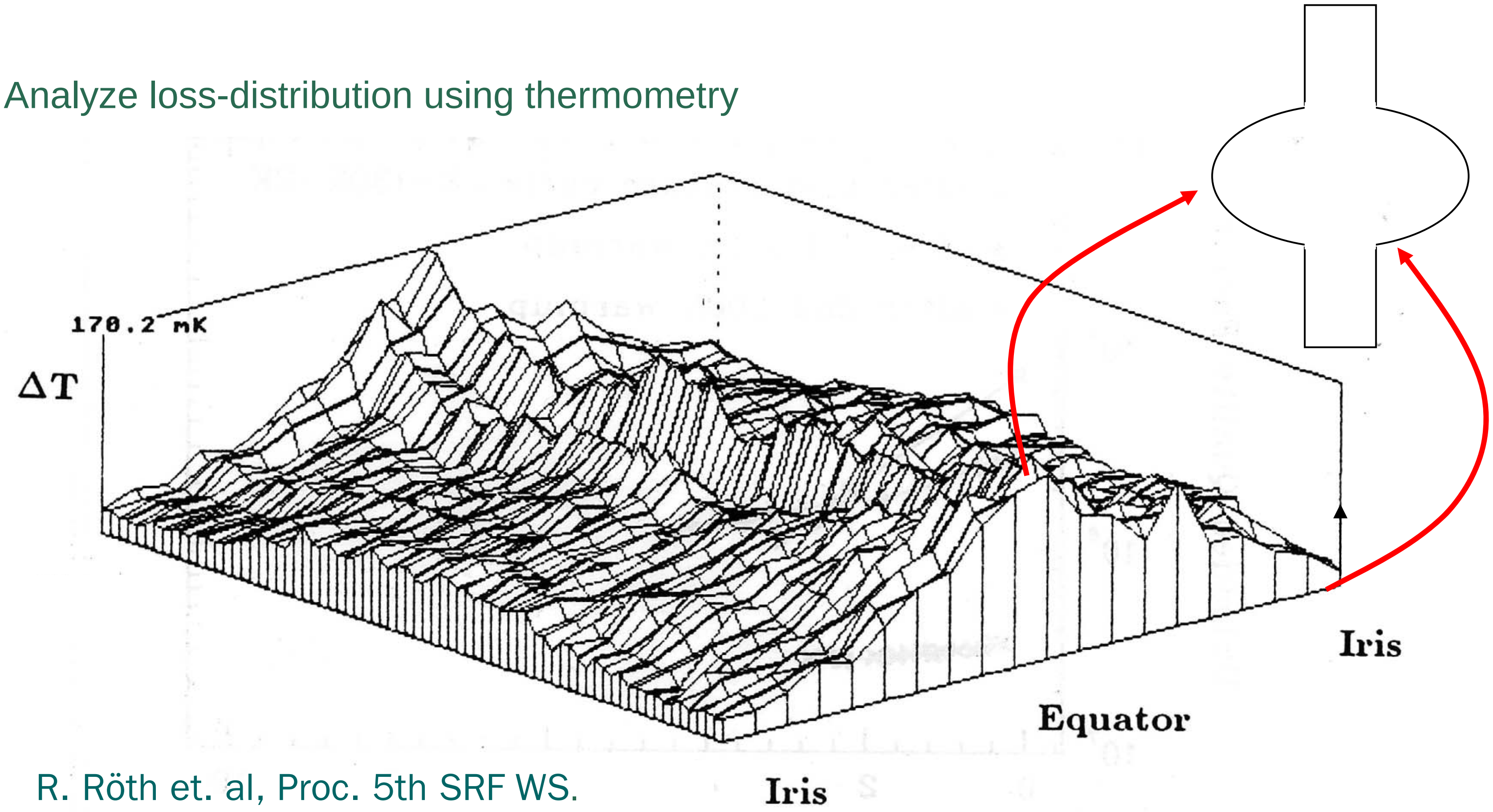
- In the 80‘s and early 90‘s it was found that sometimes a good cavity could go „bad“ when tests were repeated.
- This was especially the case when cavities were installed in „real“ accelerator modules
- This became known as the Q-disease
- What follows are some of the observations

# Losses due to hydrides: The *Q* disease




B. Aune et. al

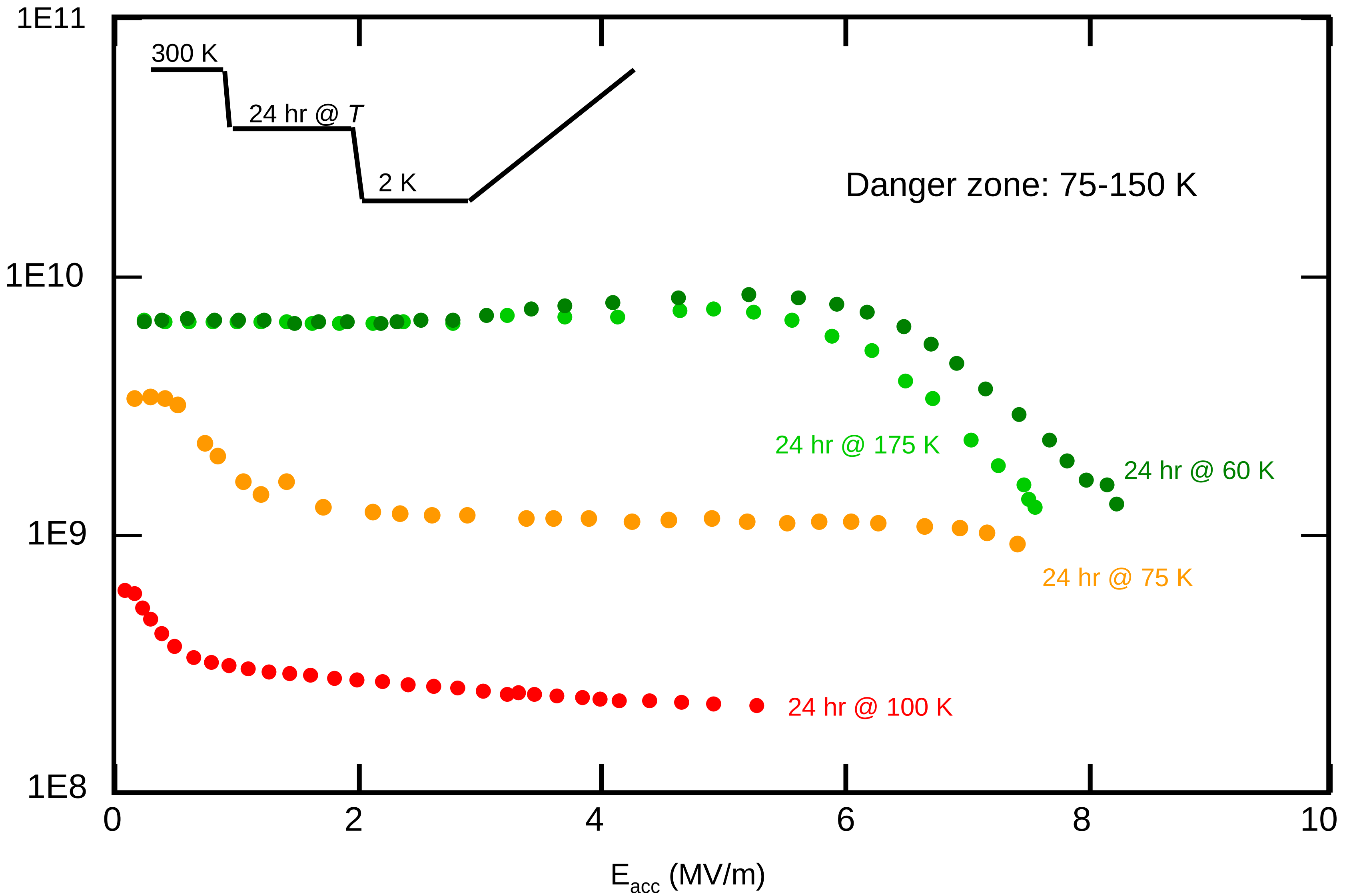
B. Aune et. al, *Proc. 1990 Linac Conference*, Analyze loss-distribution using thermometry




R. Röth et. al, Proc. 5th SRF WS.

Increased surface resistance is uniformly distributed (losses proportional to $H^2$)

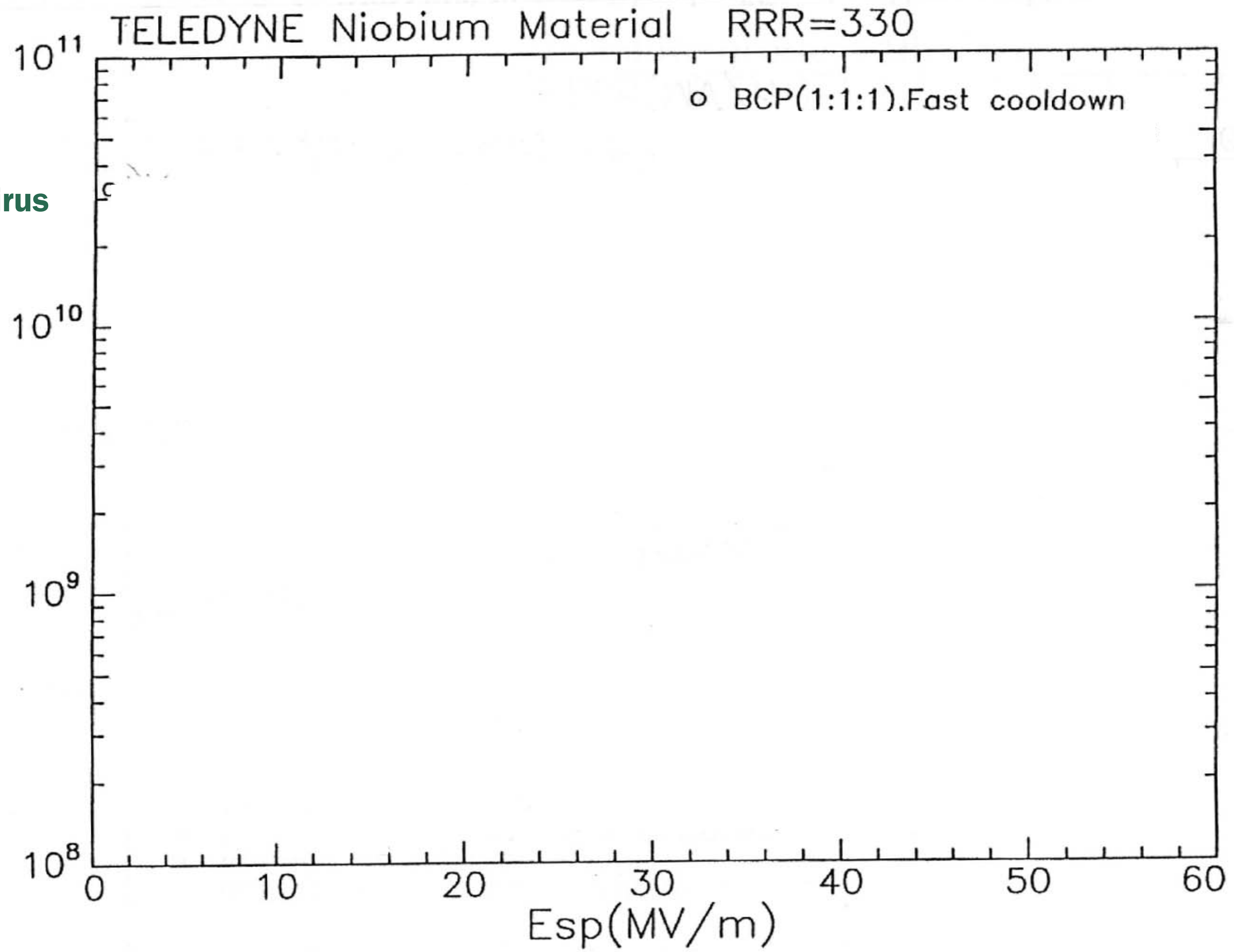
# The Danger Zone—100 K




J. Halbritter, P. Kneisel, K. Saito, Proc 6th SRF WS

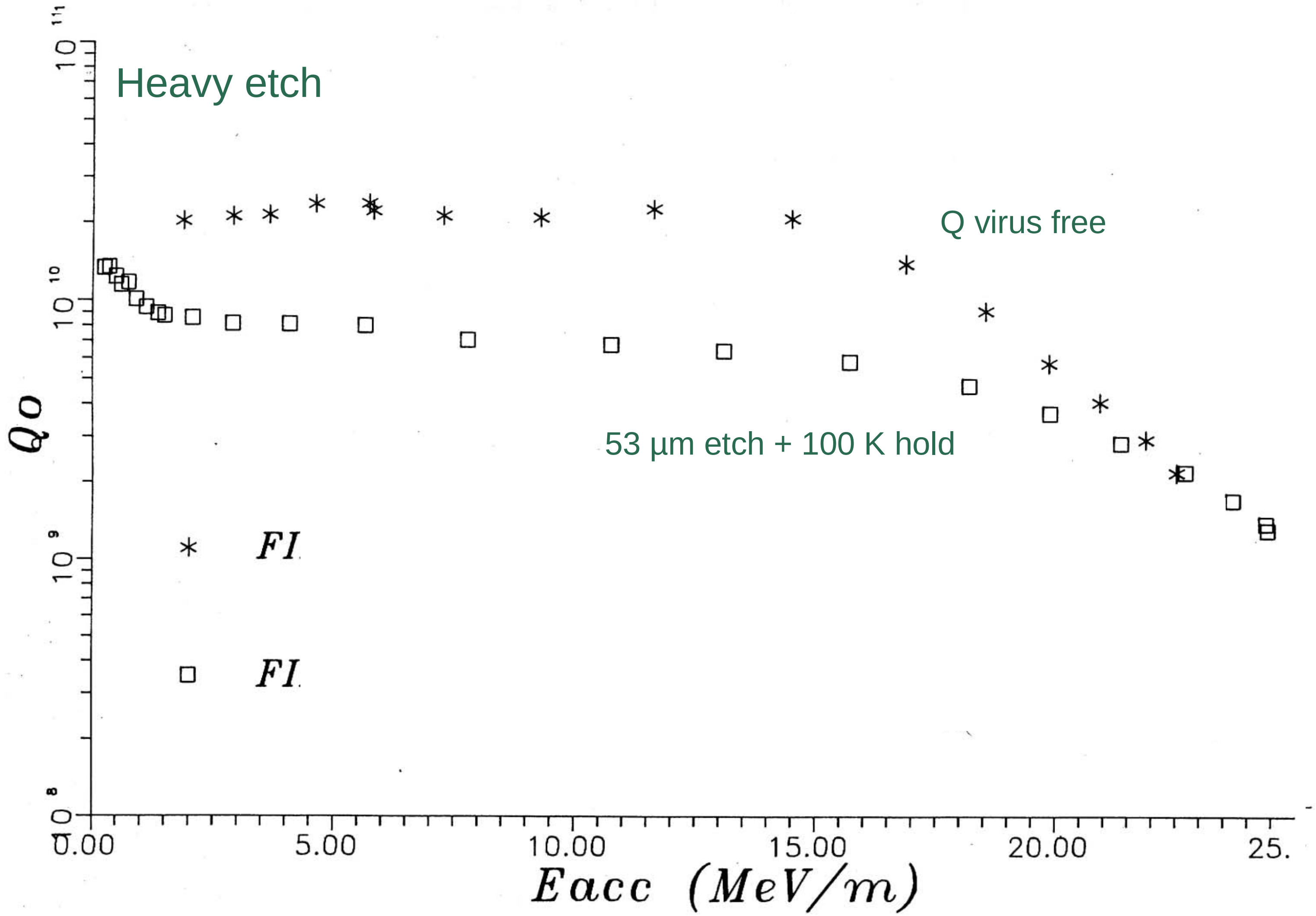
# Room-Temperature Cycle

**A room temperature cycle removes the Q-virus**




K. Saito & P. Kneisel

- **A heavy etch enhances the danger of the Q-virus**

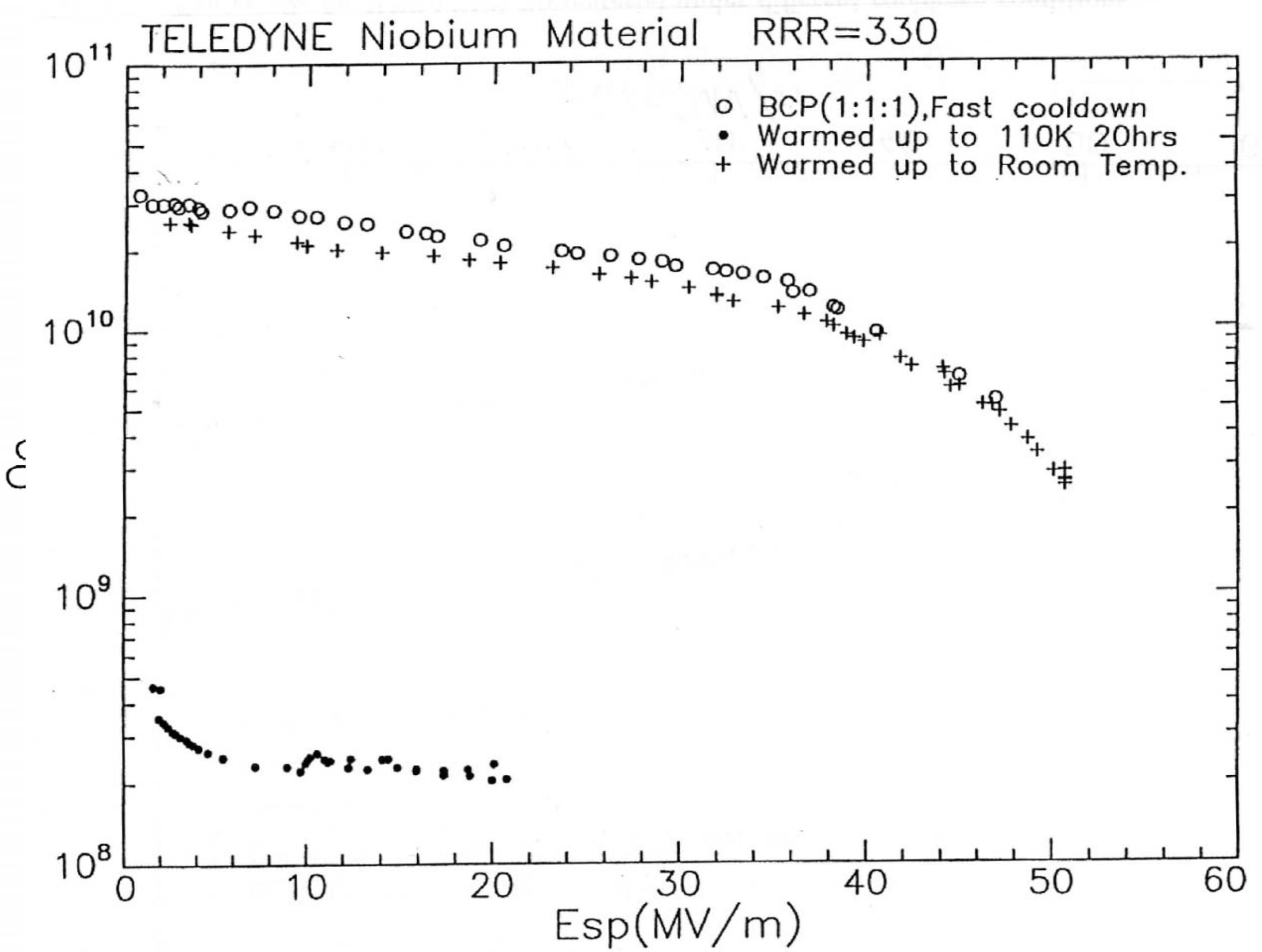




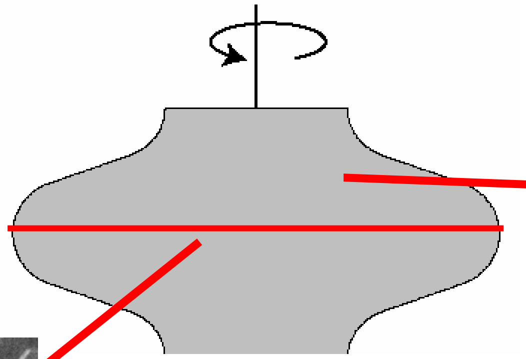

B. Bonin, B., and R. Röth, Proc. 5th SRF WS

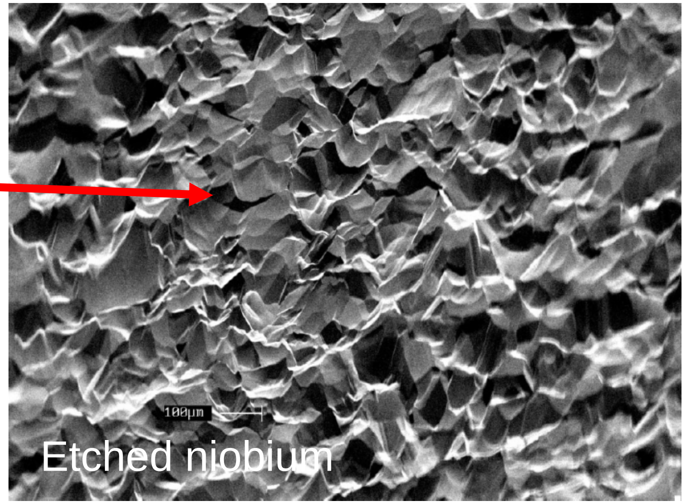
# Effect of Niobium Purity

- **High purity material is more susceptible**

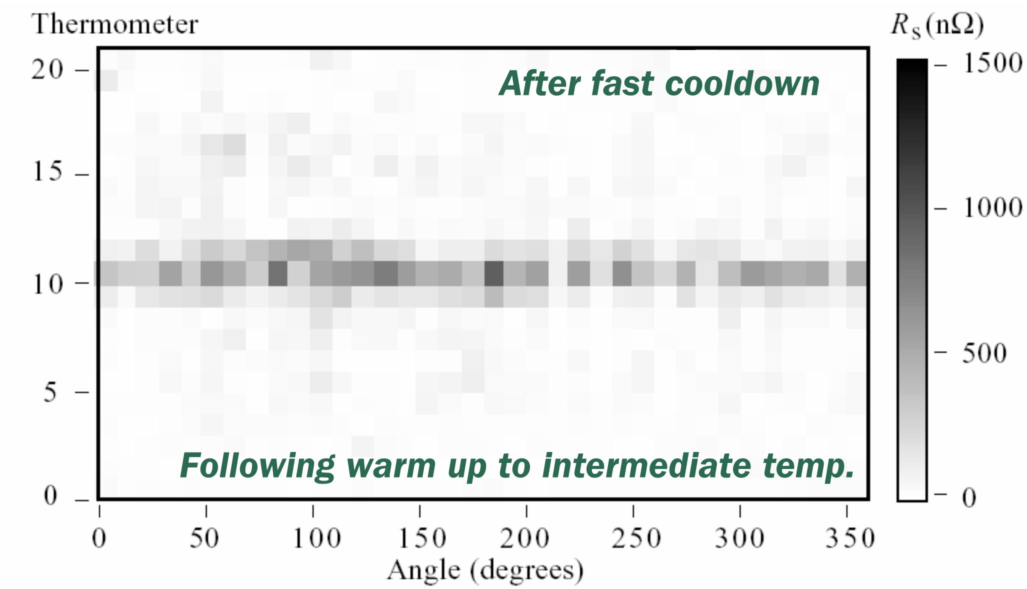



K. Saito & P. Kneisel




100µm
Etched niobium


Equator-weld region


Thermometer
$R_S$ (nΩ)
After fast cooldown
Following warm up to intermediate temp.
20
15
10
5
0
0
50
100
150
200
250
300
350
Angle (degrees)
1500
1000
500
0

**The most likely culprit is hydrogen:**

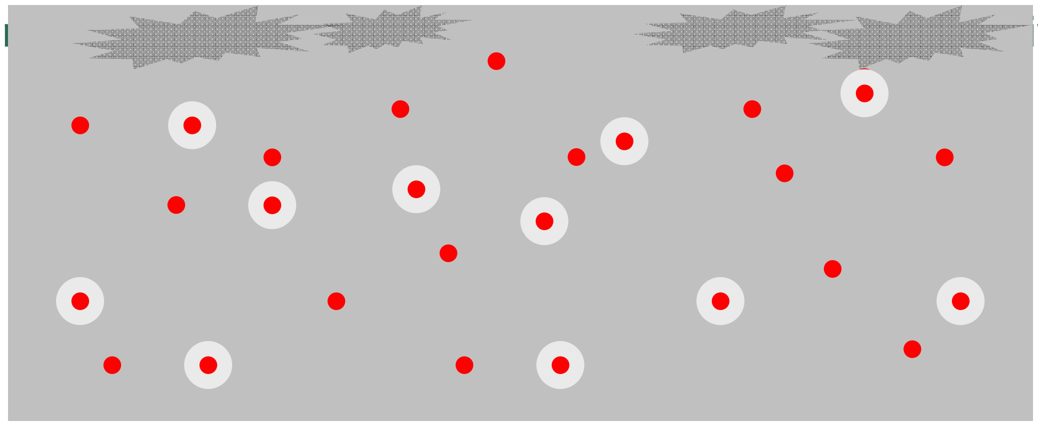
- **Nb-H system undergoes several phase transitions at low temperature, problems arise for concentrations greater than 2 wt ppm**
- **Mobile even at 120 K (300 µm in 1 hour!); not so other impurities**

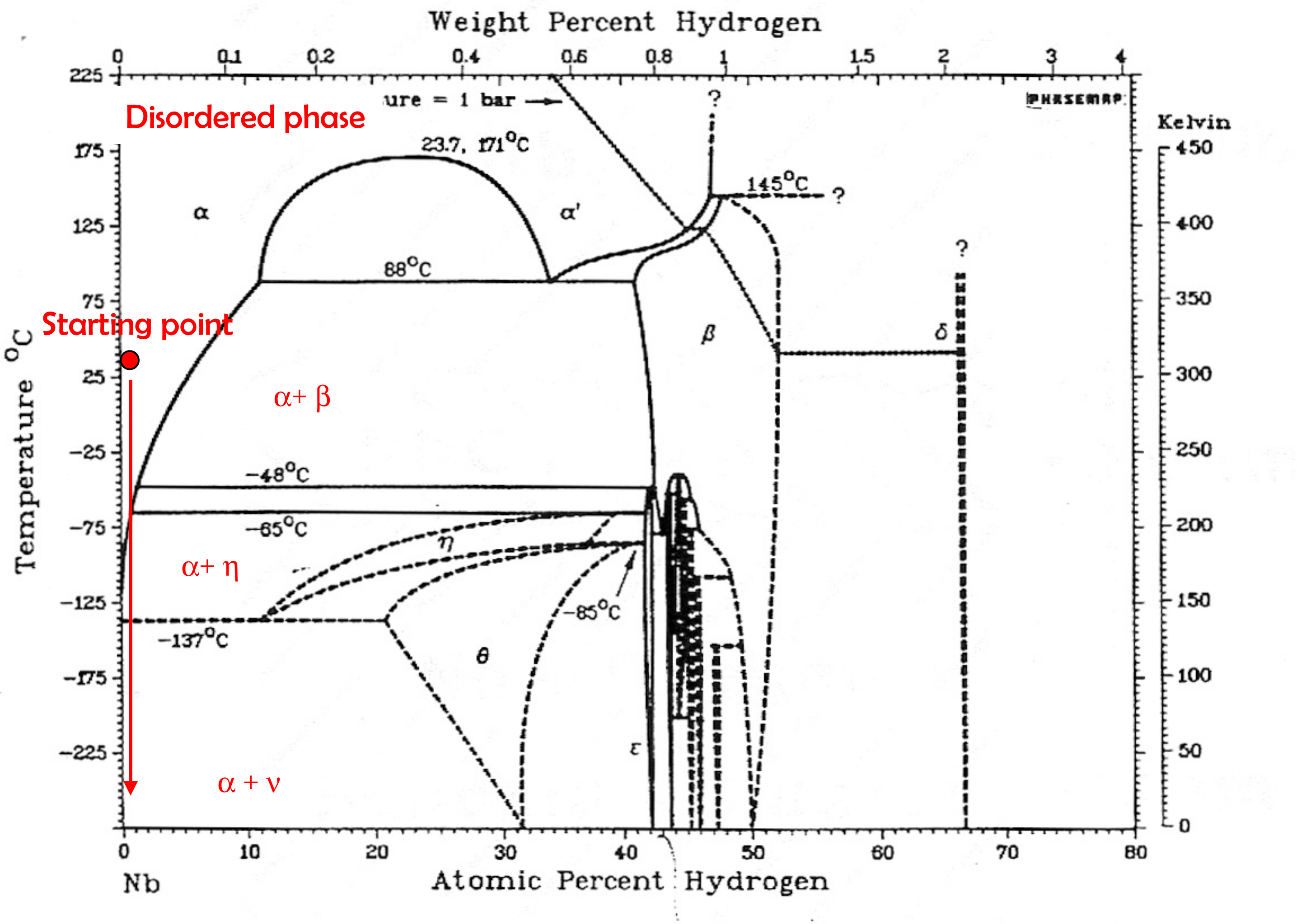
  → During cooldown hydrogen moves to form high-concentration islands that precipitate to bad SC hydrides → “weak superconductor”

  - Cool quickly to < 100 K to “freeze” hydrogen in place
- **Hydrogen likes to sit at “low-electron-density” sites in the niobium**

  → near the surface or at interstitial impurities

  → for impure niobium, much hydrogen is “bound” and cannot precipitate at the surface
- **Copious hydr ... ive oxide layer is missing**



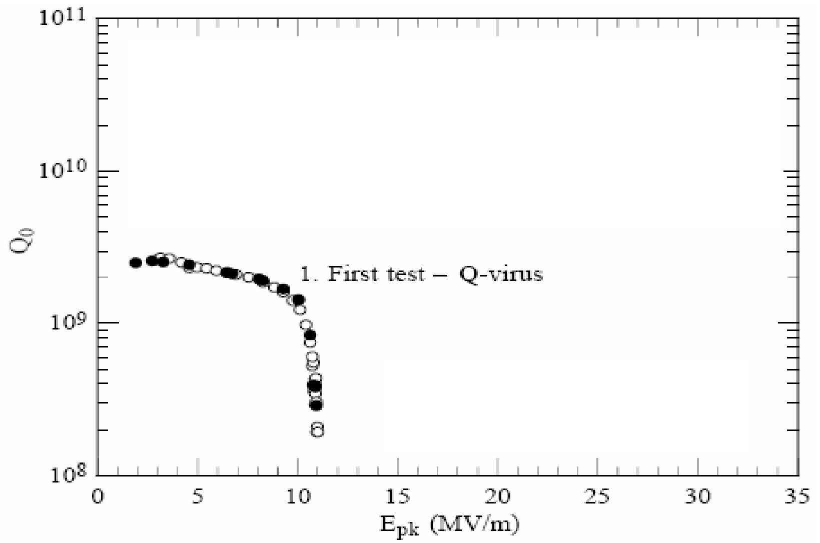



Back

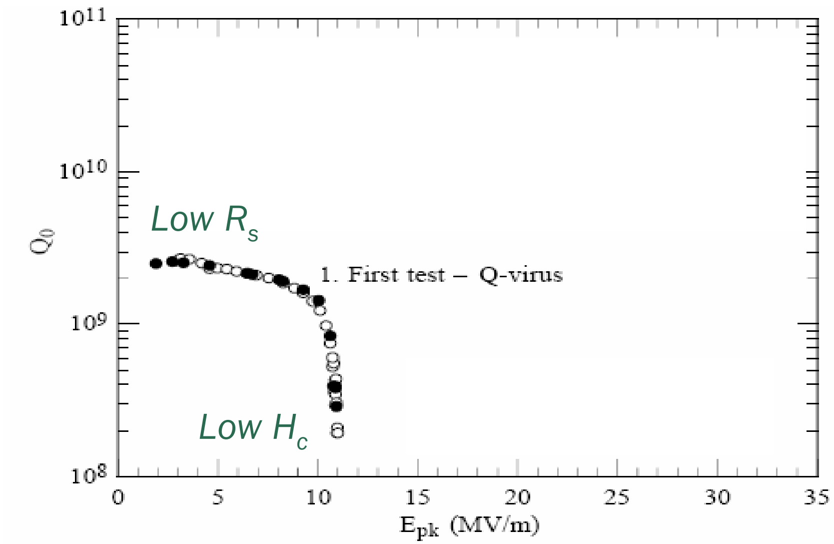
J.F. Smith, *Bulletin of Alloy Phase Diagrams*, **4**, 39–46 (1983).

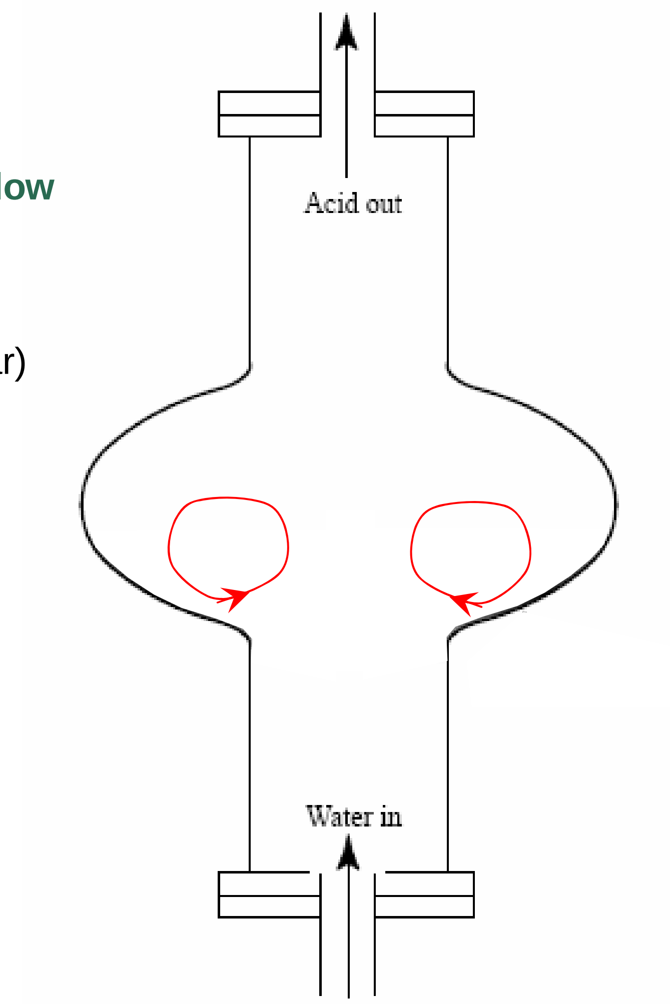
**How do we „vaccinate“ cavities against the *Q* disease?**

- **Buy niobium with little hydrogen to begin with (< 1 wt ppm)**
- **Etch your cavities with cold (< 15 C) acid**
- **Use a large acid volume to stabilize the temperature (exothermic reaction!)**
- **Vaccum-furnace bake at 700-900 C (*P* < $10^{-6}$ mbar) to drive out the hydrogen**

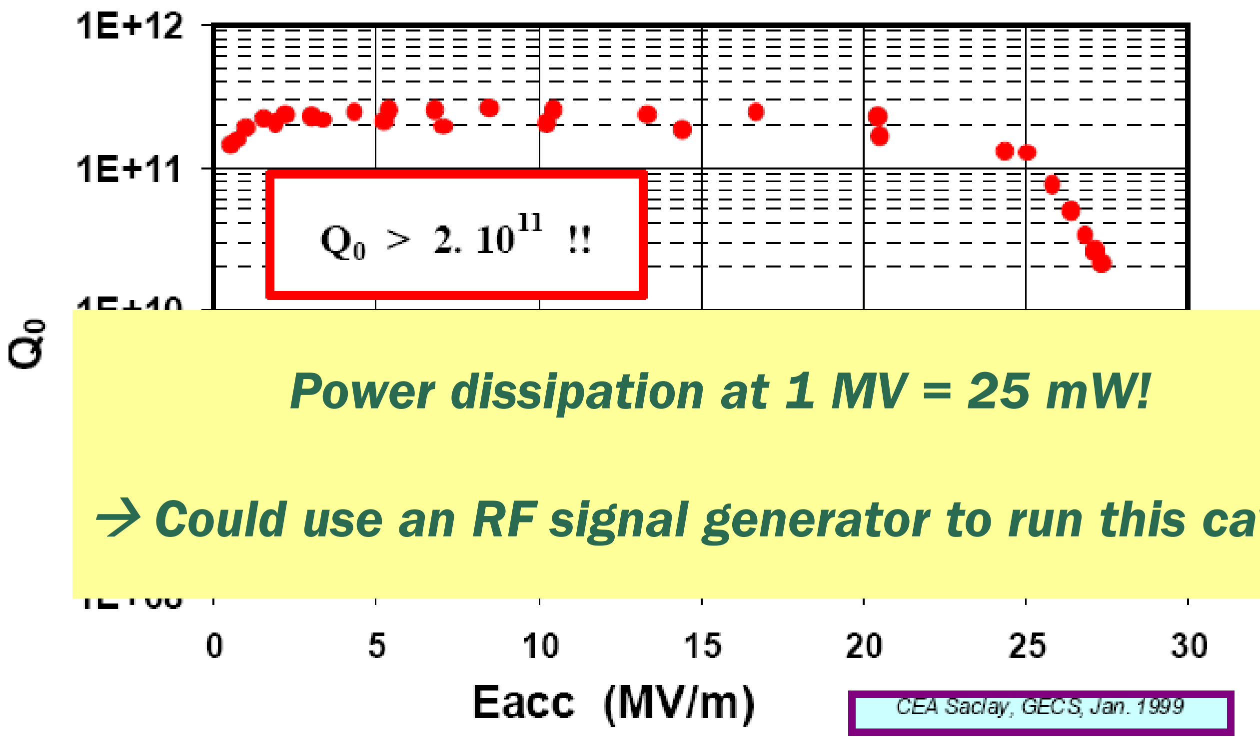


- **Tried a new scheme to remove acid without exposure to air**
  - Was supposed to reduce field emission
- **Following RF test showed VERY strong heating in the lower portion of the cavity**
- **Presumably, acid removal in lower portion was probably slow**
- **Rather it diluted the acid slowly → increased reaction rate**
- **How to solve the problem?**
  - Heat the cavity to 900 C in a vacuum furnace (P < $10^{-6}$ mbar)
  - Hydrogen is removed and cavity performance recovered



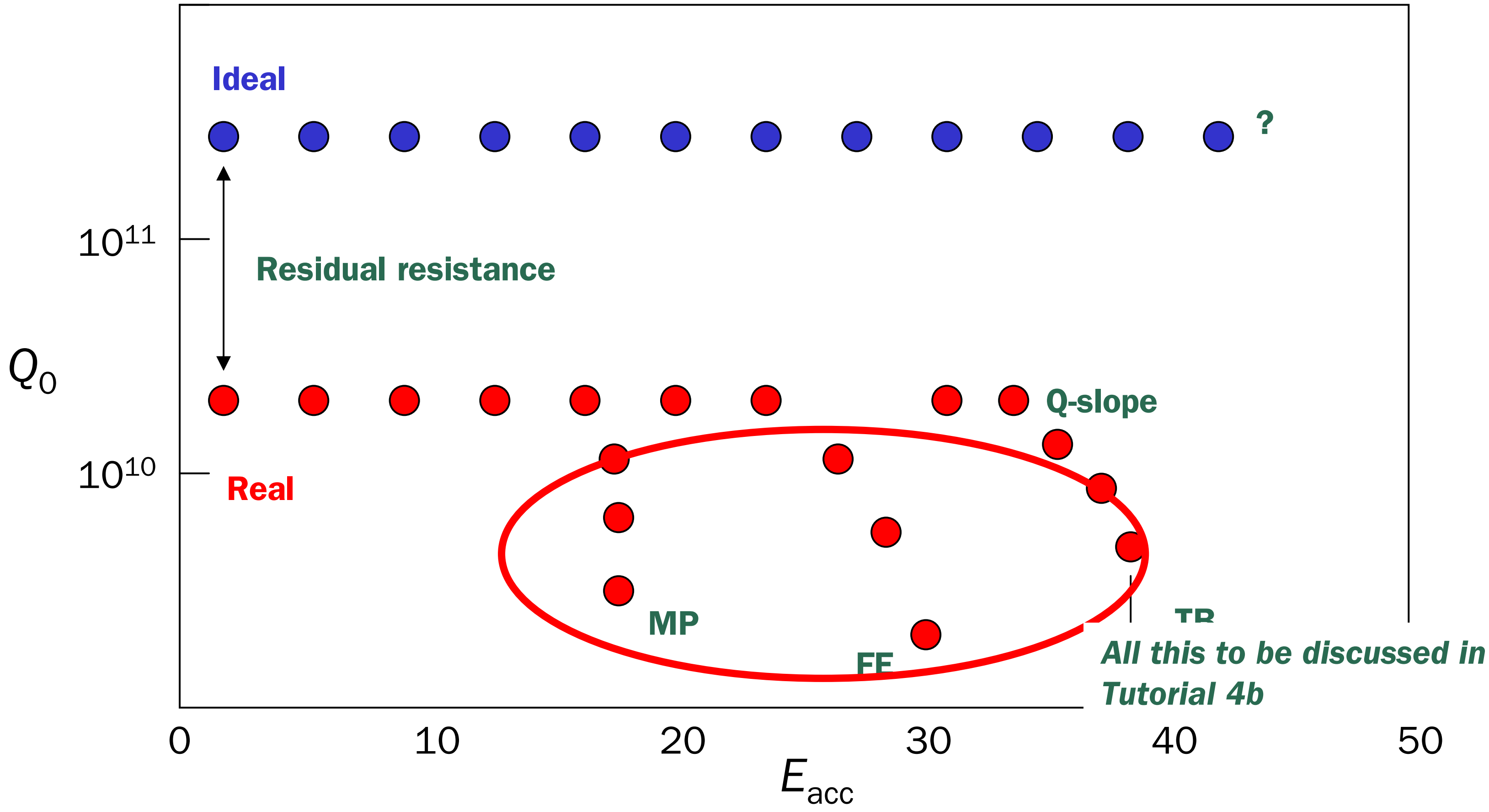
- With a carefully prepared cavity, well shielded from the earths field, one can achieve a very high Q factor
- Surface resistance is around 1.3 nΩ
- SHRIMP predicts a value around 1.8 nΩ for BCS losses




***Power dissipation at 1 MV = 25 mW!***

***→ Could use an RF signal generator to run this cavity!***

- What is the intrinsic field limitation of niobium cavities




*All this to be discussed in Tutorial 4b*

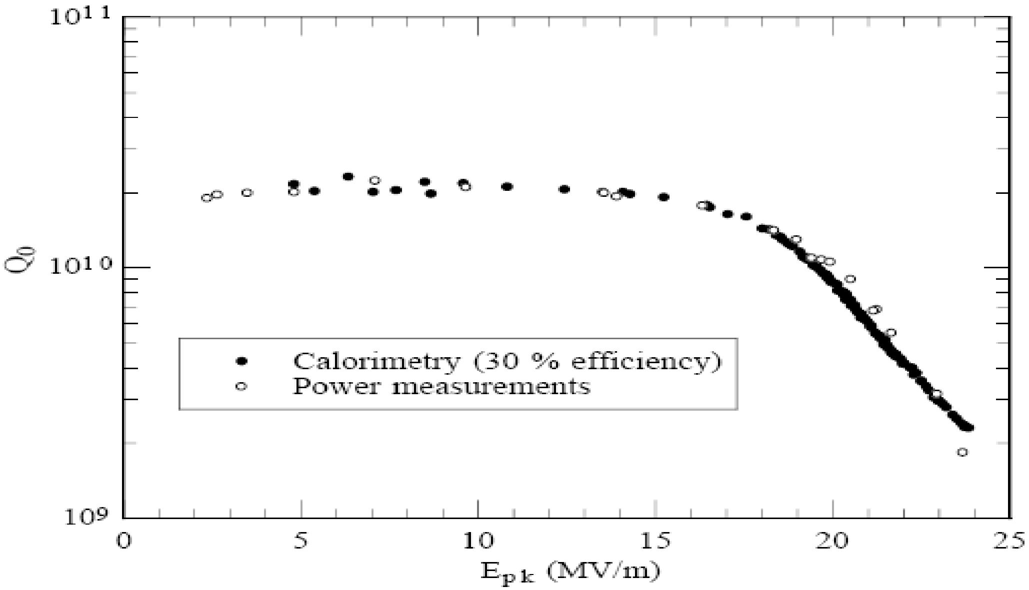
- **Two field limits possible:**
  - Electric field
  - Magnetic field
- **Peak fields rather than accelerating field will be the limit**
- **→ Ratios play a vital role:**

$$
\begin{aligned}
\frac{E_{\rm pk}}{E_{\rm acc}} &= \frac{\pi}{2} = 1.6 \\
\frac{H_{\rm pk}}{E_{\rm acc}} &= 2430\ \frac{\rm A/m}{\rm MV/m}
\end{aligned}
$$

For pillbox

# Electric field limit

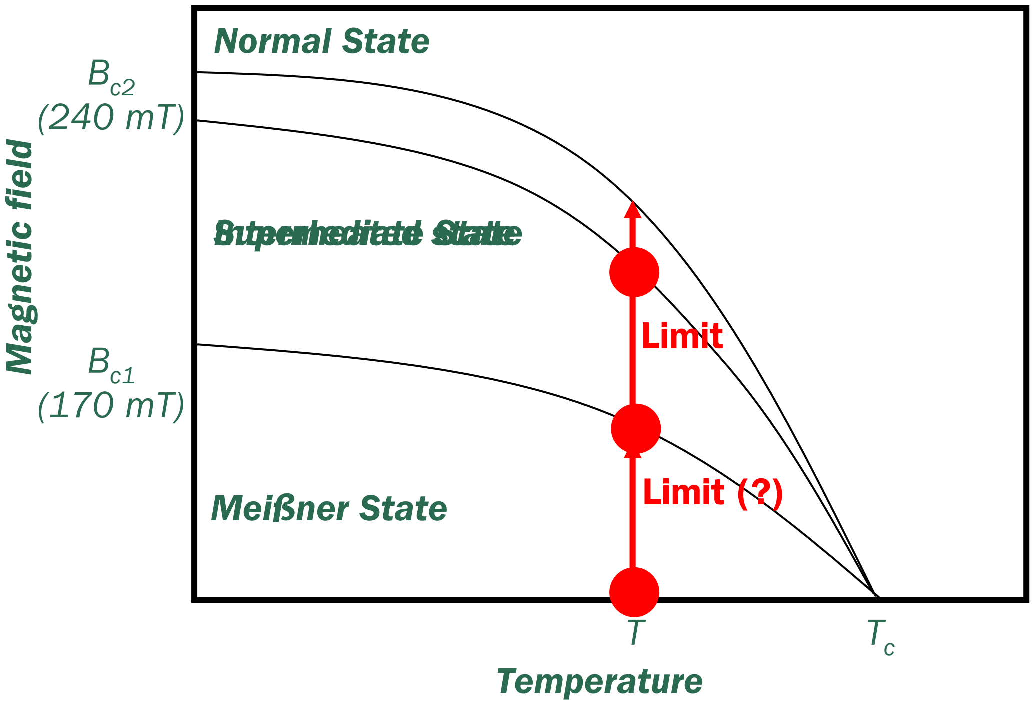
- **BCS theory does not predict an electric field limit**
- **In real cavities, a practical electric field limit clearly exists: Field emission in high electric field regions (Tutorial 4b)**
- **To test whether there is a fundamental field limit:**
  - Design a cavity with relatively small $H_{pk}/E_{acc}$ → to eliminate any magnetic field limit
  - Pulse the cavity with high power (MW) in short time (µs) → reach high field before field emission can cause cavity quenches
- **That way 145 MV/m (CW) and 220 MV/m (pulsed) peak fields have been achieved**
- **So far no fundamental electric-field limit observed**

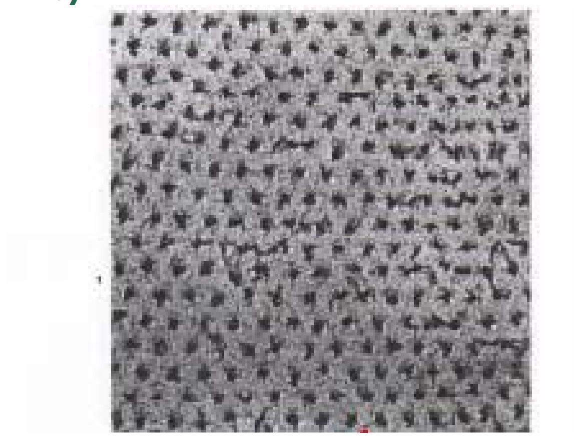



D. Moffat et al., Proc. 4th SRF WS
J. Delayen and K. W. Shepard, *Appl. Phys. Lett.*, **57(5)**:514

- BCS superconductivity does predict a magnetic field limit
- Intermediate state is lossy in RF field → quenches cavity (see discussion on trapped flux)
  → must remain below $H_{c1}$?
- Not quite: Phase transition is first order (latent heat) → it takes time to nucleate this (≈ 1μs)
- → for short times can „superheat“ the field and remain in the Meißner State
- Theory predicts a superheating field $H_{sh}$ = 240 mT (@ 0 K for Nb)

*Type-II superconductors*



- Temperature dependance of critical field is given by

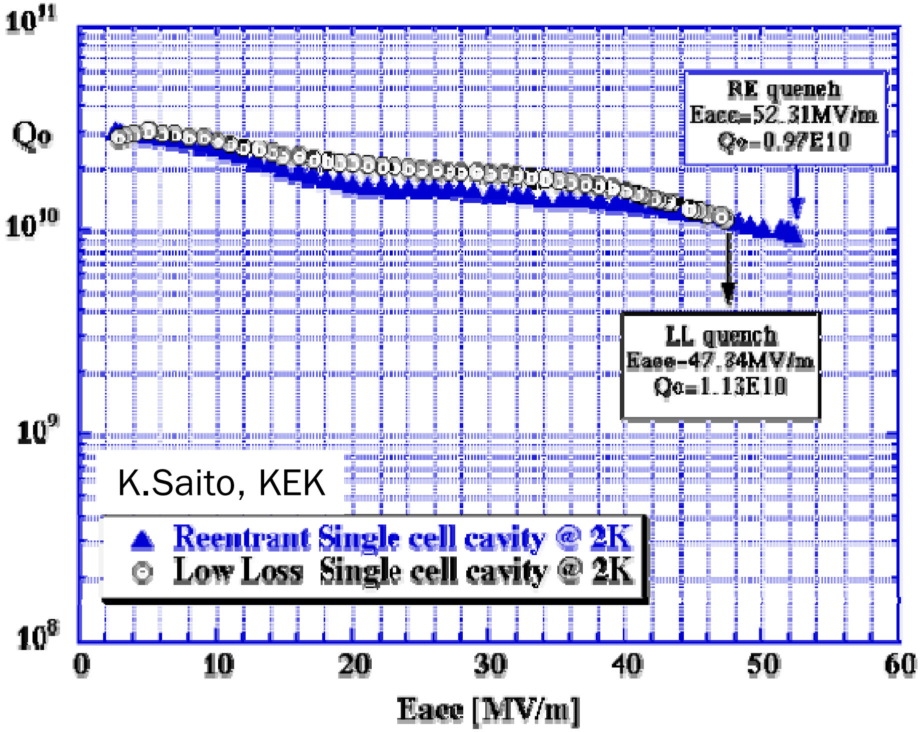
$$H_{sh}(T) = H_{sh}(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

- **Let's calculate an example with out Pillbox:**
  - For Nb at 2 K: $B_{sh} \approx$ 231 mT
  - → $E_{acc}$ = 75 MV/m when $B_{pk}$ = $B_{sh}$
  - → $E_{pk}$ = 120 MV/m ← already exceeded with other cavities

$$\frac{E_{\mathrm{pk}}}{E_{\mathrm{acc}}} = \frac{\pi}{2} = 1.6$$

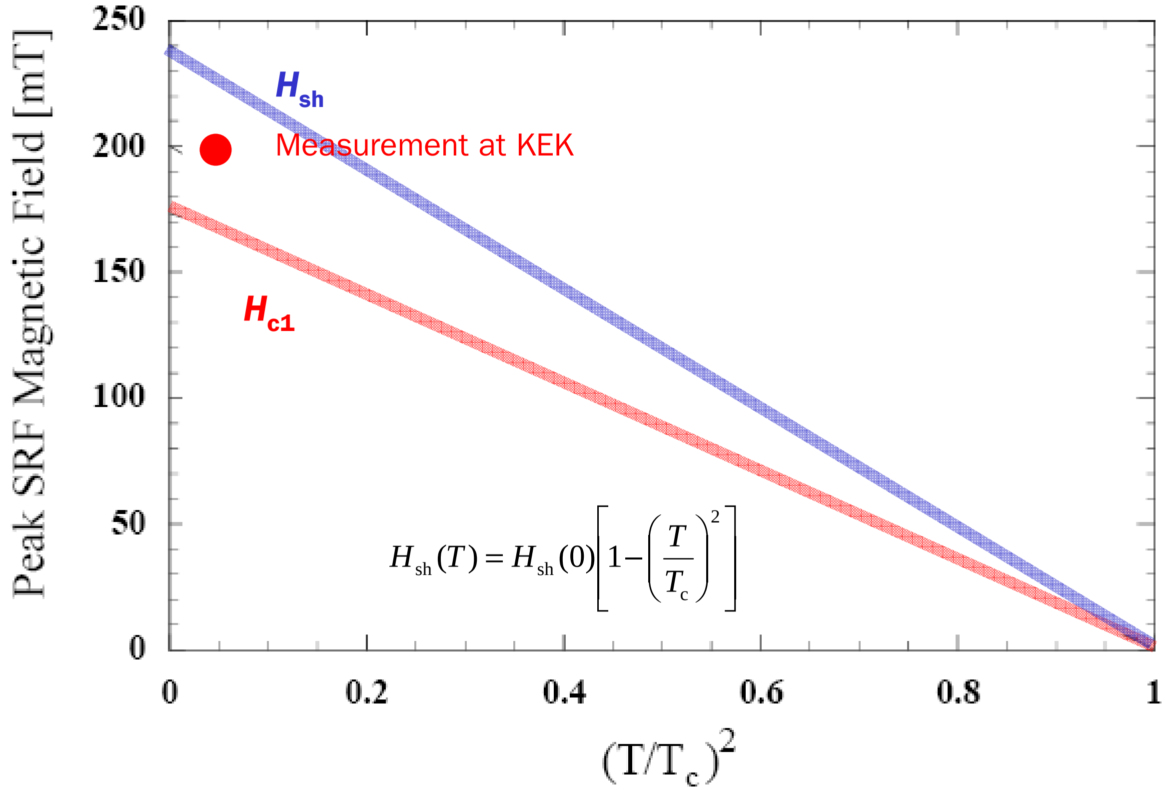
$$\frac{H_{\mathrm{pk}}}{E_{\mathrm{acc}}} = 2430 \frac{\mathrm{A/m}}{\mathrm{MV/m}}$$

- **„Best real cavity results" (@ 2 K)**
  - $E_{acc}$ = 52,3 MV/m
  - $E_{pk}$ = 116 MV/m
  - $B_{sh}$(229 mT) > $B_{pk}$ = 197 mT > $B_{c1}$ (162 mT)
  - At $B_{sh}$: $E_{acc}$ = 60.9 MV/m



# Reaching the magnetic field limit

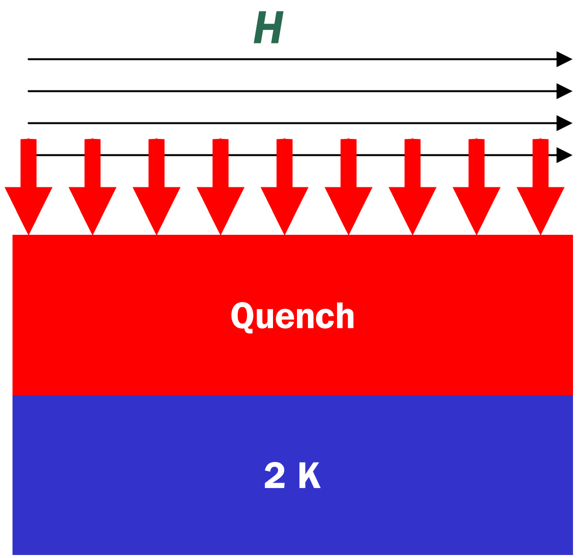
- **To demonstrate the field limit $B_{pk} = B_{sh}$**
  - Apply short, high-power pulses to reach the maximum field before anomalous losses like thermal breakdown (due to particles) or field emission can kick in.
  - Measure closer to $T_c$ so that the superheating field is lower
- **Clearly $H_{c1}$ has been exceeded and $H_{sh}$ reached at higher temperatures**

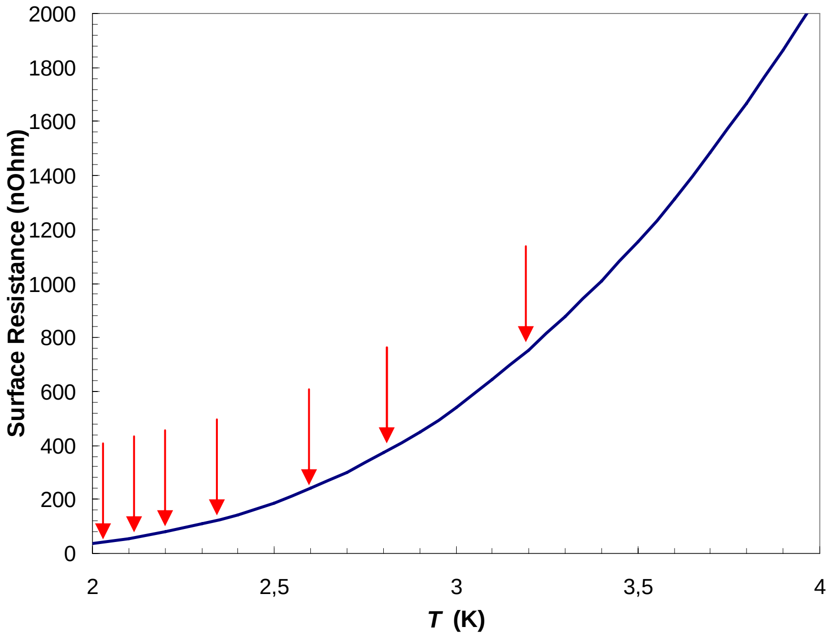





T. Hays et. al, Proc. 8th SRF WS

# Another „fundamental limit“: Global Thermal Instability

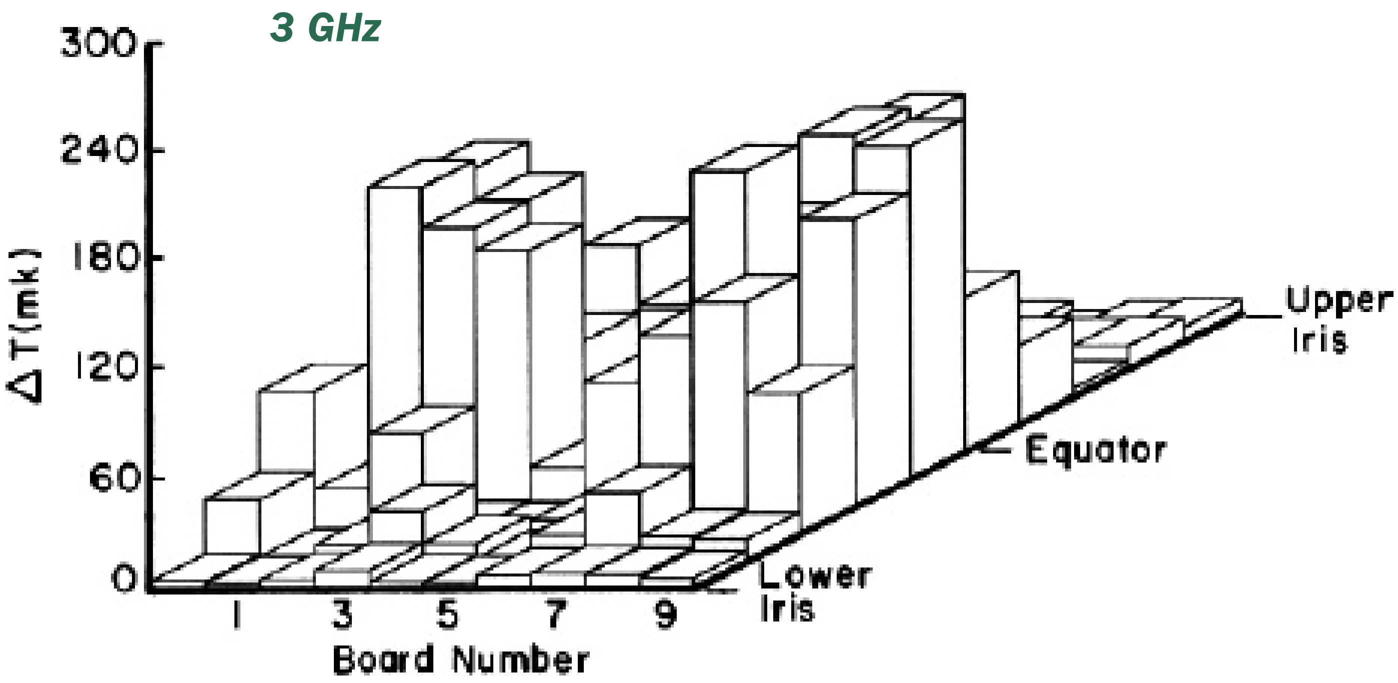



- **Exponential increase of BCS surface resistance with temperature**
  **→ Danger of thermal runaway (global quench, contrast with local quench)**

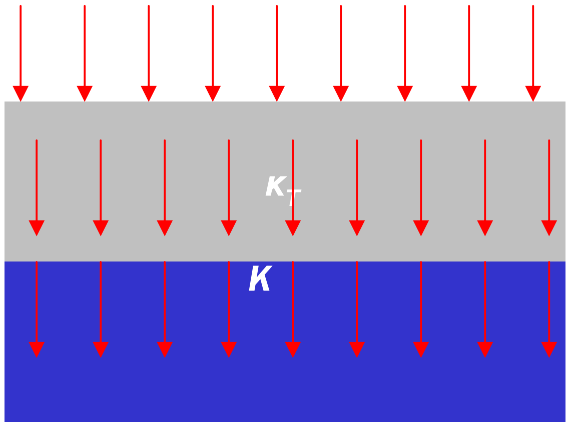
**→ Field limit is a direct consequence of RF superconductivity**

# GTI in 3 GHz Cavity




J. Graber, PhD thesis, Cornell University, 1993

- **To calculate onset need:**
  - Surface resistance
  - Thermal conductivity of niobium
  - Kapitza conductivity into the helium bath



- **Thermal energy in a superconductor can be carried by electrons and lattice vibrations (phonons)**
  - Cooper pairs do not scatter off lattice → cannot transfer heat
    → only the normal „fluid“ is involved in heat transfer
    → Largest near $T_c$, then drops exponentially
    → Specific heat due to electrons drops as $T \exp(-\Delta/k_B T)$
  - Only few phonons present at low temperature
    → Electronic contribution dominates near $T_c$
  - Specific heat due to phonons only drop as $T^3$
    → Phonons dominate at lowest temperatures
- **Electronic contribution limited by:**
  - NC electrons scattering off impurities (concentration determined by the RRR)
  - NC electrons scattering off phonons
- **Phonon contribution limited by:**
  - Phonons scattering off electrons
  - Phonons scattering off lattice defects, in particular grain boundaries

$$\kappa_T = \quad + $$

*Electron contribution* *Phonon contribution*



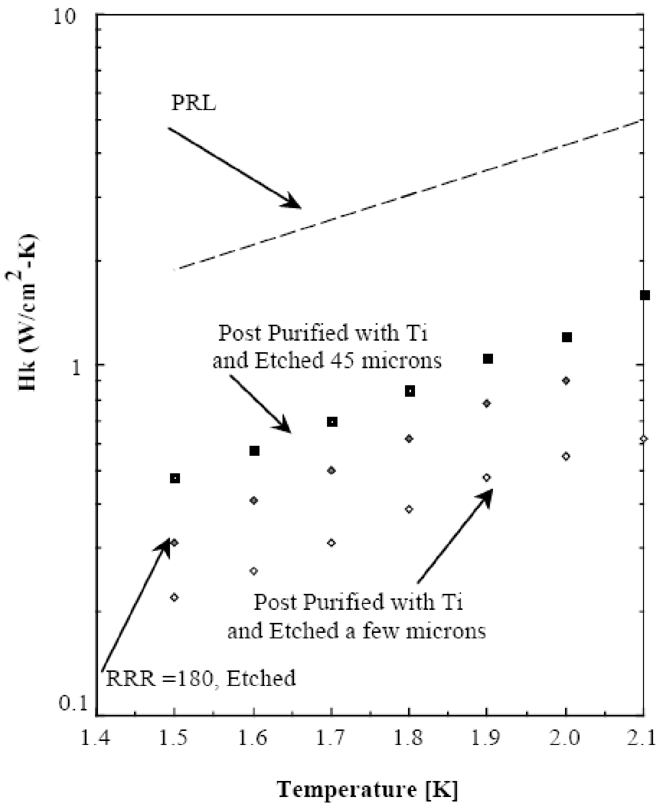
F. Koechling and B. Bonin, Supercond. Sci. Techn. 9 (1996)

→ **To maximize thermal conductivity:**

- Decrease impurities of Nb (high RRR material)
- Increase the size of the crystal grains

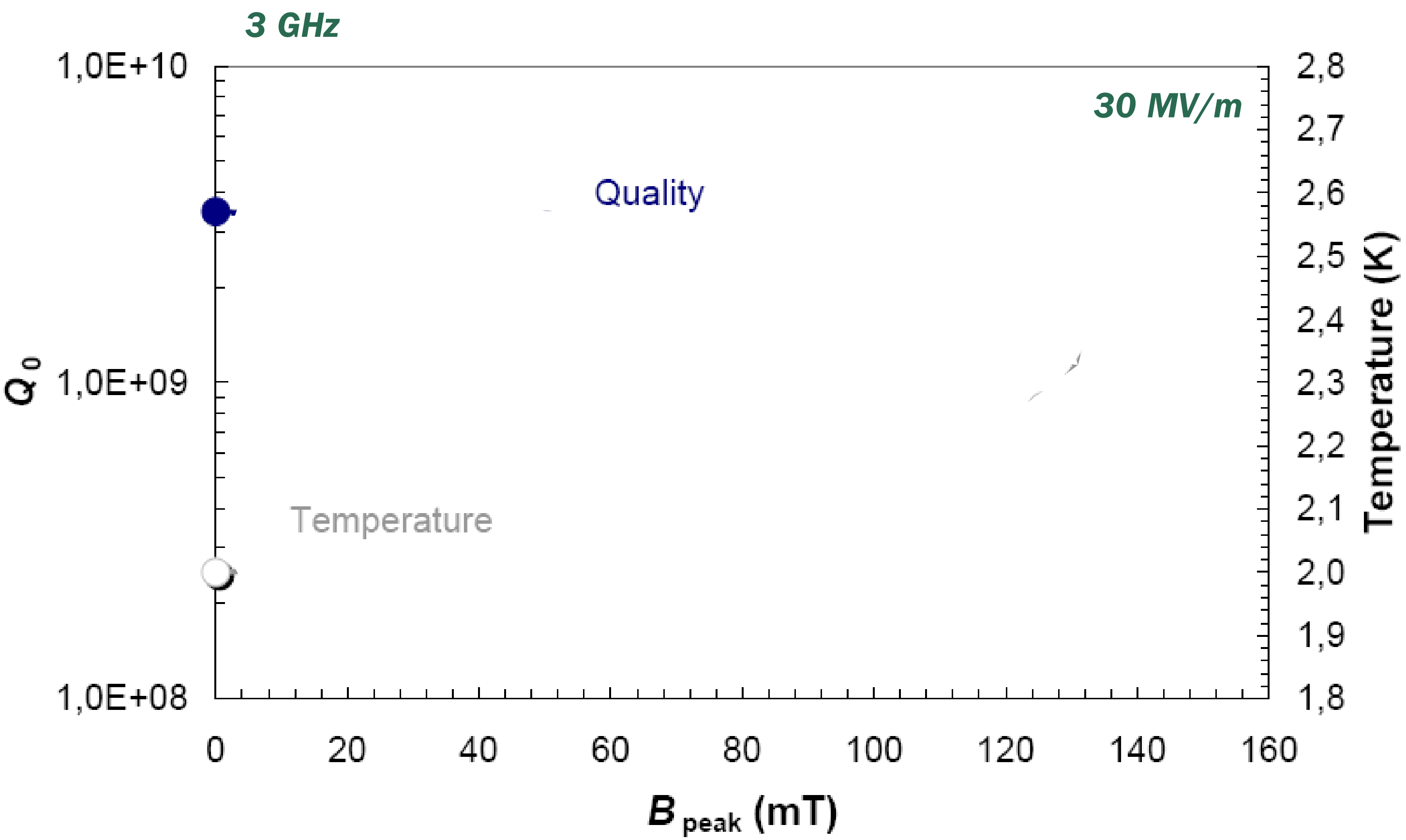
*ACCEL* *P. Kneisel*

- At the interface from the niobium to the helium, heat is transferred by phonons
- Theory not well understood, but generally dependance follows a $T^3$ to $T^4$ law
- Depends on the surface condition of the niobium
- Typical values are in the range 0.1-1 W/cm² K




A. Bouchea et al., *Proc. 7th SRF WS*

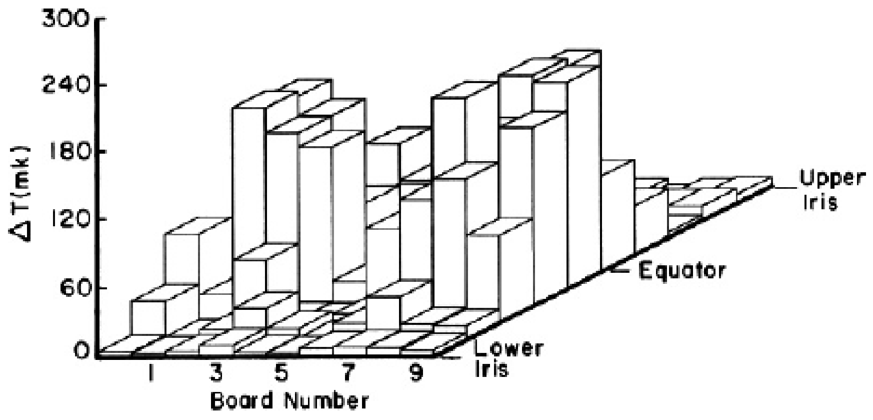
- **Surface resistance, thermal conductivity and Kapitza conductivity are all non-linear**
  → Must simulate GTI

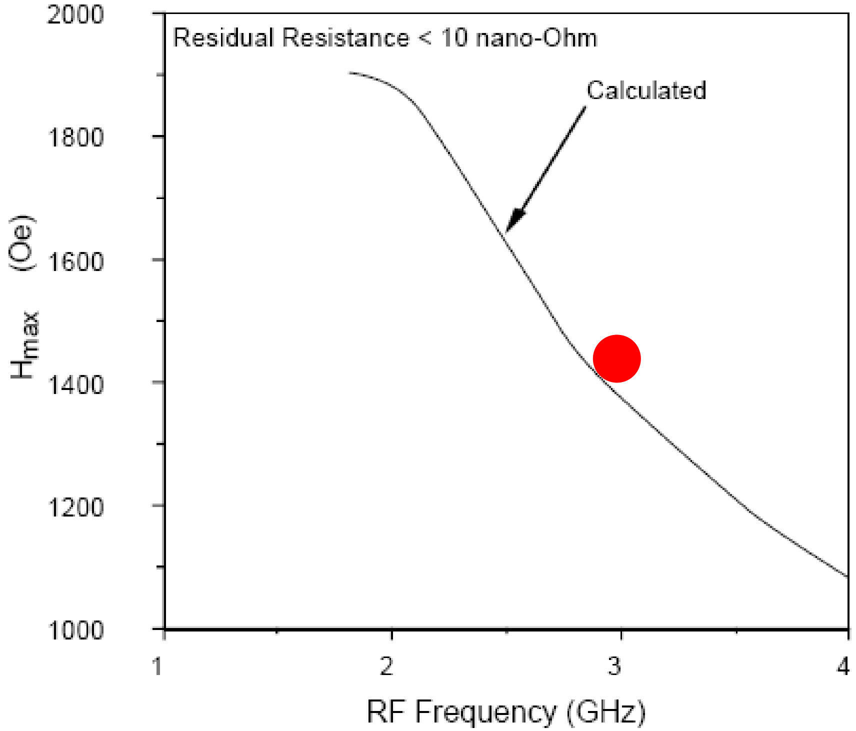


# Typical thermal conductivity of niobium

→ **Problem largest for**
- Cavities made of low thermal conductivity
- Operation at temperatures where the BCS resistance is significant
- High-frequency cavities, > 2 GHz (recall $\omega^2$ dependance of resistance)
- Simulations at least partially validated by experiment

→ for highest gradients will need to stay at lower frequencies



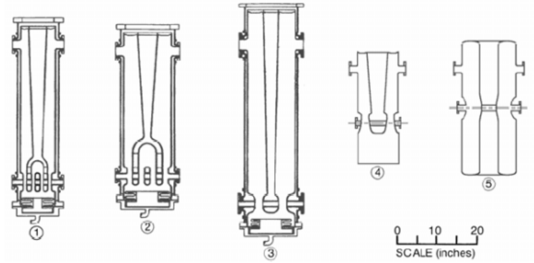



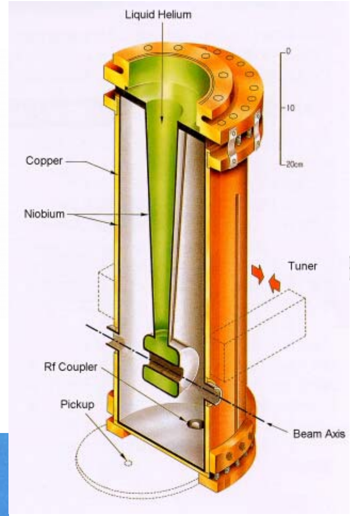

J. Graber, PhD thesis, Cornell University, 1993

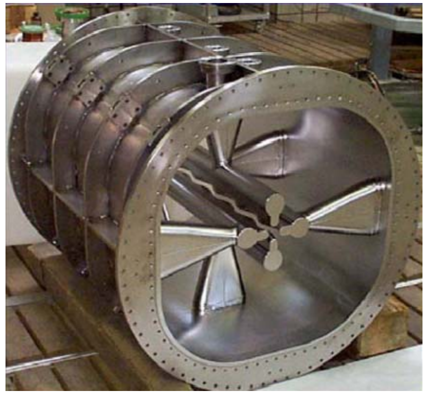
# Cavity modes in real cavities

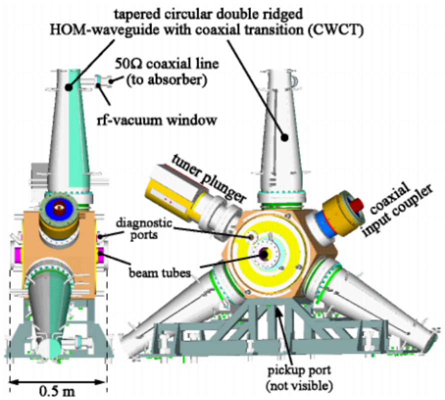
- In reality one has to calculate the modes with field solvers. Simply adding beam tubes already means there are no analytic solutions
- But can still identify the modes
- Length is still chosen according to the previous criterion
- Show a field map in a real cavity

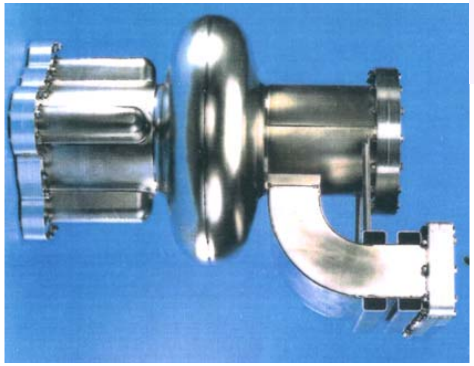
# Different Cavity Designs: Trying to See the Forest for the Trees

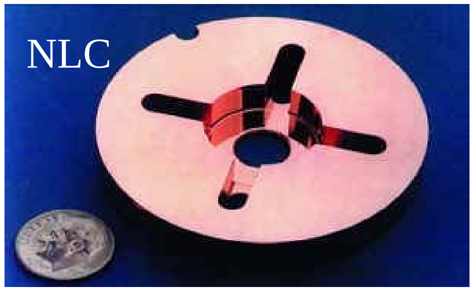


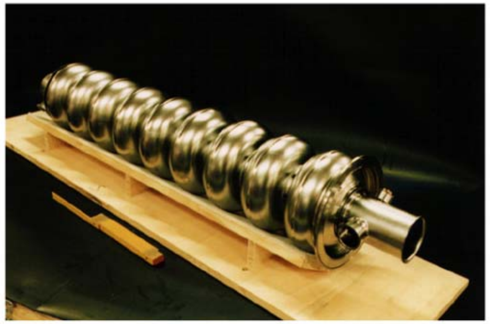





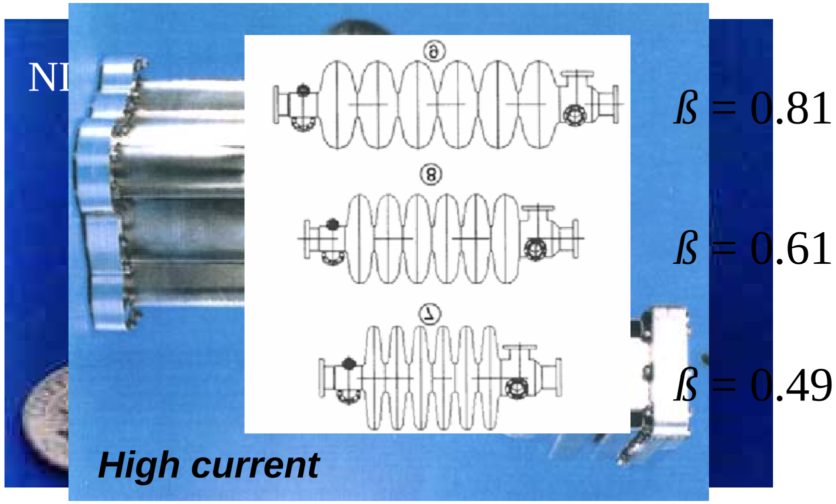



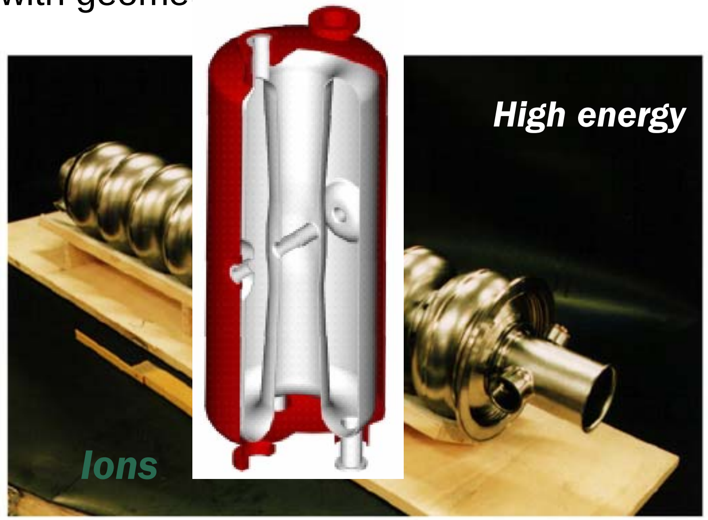


$$L_{acc} = \frac{\beta c T_{RF}}{2} \qquad P_{diss} = \frac{\left(E_{acc} L_{acc}\right)^2}{\frac{R_a}{Q_0} \times Q_0} \qquad I_{th} \propto \frac{E_{acc}^2}{\frac{R_a}{Q_0} \times Q_L \omega^{3/2}}$$

- **First consideration is the speed of the particles to be accelerated**
  - The slower the particles (ions) the shorter the gap:
- **Application plays an important role (e.g., high current v. high energy)**
  - Peak fields will play a role
  - + other issues that affect cavity geometry and the frequency
- **Then consider SC or NC cavities**
  - For NC cavities must reduce power dissipation with geometry



- **Measurement of surface resistance reveals a SC transition for degraded cavities**